# Control of Corporate Decisions: Shareholders vs.

# Management

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## ABSTRACT

Activist shareholders have lately been attempting to assert themselves in a struggle with management and regulators over control of corporate decisions. These efforts have met with mixed success. Meanwhile, shareholders have been pressing for changes in the rules governing access to the corporate proxy process, especially in regard to nominating directors. The key issue which these events have brought to light is whether, in fact, shareholders will be better off with enhanced control over corporate decisions. Proponents of increased shareholder participation argue that such participation is needed to counter the agency problems associated with management decisions. Opponents counter that shareholders lack the requisite knowledge and expertise to make effective decisions or that shareholders may have incentives to make value-reducing decisions. In this paper, we investigate what determines the optimality of shareholder control, taking account of some of the above arguments, both pro and con. Our main contribution is to use formal modeling to uncover some factors overlooked in these arguments. For example, we show that the claims that shareholders should not have control over important decisions because they lack sufficient information to make an informed decision or because they have a non-valuemaximizing agenda are flawed. On the other hand, it has been argued that, since shareholders have the "correct" objective (value maximization) and can always delegate the decision to management when they believe management will make a better decision, shareholders should control all major decisions. We show that this argument is also flawed.

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# Control of Corporate Decisions: Shareholders vs. Management

Activist shareholders have lately been attempting to assert themselves in a struggle with management and regulators over control of corporate decisions. Under the current rules, these efforts have met with mixed success.<sup>1</sup> Consequently shareholders have also been pressing for changes in the rules governing access to the corporate proxy process, especially in regard to nominating directors.<sup>2</sup>

The key issue that these events have brought to light is whether, in fact, shareholders will be better off with enhanced control over corporate decisions. Proponents of increased shareholder participation argue that such participation is needed to counter the agency problems associated with management decisions. The leading proponent, at least in the academy, is Lucian Bebchuk (2005). In this view, boards of directors do not exercise sufficient control over self-interested managers because they are typically hand-picked by management insiders who control the proxy process. Moreover, it has been argued that, since shareholders have the "correct" objective (value maximization) and can always delegate the decision to management when they believe management will make a better decision, shareholders should control all major decisions.<sup>3</sup> An array of legal scholars opposes Bebchuk's conclusion, offering several arguments such as that shareholders lack sufficient information to make an informed decision or that some shareholders may have social, political or environmental agendas, or private benefits of

<sup>&</sup>lt;sup>1</sup> For example, Carl Icahn attempted, without success, to force a breakup of Time Warner. On the other hand, Nelson Peltz succeeded in getting himself and an ally elected to the board of H.J. Heinz Co. and in getting management to implement accelerated cost cuts and restructuring.

<sup>&</sup>lt;sup>2</sup> For example, AFSCME, as a stockholder in AIG, demanded that AIG shareholders be allowed to vote on a measure to give them a greater voice in the selection of directors. After a lengthy legal process, the SEC decided not to change the rule.

<sup>&</sup>lt;sup>3</sup> For example, Bebchuk (2005, pp. 881-882), claiming that shareholder ignorance is no excuse for denying shareholders control, states, "After balancing the considerations for and against deference [to management], rational shareholders might often conclude that deference would be best on an expected-value basis. Other times, however, they might reach the opposite conclusion. Although shareholders cannot be expected to get it right in every case, it is their money that is on the line, and they thus naturally have incentives to reach decisions that would best serve their interests."

control.<sup>4</sup> This literature takes the information of the parties as given and investigates the effect of the information structure on the efficacy of particular corporate governance structures. We refer to this approach as the "information approach." Another literature that largely predates Bebchuk (2005) addresses a different set of objections to the idea of shareholder control of corporate decisions. This literature, which is surveyed in the next section, focuses on the potential for increased shareholder influence over decisions to weaken incentives for management to engage in various value-increasing activities, including information acquisition. We refer to this approach as the "incentive approach."<sup>5</sup>

In this paper, we investigate what determines the optimal control of corporate decisions. In particular, we take a normative perspective and ask which allocation of control (to shareholders or management) maximizes share value.<sup>6</sup> To analyze this issue, we adopt the information approach as opposed to the incentive approach. This is partly because the incentive approach has been more thoroughly investigated, but mainly because we want to address more directly the recent debate based on the information approach. Our main contribution is to use formal modeling to uncover some factors overlooked in the information-approach arguments, both pro and con.

Our modeling strategy incorporates the following key features. First, we assume management's preferences result in their making decisions that are biased relative to value maximizing decisions. Second, we assume *both* parties have private information relevant to a decision and consider strategic communication of this private information. Third, we emphasize that control over a decision by a given party does not require that party actually to make the decision. The controlling party may make the

<sup>&</sup>lt;sup>4</sup> See Bainbridge (2006), Lipton (2002), Stout (2006), and Strine (2006).

<sup>&</sup>lt;sup>5</sup> The phrases "information approach" and "incentive approach" are not meant to suggest that the information approach excludes incentive considerations or the incentive approach excludes information issues. We choose these phrases as shorthand for referring to the two strands of literature, because, in the information approach, the key driver is communication or utilization of information rather than incentives and *vice versa* for the incentive approach.

<sup>&</sup>lt;sup>6</sup> We will use the word "optimal" to mean value-maximizing throughout the paper, even in cases where some shareholders have other objectives (see section 5 for a discussion). We also assume that maximizing share value and maximizing firm value are equivalent. We revisit the issue of who should allocate control briefly in Sections 4 and 6.

decision itself but also may delegate the decision to the other party. Finally, to address the opponents of shareholder power in the information approach, we consider situations in which shareholders are *misinformed* and situations in which non-value-maximizing shareholders hijack control whenever shareholders are allocated control.

In general, our analysis highlights the complicated interaction among control rights, who actually makes the decision, and the extent of communication between the parties. In particular, control and the delegation choice of the controlling party determine not only the decision maker but also, in part, how much information will be used to make the decision. The decision maker's own private information will be fully utilized, but the other party's information will be only partially communicated to the decision maker. The extent of this communication is limited by the importance of the managerial agency problem, as well as by the extent of non-value-maximizing behavior on the part of shareholders.<sup>7</sup> The result is that whether shareholder control is optimal depends on such characteristics of the decision as the extent of private information on both sides and the extent of agency problems (potentially on both sides).

A naïve approach to the allocation of control, often taken by opponents to greater shareholder participation, suggests that, shareholders should not be in control, since they lack management's superior information. This approach, however, ignores the possibility that shareholders will delegate the decision to better-informed managers. We show that if shareholders have no private information, recognizing that fact, they will delegate the decision to management if and only if management's private information is sufficiently valuable that it outweighs the cost due to the managerial agency problem. That is, shareholders delegate the decision to managers in precisely the correct situations to maximize share value. This raises the question of why, given that shareholders are fully aware of their information limitations and their preferences are perfectly aligned with our criterion for optimality, they should not control all important corporate decisions, a position taken by some proponents of shareholder control. The reason,

<sup>&</sup>lt;sup>7</sup> This part of our model draws heavily on Crawford and Sobel (1982) and is similar to the models in Dessein (2002), and Harris and Raviv (2005, 2008a). See section 2 for a detailed comparison.

as our analysis shows, is that, when shareholders have private information, they will fail to delegate the decision to managers in some situations in which such delegation would increase share value. This stems from a commitment problem that we discuss in detail below.

Opponents of shareholder control also argue that shareholders may not only be ill-informed but also overconfident in their ability to understand the issues involved in some decisions, i.e., believe they have more information than they, in fact, do. We show that even in this case, for some decisions, firm value is maximized if these decisions are controlled by shareholders. Shareholders' misperception introduces a bias like that of management. Optimal control trades off the cost due to management's bias when they are in control against the cost of shareholders' bias and the cost of imperfect communication of management's information when shareholders are in control. It is not hard to see that this tradeoff could go either way. Moreover, the communication cost is attenuated if the shareholders' misperception bias is in the same direction as management's bias, further strengthening the case for shareholder control.

Finally, opponents of shareholder control fear that some shareholders want to use corporate resources to further a social, personal or political agenda at the expense of profits. Some examples might include environmentally friendly production techniques, wealth redistribution (*e.g.*, to workers), support for certain political candidates, boycotts of products or countries, etc.<sup>8</sup> Thus, similar to management, decisions made by non-value maximizing shareholders entail an agency cost. Nevertheless, we find that shareholders should control some decisions, even though we continue to assume that the objective is share value maximization (more on this in section 5) and that the non-value-maximizing shareholders control any decision assigned to the shareholders.

The next section discusses the relationship of our paper to the "incentive approach" literature on corporate governance. Section 2 presents our model. In section 3, we analyze the "base case" in which shareholders want to maximize the value of their shares and accurately assess their private information.

<sup>&</sup>lt;sup>8</sup> See, for example, Agrawal (2007), which provides evidence that some union pension funds vote differently in shareholder elections in firms that employ members of that union than they do in elections in firms that do not employ members.

The case in which shareholders are over-confident about their information is analyzed in section 4. We consider shareholders with non-value-maximizing objectives in section 5. Section 6 concludes.

# 1 Relation to the "Incentive Approach" Literature

There is a vast literature on corporate governance. Clearly we cannot do justice to this literature here.<sup>9</sup> Instead, we focus on the key issue of this paper, namely the optimality of shareholder control, using the "incentive approach." This literature focuses on the effect of the allocation of control on the incentives of various parties to engage in value-increasing activities, e.g., invest in firm-specific capital, to acquire information, etc. In this approach, the cost of shareholder control is that it reduces the incentives of management.<sup>10</sup> In contrast, our approach emphasizes the effect of the allocation of control on the extent to which exogenous private information is used in making decisions, taking account of the possibilities of communication of information and delegation of decision-making authority.

The paper in this literature that is, perhaps, closest to ours is Aghion and Tirole (1997), hereafter referred to as AT, which explicitly considers the *allocation of control* over a decision as we do (and not just the costs of shareholder control). <sup>11</sup> AT is primarily concerned with the allocation of "real" and "formal" authority. "Formal" authority is the authority to make the decision, while "real" authority refers to the party who effectively makes the decision. The AT model includes a principal and an agent who must choose a project from a set of projects with unobserved characteristics. Both can exert effort to learn the characteristics of all the projects. An important feature of the AT model is that if both parties are informed, they have the *same* information. Preferences of the two parties are as follows. If both are

<sup>&</sup>lt;sup>9</sup> For an excellent survey of the literature on corporate governance, see Becht, Bolton and Röell (2003). Hermalin and Weisbach (2003) and Adams, Hermalin and Weisbach (2008) provide thorough surveys of the literature on boards of directors.

<sup>&</sup>lt;sup>10</sup> Shleifer and Vishny (1997) discusses this idea in a survey of corporate governance.

<sup>&</sup>lt;sup>11</sup> Almazan and Suarez (2003) analyze the allocation of control over a particular decision, namely CEO replacement. If shareholders control the replacement decision, the CEO has less incentive to invest in firm-specific human capital. Hermalin and Weisbach (1998) also model a board's decision about whether to retain or replace the CEO. Another branch of the literature assumes shareholders control the decision and asks what are the potential costs of this allocation of control. Notable papers in this branch include Burkhart, Gromb and Panunzi (1997), Adams and Ferreira (2007), Myers (2000), and Shleifer and Summers (1988).

informed, they prefer different projects. If one party is informed, and the other is not, the parties agree that the best choice is the informed party's most preferred project. If neither party is informed, they agree that the best project is "no project." Therefore, if the principal has formal authority, and she learns the projects' characteristics (regardless of whether the agent also learns them), the principal chooses her most preferred project. If only the agent learns about the projects, the principal prefers to accept the project recommended by the agent. In this case, the principal has formal authority, but the agent has real authority. If the agent has formal authority, the same description applies with the roles of the parties reversed. Thus the party with formal authority delegates real authority if and only if the party with formal authority does not have the information and the other party does. Clearly, not having formal authority reduces one's benefit from learning about the projects, since he or she may not get to use the information. Consequently, the party without formal authority will expend less effort to obtain information, other things equal.

To compare the results of AT with those of the current model, we interpret "formal authority" as what we call control. Giving "real authority" corresponds in our model to delegating the decision, since in both models, this results in the non-controlling party's most preferred outcome. We also identify AT's principal with our shareholders and AT's agent with our management. Using our terminology, the allocation of control in AT is determined by its effect on the incentives of the parties to produce information or by the relative importance of the decision to the two parties. In the current paper, the allocation of control is determined by the relative importance of the private information of the parties and the extent to which the private information of the controlling party distorts that party's delegation decision. Moreover, in AT, delegation is determined by who has information: the controlling party will delegate to the other if and only if the other party has information and the controlling party does not. In our paper, both parties (generally) have information, and delegation is determined by the *realization* of the information. Finally, in AT, the actual decision (project choice) is determined by who turns out to be the decision maker and

whether or not she has information. In our paper, the actual decision is also determined in part by who turns out to be the decision maker but, in addition, depends on the decision-maker's own private information, the other party's private information and what that party communicates to the decision maker about this information. In our paper, the information communicated is different from the information itself. The extent of this difference depends on the importance of the two parties' private information and the extent of their differences in preferences.

With the above interpretation, we can compare the specific results of the two papers. First, in the current paper, if shareholders have no private information, shareholder control is always optimal (strictly in some cases). Recall that, the only reason in our model for management control is that sometimes shareholders delegate suboptimally. Suboptimal delegation occurs only when shareholders have private information, however, so shareholder control is always optimal when they have no private information. In AT, control is irrelevant when the marginal cost of effort for shareholders is sufficiently high that they never acquire information. In this case, either only management has information or neither does. Either way and regardless of who has control, the project chosen is the one most preferred by management (which may be "no project").

Second, if managers have little private information, again, shareholder control is strictly optimal in our model, because the loss due to management's agency cost exceeds the benefits of using fully their information. In AT, however, if the marginal cost of effort for managers is sufficiently high that their equilibrium effort is zero even when they are in control, control is again irrelevant for the same reason as when shareholders never have information.

Third, in the current paper, if shareholders are misinformed, it is still sometimes strictly optimal for them to have control. Recall that shareholders' misperception introduces a bias like that of management. But this bias may not be sufficient to offset management's bias for larger projects. Moreover, management's information is more precisely communicated to shareholders if the shareholders' misperception bias is in the same direction as management's bias, further strengthening the case for shareholder control. In AT, it is never optimal for misinformed shareholders to have control.

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Strictly speaking, AT does not consider misinformed agents, but suppose that shareholders in their model, as in our model with misinformed shareholders, receive a signal with positive probability (which may depend on their effort) that they interpret as the characteristics of the projects but which is, in fact, pure noise. If shareholders are in control and receive the signal, they will essentially choose a project at random, an outcome AT assumes to be suboptimal.

Fourth, if management's preferences become better aligned with shareholders', management control becomes more attractive for some parameter values in our model. A decrease in management's bias directly reduces the agency cost of management control and may result in a more highly suboptimal delegation decision by shareholders. These effects strengthen the case for management control. On the other hand, better-aligned preferences cause management to communicate more of their information whenever shareholders do not delegate. Consequently, management control is not more attractive in all cases. In AT, preferences become more similar when the fraction of his private benefits the manager receives if the shareholders' preferred project is chosen increases. AT's result is that shareholder control becomes more attractive in that case, because the increase has no impact on shareholders' payoff when they are in control but reduces management's effort when they are in control (AT, p. 15).

The incentive approach literature focuses on the distortion of incentives as the cost of shareholder control while we stress the importance of managerial private information and shareholders' inability to delegate efficiently when they also have private information. As discussed above, the two approaches lead to somewhat different conclusions.

# 2 The Model

We consider a firm that consists of two groups of actors, management and shareholders. Each group is assumed to act as if it were a single individual.<sup>12</sup> One of these groups must make a decision,

<sup>&</sup>lt;sup>12</sup> Thus, we do not consider how conflicts among the members of either group are resolved, nor do we model the sharing of information among the members of a group. In particular, we do not analyze voting by shareholders or other schemes for aggregating their preferences and information into decisions. Our aim is simply to understand, assuming the difficult issue of preference and information aggregation can be successfully resolved,

choosing a value  $s \in \mathbb{R}^{13}$  Some examples of the kinds of decisions we have in mind are the reservation price for sale of the firm or some of its assets, the size of a major investment, executive compensation, etc. Either group may control the decision. The group in control of a decision may make the decision itself or delegate the decision to the other party. The issue we analyze is which group should control the decision, assuming that the goal is to maximize expected firm value.<sup>14</sup>

We assume control is allocated, once and for all, *ex ante*, i.e., before any private information is revealed to the parties. Thereafter, the party in control can delegate the decision *voluntarily* but cannot transfer control in exchange for compensation. One can justify these assumptions by noting that we do not seem to observe transactions in which managers buy, with their own money, control of particular decisions from shareholders without also buying the stock or vice versa. Neither do we observe litigation over such transactions (something one would expect to see if they were prevalent). We do not attempt to explain this but, rather, take it as given.

Our model is similar to those in Crawford and Sobel (1982), hereafter CS, Dessein (2002), and Harris and Raviv (2005, 2008a). In the CS "cheap-talk model," a principal makes a decision based on an agent's signal of his private information. CS fully characterizes the equilibria of the resulting game and

which decisions it makes sense for shareholders to control. Aggregating shareholder information and preferences may not be as great an issue as commentators think. Holderness (2009) offers evidence that the vast majority of U.S. public firms have large blockholders, similar to firms in the rest of the world.

<sup>&</sup>lt;sup>13</sup> We assume that the board of directors is controlled by management insiders and hence always acts in their interests. Obviously, if the interests of shareholders are perfectly represented by independent directors on the board whose information includes any private information of shareholders, then the issue becomes who should control the board or various decisions made by the board, not whether shareholders should directly control these decisions. This is the topic addressed in Harris and Raviv (2008a). Here, we make the opposite assumption that the board does not effectively represent the interests of shareholders. For evidence on the extent to which CEO involvement in the selection of new board members results in appointments of less independent directors, see Shivadasani and Yermack (1999).

<sup>&</sup>lt;sup>14</sup> Alternatively, we could assume that control is assigned to maximize the sum of expected firm value and expected payoff to management. This would be appropriate if control is assigned in return for a monetary payment. It is difficult to imagine, however, that, in any real-world situation, management's gain from obtaining control is greater than shareholders' loss from management control (formally, this would restrict the feasible parameter values in the model; a formal analysis of these restrictions is available from the authors on request). On the other hand, if our model implies that management control maximizes share value, shareholders could not extract any compensation from management for allowing them control, since management knows that shareholders are better off with management control. In either case, maximizing the sum of firm and management values or, equivalently, allowing monetary transfers contingent on control would not affect the allocation of control.

works out closed-form solutions for the uniform-quadratic case that we adopt here. Dessein introduces into the CS model the possibility that the principal, instead of taking the decision herself, delegates the decision to the agent. Harris and Raviv (2005) extends the model of Dessein by introducing private information on the part of the principal, as well as on the part of the agent, and by allowing the principal's delegation decision to depend on her private information. The CS, Dessein and Harris-Raviv (2005) models assume that the principal has control over the decision. Harris and Raviv (2008a) analyzes the issue of whether "insiders" or "outsiders" on a board of directors should control corporate decisions. It starts with the model of Harris and Raviv (2005) and introduces endogenous, costly information acquisition by outside board members to determine board size endogenously. The current model analyzes the control issue using a specialized version of the Harris-Raviv (2008a) model. By eliminating endogenous information acquisition and assuming that, while shareholders have private information, their private information is relatively unimportant (see below for a precise definition), we obtain more precise results, especially regarding delegation, for the base case in which shareholders are not misinformed and are value-maximizing. The specialization also allows us to extend the model to address the issues of misinformed and non-value-maximizing shareholders.

Firm value is maximized by choosing a decision,  $s = \tilde{a} + \tilde{p}$ , where  $\tilde{a}$  and  $\tilde{p}$  are random variables discussed below. To the extent that *s* differs from  $\tilde{a} + \tilde{p}$ , there is a loss in value given by

$$\left(s - \left(\tilde{a} + \tilde{p}\right)\right)^2. \tag{1}$$

Thus, the problem is to assign control of the decision so as to minimize the expectation of (1).

After control is assigned but before any decisions, including delegation decisions, are made, management privately observes  $\tilde{a}$ , and shareholders privately observe  $\tilde{p}$ . One might reasonably ask what relevant information would shareholders have about firm decisions that management does not have. Who we have in mind here are activist shareholders such as Kirk Kerkorian or various fund managers. As evidenced by their attempts to affect corporate decisions, these individuals have, or at least believe they have, such information, perhaps gleaned from their experience with other firms. Note that we are not assuming shareholders have better information than management, only that they may have different relevant information. We make the following assumptions regarding the distributions of  $\tilde{a}$  and  $\tilde{p}$ :

Assumption 1. The variables  $\tilde{a}$  and  $\tilde{p}$  are independent with  $\tilde{a}$  uniformly distributed on [0, A]and  $\tilde{p}$  uniformly distributed on [0, P].

We assume that shareholders want to maximize expected firm value,<sup>15</sup> Thus shareholders would choose *s* to minimize the expectation of the expression in (1), given their observation *p* of  $\tilde{p}$ , and their information about  $\tilde{a}$ . It follows that shareholders' optimal decision is given by  $s = E(\tilde{a} + \tilde{p}) = p + E(\tilde{a})$ , where the expectation is conditional on whatever information shareholders have about  $\tilde{a}$ .

To make the problem interesting, we assume an agency problem. In particular, in addition to caring about firm value, management also prefers larger values of s. In particular, management chooses s to minimize the expectation of the loss function

$$\left(s - \left(\tilde{a} + \tilde{p} + b\right)\right)^2,\tag{2}$$

where *management's bias*, *b*, is a positive parameter that measures the extent of the agency problem between shareholders and management. Management's optimal decision, given their observation *a* of  $\tilde{a}$ , is given by  $s = E(\tilde{a} + \tilde{p}) + b = a + E(\tilde{p}) + b$ , where the expectation is conditional on whatever information management has about  $\tilde{p}$ .<sup>16</sup>

Because of the quadratic loss functions in (1) and (2), the difference between the expected loss that results when management makes the decision and the expected-loss-minimizing decision, for any given information, is  $b^2$ . We therefore refer to  $b^2$  (and sometimes *b*) as the *agency cost*. Also, it turns out

<sup>&</sup>lt;sup>15</sup> In section 5, we consider non-value-maximizing behavior on the part of shareholders.

<sup>&</sup>lt;sup>16</sup> Our results depend only on the widths, *A* and *P*, of the supports of  $\tilde{a}$  and  $\tilde{p}$ , not on their locations. The specific assumptions that the optimal decision is the sum of  $\tilde{a}$  and  $\tilde{p}$  and that the loss is quadratic in equations (1) and (2) and Assumption 1 are used in Harris and Raviv (2005, 2008a). The quadratic loss function case together with Assumption 1 was originally solved by CS and used by Dessein (2002), assuming one-sided private information. These can be generalized somewhat for some of our results, but they greatly simplify the analysis. We believe that the insights derived from this model do not depend on these assumptions.

that if an unbiased decision-maker chooses *s*, the difference in firm value between knowing  $\tilde{p}$  (respectively,  $\tilde{a}$ ) and having no information about  $\tilde{p}$  (respectively,  $\tilde{a}$ ) is exactly the variance of  $\tilde{p}$ , denoted  $\sigma_p^2$  (respectively,  $\sigma_a^2$ ). We will therefore refer to  $\sigma_p^2$  and  $\sigma_p$  ( $\sigma_a^2$  and  $\sigma_a$ ) as the *importance of shareholders'* (*management's*) *information*. We focus on the case in which the agency problem (as measured by *b*) is severe relative to the importance of shareholders' information. It can be shown (see Harris and Raviv (2005)) that this assumption implies that management will not want to delegate to shareholders, which we believe is realistic, and that shareholders will never directly reveal any of their private information to management. These implications simplify the analysis relative to the more general case. They also allow us to focus on what we think are the more realistic cases and to obtain a uniqueness result on the equilibrium when shareholders are in control, as well as additional comparative statics results. Formally, we assume

Assumption 2:  $\sigma_p < b$ .

The sequence of events is assumed to be the following. After observing its private information, the controlling party decides whether to delegate to the other party or not. The party not making the decision may communicate some or all of its private information to the decision maker. Finally, the decision maker chooses s and firm value is realized.<sup>17</sup>

# 3 Base Case Analysis

In this section, we analyze the case in which shareholders are value-maximizers and understand perfectly the extent of their private information as well as all other parameters of the model. Since the current model is a special case of the model in Harris and Raviv (2008a) as explained in section 2, we will

<sup>&</sup>lt;sup>17</sup> As in CS and the literature that uses this model, we do not allow monetary transfers contingent on control, reports, decisions, or outcomes. With respect to contracts contingent on reports, see Krishna and Morgan (2008). Note that the assumption that contracts contingent on the decision-maker's decision are not feasible rules out the possibility of constrained delegation, i.e., delegating the decision but constraining the decision-maker to a specific subset of the possible decisions. On the issue of constrained delegation, see Alonso and Matouschek (2007, 2008). Also as in CS, we do not consider multi-stage communication between management and shareholders or intermediaries in the communication process. These elements are discussed briefly in the conclusions.

borrow liberally from the results in that paper. We first discuss separately the two cases in which shareholders are assumed to be in control of the decision and management is assumed to be in control. We then determine optimal control by comparing the equilibrium firm values for the two cases.

When shareholders are in control and do not delegate, they make the decision using their own information about  $\tilde{p}$  and whatever they infer about  $\tilde{a}$  from management's report, *r*. As noted above, if shareholders observe  $\tilde{p} = p$ , their optimal decision is given by

$$s(p,r) = \overline{a}(r) + p, \qquad (3)$$

where  $\overline{a}(r) = E(\tilde{a}|r)$  is the mean of shareholders' posterior beliefs, given r.

Because of the agency problem, management will not fully reveal their information. In the Pareto best equilibrium of the game between management and shareholders, management will partition the support of  $\tilde{a}$  into cells  $[a_0, a_1], [a_1, a_2], \dots, [a_{N-1}, a_N]$ , with  $0 = a_0 < a_1 < \dots < a_N = A$ , where the number of cells, N = N(b, A), is determined by management's bias, b, and the width of the support of  $\tilde{a}$ , A. Management then reports a uniformly distributed random draw from the cell that contains  $\tilde{a}$ . Thus, if management's report  $r \in [a_{i-1}, a_i]$ ,  $\overline{a}(r) = \frac{a_{i-1} + a_i}{2}$ . Since shareholders do not obtain full information about  $\tilde{a}$ , there is a consequent loss of *ex ante* expected firm value, which we denote by L(b, A). That is, L(b, A) is the expected loss in firm value due to having only information about  $\tilde{a}$  that is transmitted by management in equilibrium (as opposed to full information).<sup>18</sup>

Now suppose shareholders are in control and do delegate. Because shareholders' information is less important than the agency cost (Assumption 2), shareholders' report to management reveals nothing about  $\tilde{p}$  (the proof of this can be found in Harris and Raviv (2005)). Management can, however, infer something from the fact that shareholders have chosen to delegate. Consequently, if management

<sup>&</sup>lt;sup>18</sup> Explicit formulas for N(b, A),  $\{a_i\}_{i=1}^{N(b,A)}$ , and L(b, A) are given in the appendix.

observes  $\tilde{a} = a$ , they choose

$$s(a) = a + \hat{p} + b , \qquad (4)$$

where  $\hat{p} = E(\tilde{p}|delegation)$  is the mean of management's posterior belief about  $\tilde{p}$  given that shareholders have chosen to delegate. To analyze the equilibrium in this case, one must first understand for which values of  $\tilde{p}$ , shareholders will delegate. This is determined by comparing the expected loss if shareholders delegate against the expected loss if they do not.

If shareholders have no private information ( $\sigma_p = 0$ ), there is no loss due to imperfect communication about shareholders' private information if they delegate, so the loss from delegating is simply the direct agency cost,  $b^2$ . The expected loss from not delegating is L(b,A). Consequently, shareholders delegate in this case if and only if  $L(b,A) \ge b^2$ . It is shown in Lemma 1 of Harris and Raviv (2008b), however, that  $L(b,A) \ge b^2$  if and only if  $\sigma_a \ge b$ . Therefore, when shareholders have no private information, they will delegate if and only if management's information is more important than agency costs. This result is quite intuitive, and the delegation policy maximizes firm value. This is not the case when shareholders have private information.

If shareholders have private information, the expected loss if they do not delegate is still L(b, A). The expected loss if they do delegate now depends on the realization of  $\tilde{p}$ , since shareholders observe  $\tilde{p}$  before choosing whether to delegate. This loss is given by  $(\hat{p}+b-\tilde{p})^2$ . Thus in equilibrium, the set of values of  $\tilde{p}$  for which shareholders delegate, the "delegation region," must be such that  $\tilde{p}$  is in the delegation region if and only if  $(\hat{p}+b-\tilde{p})^2 \leq L(b,A)$  and  $\hat{p} = E(\tilde{p}|\tilde{p} \in \text{delegation region})$ . Proposition 1 shows that the delegation region must be of the form  $[p^*, P]$  for some threshold  $p^*$  and characterizes how  $p^*$  depends on the parameters  $\sigma_a, \sigma_p$  and b.

Proposition 1: Suppose shareholders are in control of a decision. The unique pure-strategy

Perfect Bayes' Equilibrium of the resulting game is as follows:<sup>19</sup>

- If shareholders have no private information ( $\sigma_p = 0$ ) they will delegate the decision if and only if
  - $\sigma_a \ge b$ . If shareholders delegate, management chooses  $s = \tilde{a} + b$ . [This is shown in Dessein (2002).]
- If shareholders have private information ( $\sigma_p > 0$ ), they will delegate the decision if and only if

 $\tilde{p} \ge p^*$ , where  $p^*$  and the width of the delegation region,  $d = P - p^*$ , are as follows.

- Case 1:  $\sigma_a \leq b$ . In this case, shareholders never delegate, i.e.,  $p^* = P$  and d = 0. If shareholders delegate (this is an off-equilibrium-path move, unless  $\sigma_a = b$  and  $\tilde{p} = P$ ), management infers that  $\tilde{p} = P$  with probability one and chooses  $s = \tilde{a} + P + b$ .
- Case 2:  $P \le 2\left[\sqrt{L(b,A)} b\right]$ . In this case, shareholders always delegate, i.e.,  $p^* = 0$  and d = P. Management infers nothing if shareholders delegate (this is their equilibrium move) and chooses

$$s = \tilde{a} + \frac{P}{2} + b \; .$$

• Case 3: 
$$\sigma_a > b$$
 and  $P > 2\left[\sqrt{L(b,A)} - b\right]$ . In this case,  $p^*, d \in (0,P)$ , and are given by  
 $P - p^* = d = 2\left[\sqrt{L(b,A)} - b\right]$ . (5)

If shareholders delegate, management's posterior belief about  $\tilde{p}$  is uniform on  $[p^*, P]$ , and it

chooses 
$$s = \tilde{a} + \frac{P + p^*}{2} + b$$
. There are no off-equilibrium-path moves in this case.

It follows from Proposition 1 that, when shareholders are in control, the expected loss in firm value is given by

$$L_{s} = \frac{d}{P} \left( b^{2} + \sigma \left( d \right)^{2} \right) + \left( 1 - \frac{d}{P} \right) L(b, A), \qquad (6)$$

where  $\sigma(x)$  is the standard deviation of a random variable uniformly distributed on an interval of width

<sup>&</sup>lt;sup>19</sup> Harris and Raviv (2005) show that there is an equilibrium of the form described in this proposition. Here, by simplifying the problem somewhat, we show that this is the only pure-strategy equilibrium.

*x*, i.e.,  $\sigma(x) = x/\sqrt{12}$  The first term on the right hand side of (6) is the probability that shareholders delegate, d/P, times the expected loss if they do, namely the agency cost,  $b^2$ , plus the loss from knowing only that  $\tilde{p} \in [p^*, P]$ ,  $\sigma(d)^2$ . The second term is the probability that shareholders do not delegate, 1-d/P, times the expected loss if shareholders make the decision, L(b, A).<sup>20</sup>

It is important to note that the equilibrium delegation threshold,  $p^*$ , is not an optimal threshold from an *ex ante* point of view (before private information is observed). Suppose shareholders could commit *ex ante* to an optimal threshold, say  $p^{**}$ . Then, if shareholders delegate, management correctly infers that  $\tilde{p} \ge p^{**}$ . This gives shareholders an incentive to commit to a lower threshold than  $p^*$ , because, by so doing, they induce management to infer that  $\tilde{p}$  is lower than if the threshold is  $p^*$ , partly counteracting management's bias toward larger *s*. Sticking to a policy of delegating when  $\tilde{p} \ge p^{**}$ , however, requires shareholders to delegate for  $\tilde{p} \in [p^{**}, p^*)$  even though, for such values, *ex post*, the cost of delegating is greater than the cost of not delegating. Nevertheless, the *ex ante* expected cost of this policy is lower than for the equilibrium threshold because of the effect on management's inference.

If, as we assume, shareholders cannot commit *ex ante* to a delegation threshold, there is no benefit to choosing a lower threshold *ex post*, since doing so will not affect management's equilibrium inference regarding  $\tilde{p}$ . The result that the *ex-ante-optimal* delegation threshold is lower than the equilibrium threshold is shown formally in Lemma 1 (proved in the appendix).

**Lemma 1.** (a) Let  $d^*$  minimize

$$\frac{x}{P}\left(b^{2}+\sigma\left(x\right)^{2}\right)+\left(1-\frac{x}{P}\right)L\left(b,A\right).$$
(7)

with respect to x, subject to  $x \in [0, P]$ , and let  $p^{**} = P - d^*$ . Then  $p^{**}$  is an *ex-ante*-optimal

<sup>&</sup>lt;sup>20</sup> In the special case in which shareholders have no private information, d = P = 0. Equation (6) is still correct if we take d/P = 1 if  $b \le \sigma_a$  and d/P = 0 otherwise.

delegation threshold, and  $d^*$  and  $p^{**}$  are given by

If 
$$\sigma_a \le b$$
,  $d^* = 0$ ,  $p^{**} = P$ ; (8)

if 
$$P \le 2\left[\sqrt{L(b,A) - b^2}\right], d^* = P, p^{**} = 0;$$
 (9)

otherwise, 
$$d^* = P - p^{**} = 2 \left[ \sqrt{L(b,A) - b^2} \right] \in (0,P).$$
 (10)

Moreover,  $d^* \ge d$  and  $p^{**} \le p^*$ , with strict inequalities whenever  $\sigma_a > b$  and d < P.

(b) The loss from shareholder control if shareholders can commit to delegate if and only if  $\tilde{p} \in [p^{**}, P]$  is smaller than the loss from management control, strictly if  $P > 2 \left\lceil \sqrt{L(b, A) - b^2} \right\rceil$ .

The reason this result is so important is that it explains why the intuition described in the introduction that, since they can delegate, it is always optimal for shareholders to be in control is not correct.<sup>21</sup>

Now consider management control of the decision. From the point of view of management, shareholders are biased toward smaller choices of s by -b. Consequently, when management is in control, their delegation decision is the mirror image of shareholders' delegation decision. The analysis of the previous subsection applies with the obvious renaming of players. In particular, management never delegates if  $\sigma_p < b$ . Assumption 2 therefore implies that management will never delegate the decision to shareholders. As before, Assumption 2 also implies that shareholders will refuse to share any information about  $\tilde{p}$  with management. Consequently, the expected loss in firm value under management control is the agency cost,  $b^2$ , plus the loss from knowing only that  $\tilde{p} \in [0, P]$ ,  $\sigma_p^2$ , i.e.,

$$L_M = b^2 + \sigma_p^2. \tag{11}$$

Optimal control can now be determined by whether the net gain to shareholder control,

<sup>&</sup>lt;sup>21</sup> The fact that suboptimal delegation by outsiders leads to the conclusion that insider-control is sometimes optimal was present in Harris and Raviv (2008a) but was not fully explained there. In particular, Harris and Raviv (2008a) does not present a proof of the suboptimality of the delegation decision as we do here in Lemma 1.

 $\Delta \equiv L_M - L_s$ , is non-negative, where  $L_M$  is given by (11) and  $L_s$  is given by (6). The main result of this section is stated in the following proposition and depicted in Figure 1 (a more formal statement of this result and the proof are given in the appendix).

**Proposition 2.** For any given value of agency costs, *b*, the possible combinations of the importance of management's and shareholders' information,  $(\sigma_a, \sigma_p)$ , can be divided into three regions as depicted in Figure 1. In particular:

- Shareholder control is strictly optimal for all decisions for which  $\sigma_a < b$ . For decisions for which  $\sigma_a \ge b$ , shareholder control is strictly optimal for those decisions for which  $\sigma_p > \sigma_p^U(\sigma_a)$ , where  $\sigma_p^U(\sigma_a) < \sigma_a$  and increasing.
- Management control is strictly optimal for decisions for which  $\sigma_p^L(\sigma_a) < \sigma_p < \sigma_p^U(\sigma_a)$ , where  $\sigma_p^L(\sigma_a) < \sigma_p^U(\sigma_a)$ , and  $\sigma_p^L(\sigma_a)$  is also increasing.
- Control is irrelevant for decisions for which  $\sigma_p < \sigma_p^L(\sigma_a)$ .



#### Figure 1

This graph shows how optimal control varies with the two information parameters,  $\sigma_a$ , the importance of management's information, and  $\sigma_p$ , the importance of shareholders' information. For this figure, b = 8.

The first bullet of Proposition 2 is quite intuitive. It states, essentially, that shareholders should control all decisions for which management's information is less important than agency cost, all decisions for which shareholders' information is at least as important as management's, and some decisions for which management has an informational advantage relative to its agency cost and relative to shareholders. For shareholder control to be optimal in these cases, shareholders' information must be sufficiently more important than management's information, and the hurdle that  $\sigma_p$  must clear increases as management's information becomes more important. The second bullet states that management should control decisions for which shareholders' information is less important than the hurdle mentioned above but not too much less important. The reason for this last caveat is that if shareholders' information is too much less

important than management's, shareholders will always delegate to management, so control is irrelevant (this is the content of the third bullet).

Proposition 2 has several important implications. First, it is optimal for biased management to control some decisions, even though shareholders' objective is to maximize firm value and they may delegate the decision to management. This follows from the fact that, when shareholders have private information, they do not delegate optimally as was discussed above. In such situations, it can be better for shareholders to have management in control, which is essentially a commitment to delegate. From the point of view of assigning control, which is done *ex ante*, if shareholders were able, *ex ante*, to commit to a delegation policy, it would always be optimal to assign them control (see Lemma 1).<sup>22</sup>

A second important implication of Proposition 2 is that, when shareholders have no private information ( $\sigma_p = 0$ ), it is strictly optimal for them to be in control when management's information is less important than agency cost ( $\sigma_a < b$ ) and weakly optimal when management's information is more important than agency cost ( $\sigma_a \ge b$ ).<sup>23</sup> This is because, when shareholders have no private information, they make an *ex-ante*-optimal delegation decision, namely to delegate if management's information is more important than agency cost and not otherwise. As noted in section 1, this result is in contrast to that

<sup>&</sup>lt;sup>22</sup> Given that suboptimal delegation results in suboptimality of shareholder control for some decisions, and that delegation is suboptimal because the delegation decision conveys information, one might ask if the optimality of shareholder control can be restored, at least for some decisions, by requiring shareholders, when in control, to make the delegation decision before observing their private information. This is equivalent to choosing between management control and shareholder control with delegation by shareholders prohibited. If shareholders are in control and must make the delegation decision before observing their private information, they will delegate if and only if  $b^2 + \sigma_n^2 \le L(b, A)$ , and expected losses will be given by min  $\{b^2 + \sigma_n^2, L(b, A)\}$ . But, if management is in control, expected losses will be given by  $b^2 + \sigma_n^2$ , whereas, if shareholders are in control and cannot delegate, expected losses will be given by L(b,A). An optimal (for shareholders) choice between these two regimes results in exactly the same expected losses as having shareholders in control and requiring them to make the delegation decision before observing their private information. For any decision, however, prohibiting shareholders from delegating when in control is worse for shareholders than allowing them to delegate contingent on their private information, because the problem with shareholders' equilibrium delegation decision is that they delegate too little, not that they delegate too much. Consequently, requiring shareholders, when in control, to make the delegation decision before observing their private information will make shareholders worse off and result in more decisions for which management control is preferred.

<sup>&</sup>lt;sup>23</sup> As noted before, many commentators, e.g., Bainbridge (2006), have argued against shareholder control on grounds of shareholder ignorance.

of Aghion and Tirole (1997), where control is irrelevant in this case.

Third, for decisions in which shareholders and management have information of approximately equal importance, shareholders should be in control, and management should control a decision only if their information is sufficiently more important than shareholders'. This result implies that the loss due to suboptimal delegation by shareholders is less than the loss due to management's bias, when the two parties have equally important information. Shareholders should also control all decisions for which management's information is less important than the agency cost, since for such decisions, shareholders, if in control, optimally do not delegate to management (see Proposition 1).

To gain additional insights into the determinants of optimal control of corporate decisions, we examine the comparative statics of optimal control with respect to the importance of the parties' information and the extent of the agency problem. The first part of the next proposition is obvious from Figure 2 and Proposition 2. The second part is stated formally and proved in the appendix.

### **Proposition 3: Comparative Statics.**

- An increase in the importance of shareholders' (management's) information makes shareholder control more (less) attractive.
- If agency costs increase, optimal control switches from management to shareholders if management's information is not too important. The reverse may occur if management's information is sufficiently important. Both cases are depicted in Figure 2.

Some intuition for these results can be obtained by examining the forces at work when the parameters are changed. First, consider an increase in the importance of shareholders' information,  $\sigma_p$ . Such an increase affects both the cost of management control,  $L_M$ , and the cost of shareholder control,  $L_s$ . The effect on the cost of management control is straightforward: since management, when in control, receives no information about shareholders' private information, any increase in the importance of this information increases the cost of management control (see equation (11)). The effect of an increase in  $\sigma_p$  on the cost of shareholder control is a bit more complicated. An increase in  $\sigma_p$  (equivalently, P) reduces the probability of delegation, d/P, which, in turn, increases the cost of shareholder control. Proposition 3 implies that the more direct effect on management control dominates the "delegation" effect on shareholder control.





This figure shows the boundary curves  $\sigma_p^U$  and  $\sigma_p^L$  for two values of *b*, b = 5 and b = 8. For decisions for which  $(\sigma_a, \sigma_p)$  is in the yellow (or lightly shaded) region, the increase in *b* from 5 to 8 results in a switch in control from management to shareholders. For decisions for which  $(\sigma_a, \sigma_p)$  is in the blue (or darkly shaded) region, the increase in *b* from 5 to 8 results in a switch in control from management. For this example, the value of  $\sigma_a$  at which the two upper boundary curves cross is approximately 9.0794.

Second, consider an increase in the importance of management's information,  $\sigma_a$ . There are two opposing effects. First, an increase in the importance of management's information aggravates the loss

due to imperfect communication of their information whenever shareholders do not delegate. Thus, for this effect, an increase in  $\sigma_a$  makes management control <u>more</u> attractive. Second, an increase in the importance of management's information also results in (weakly) more delegation by shareholders. Since shareholders, in general, delegate too little, this effect makes management control <u>less</u> attractive. Proposition 3 implies that the first effect dominates.

Finally, consider an increase in agency cost, *b*. In this case, there are three effects. First, an increase in agency cost directly increases the cost of management control more than the cost of shareholder control, since shareholders do not always delegate. Second, an increase in *b* causes management to communicate less of their information whenever shareholders do not delegate. Thus, for this effect, an increase in *b* makes management control <u>more</u> attractive. Third, an increase in agency cost affects shareholders' delegation decision. As we see from Proposition 3, the net effect is to make management control less attractive when management's information is not too important and the reverse if management's information is sufficiently important.

## 4 Shareholders Are Misinformed but Don't Realize It

As we have seen in the previous section, if shareholders fully understand their private information, the case for shareholder control is actually stronger when shareholders are poorly informed because, in this case, they make better delegation decisions. Critics of shareholder control whose opposition is based on shareholder ignorance may argue, however, that not only are shareholders poorly informed but also overestimate the extent of their information. Consequently, in this section, we assume that shareholders misperceive their private information. In particular, we assume that while shareholders believe they observe  $\tilde{p}$ , in fact they observe a random variable  $\tilde{q}$  which is independent of  $\tilde{p}$  (allowing  $\tilde{q}$  to be positively (negatively) correlated with  $\tilde{p}$  strengthens (weakens) the case for shareholder control, relative to zero correlation).

Note that, as mentioned in the Introduction, we take a normative point of view and ask what allocation of control maximizes firm value without addressing the issue of who allocates control. In the

previous section, with shareholders who are aware of the extent of their information, if shareholders chose the control allocation before observing their private information, they would choose the allocation that maximizes value. In this section, however, we must take seriously our assumption that control is allocated to maximize value, e.g., by an unbiased regulator, since misinformed shareholders would not generally choose the value maximizing allocation of control.

Even though shareholders are misinformed, we find that it is optimal for them to control some decisions. In particular, as one would expect, we show that shareholder control is optimal when their misperception is not too extreme. More interestingly, however, we also show that, when shareholders are in control and do not delegate, shareholder misperception will be offset to some extent by management's strategically distorting their report relative to the base case of section 3. This "compensation effect" results in shareholder control being optimal in some cases when it otherwise would not be.

More specifically, in this section we assume both shareholders and management believe  $\tilde{p}$  is uniform on [0, P], but management realizes that shareholders actually observe  $\tilde{q}$ . The support of  $\tilde{q}$  is assumed to be a subset of [0, P]. All other assumptions are the same as in section 3.

## 4.1 Shareholder Control and Management Control

First, suppose shareholders are in control. Since shareholders believe they are in the situation modeled in section 3, they will delegate to management if and only if  $q \ge p^*$ , where  $p^*$  is the equilibrium delegation threshold defined in section 3. If  $q \ge p^*$ , management does not change its beliefs based on the fact that shareholders chose to delegate (shareholders have no information about  $\tilde{p}$ ). Management chooses  $s = \bar{p} + a + b$ , so shareholders' actual *ex ante* expected loss from being in control is the loss due to having no information about  $\tilde{p}$  plus the agency cost:

$$E\left(\overline{p}+\widetilde{a}+b-(\widetilde{p}+\widetilde{a})\right)^2 = \sigma_p^2 + b^2.$$
(12)

Now suppose that  $q < p^*$ , so that shareholders do not delegate, and management observes  $\tilde{a} = a$ . In the communication game when shareholders do not delegate, shareholders believe that management's report is based on the same partition of [0, A],  $\{a_0, ..., a_N\}$ , as in the base case. Consequently, all reports  $r \in [a_{i-1}, a_i]$  lead shareholders to choose  $s = \overline{a_i} + q$ , where  $\overline{a_i} = (a_{i-1} + a_i)/2$ . As shown in the proof of Lemma 3 in the appendix (see equation (42)), this results in management choosing a partition cell whose midpoint is closest to  $a + b - \overline{e}$ , where  $\overline{e} = E(\overline{q} - \overline{p} | \overline{q} < p^*)$ . When shareholders believe they do observe  $\overline{p}$ , management shifts its "ideal point" for reporting  $\overline{a}$  by  $-\overline{e}$ . This corrects for the expected error caused by shareholders' misperception. Since their misperception introduces a bias in their choice of *s* on average, we refer to  $\overline{e}$  as shareholders' *misperception bias*.

For the rest of this section we assume that for N = N(b, A) > 1,  $\overline{e}$  satisfies

$$-\frac{A}{N} + 2b(N-1) = -a_1 \le \overline{e} \le A - a_{N-1} = \frac{A}{N} + 2b(N-1).$$
(13)

If  $\overline{e}$  does not satisfy these bounds, some partition cells will never be used by management, further reducing communication. It is obvious that shareholder-control is suboptimal for sufficiently large misperception biases. Our goal here is to see if shareholder-control is optimal for a range of misperception biases.

We show in the appendix (Lemma 3) that the ex ante expected loss when shareholders control is

$$L_{s} = \varphi\left(\sigma_{p}^{2} + b^{2}\right) + \left(1 - \varphi\right)\left[\sigma_{p}^{2} + L(b, A) + \sigma_{q}^{2} + \overline{e}^{2} - \frac{N(b, A) - 1}{N(b, A)}\overline{e}\left(\overline{e} + 2b\right)\right],\tag{14}$$

where  $\varphi = \Pr(\tilde{q} \ge p^*)$  is the probability of delegation and  $\sigma_q^2 = Var(\tilde{q} | \tilde{q} < p^*)$ .

The right hand side of (14) has two main terms. The first is the probability that shareholders delegate times the loss if they do as given in (12). The second main term is the probability that shareholders do not delegate times the loss if they do not. This loss consists of five terms. The first,  $\sigma_p^2$ , is the loss due to shareholders not observing  $\tilde{p}$  and will be present no matter who controls. The second term, L(b, A), is the loss from knowing only the information about  $\tilde{a}$  that would be communicated by management in equilibrium if shareholders were aware that they do not observe  $\tilde{p}$ . The third term,  $\sigma_q^2$ ,

is due to the uncertainty about the extent of shareholders' misinformation about  $\tilde{p}$ . The fourth term,  $\overline{e}^2$ , is the loss due to shareholder-misperception if management did not change its signal relative to the base case in response to this misperception. We refer to the sum of the third and fourth terms,  $\sigma_q^2 + \overline{e}^2$ , as the *direct cost* of shareholders' misperception. The fifth term is the extent to which the direct cost is offset by the fact that management's report compensates for shareholders' misperception bias. We refer to this as the *compensation effect*.

If there is no information in management's report (N = 1), then the compensation effect is absent, since management's report is vacuous. If there is no misperception bias on average ( $\overline{e} = 0$ ), again the compensation effect is missing. As long as N > 1 and  $\overline{e} \neq 0$ , the compensation effect is present and its impact on shareholders' choice of *s* is opposite to that of the misperception bias, as mentioned above. In some cases, this effect results in the optimality of shareholder-control when this would not otherwise be the case, as will be seen presently.

Now suppose management is in control. Management will never delegate to shareholders, since shareholders have no information about  $\tilde{p}$ . Consequently, shareholders' expected loss if management controls the decision is the same as in section 3, namely  $L_M = \sigma_p^2 + b^2$ , and is the same as if shareholders were in control and delegated the decision to management.

## 4.2 Optimal Control

In this subsection we show that, even though shareholders are misinformed, there are nevertheless decisions for which shareholder control is optimal. When shareholders are in control, their misperception introduces three effects in addition to those considered in the base case. First, shareholders' delegation decision is affected. Second, if shareholders do not delegate, their misperception bias affects the information communicated by management to shareholders (the compensation effect described above). Third, if shareholders do not delegate, their decision is biased relative to the value-maximizing decision (the direct cost mentioned above). Opponents of shareholder control focus on this third effect which clearly weakens the case for shareholder control. Since shareholder control is strictly optimal for some

decisions when shareholders are not misinformed, it will still be optimal in some cases if shareholders' misperception bias is small. Moreover, in some cases, the compensation effect results in the optimality of shareholder-control when the misperception bias would otherwise be too large.

Obviously, if shareholders always delegate ( $\varphi = 1$ ), control is irrelevant, since in that case management always makes the decision with no information about  $\tilde{p}$ , regardless of who is in control. Consequently, in what follows, we assume  $\varphi < 1$ . In this case, a comparison of  $L_M$  with  $L_S$  reveals that it is optimal for shareholders to control the decision if and only if

$$b^{2} > L(b,A) + \sigma_{q}^{2} + \frac{\overline{e}}{N(b,A)} \Big[\overline{e} - 2\big(N(b,A) - 1\big)b\Big].$$

$$(15)$$

Since delegating is the same as management control in this section, the comparison reduces to the loss due to management control *versus* the loss due to shareholder control when shareholders do not delegate. Notice that the cost of knowing nothing about  $\tilde{p}$  cancels since this loss is borne regardless of control.

It can be shown that for N > 2, it is strictly optimal for management to control the decision when shareholders delegate with probability less than one, regardless of  $\overline{e}$ . It then follows from (15) that shareholder control is strictly optimal for

$$\overline{e}^2 + \sigma_q^2 \le b^2 - \sigma_a^2 \text{ and } \sigma_a \in (0, b), \qquad (16)$$

and for

$$\left(\overline{e}-b\right)^2 \le b^2 - \frac{\sigma_a^2}{2} - 2\sigma_q^2 \text{ and } \sigma_a \in \left(\frac{2b}{\sqrt{3}}, b\sqrt{2}\right).$$
 (17)

Note that N = 1 for condition (16) and N = 2 for condition (17).

Since  $\overline{e}$  and  $\sigma_q^2$ , in general, depend on  $\sigma_a$  and *b* (through  $p^*$ ), it is difficult to characterize the types of decisions that satisfy condition (17). We can, however, characterize these decisions in a simple way by making an additional assumption regarding the distribution of  $\tilde{q}$ . The assumption we need is

that, for the range of values of  $\sigma_a$  and *b* that are relevant to condition (17), i.e.,  $\sigma_a \in \left(\frac{2b}{\sqrt{3}}, b\sqrt{2}\right), \tilde{q} < p^*$ 

with probability one.<sup>24</sup> Under this assumption, shareholders never delegate, which weakens the case for shareholder control when shareholders are misinformed.

With this assumption, we can depict the combinations of parameters that lead to optimal control by shareholders and managers. There are two such regions corresponding to conditions (16) and (17), depicted in Figure 3. On the left side of Figure 3, where management's information is less important than agency costs, if shareholders are in control and do not delegate, management conveys no information to shareholders (N = 1) and, therefore, there is no compensation effect. Here, shareholder control is optimal only when the direct cost of shareholders' misperception,  $\overline{e}^2 + \sigma_q^2$ , is small. This is shown by the left yellow (or lightly shaded) triangle. The upper bound on the direct cost of shareholders' misperception for shareholder control to be optimal decreases as the importance of management's information increases. In particular, when management has no private information, shareholder control is optimal whenever the direct cost of their misperception is less than the agency cost. This threshold decreases to zero as the importance of management's information increases from zero to *b*. The threshold increases with increases in management's bias.

<sup>&</sup>lt;sup>24</sup> Formally, what is required is that the support of  $\tilde{q}$  is bounded above by  $P - 2b(\sqrt{1.5} - 1)$ . The parameter values used for Figure 3 below satisfy this condition, provided  $13.8 \approx 2b\sqrt{3} > P > 2b(\sqrt{1.5} - 1) \approx 1.8$ . Note, we do not need these assumptions for condition (16), because, in that case,  $p^* = P$ .





This figure shows the combinations of the importance of management's information,  $\sigma_a^2$ , and measures of the cost of shareholders' misperception such that shareholder-control is strictly optimal. The triangle on the left side corresponds to condition (16) and is plotted relative to the direct cost of shareholder misperception,  $\sigma_q^2 + \overline{e}^2$ , on the left vertical axis. The right triangle corresponds to condition (17) and is plotted relative to  $(\overline{e} - b)^2$  on the right vertical axis. For this

figure, 
$$b^2 = 16$$
,  $\sigma_q^2 = 2$ , so  $4b^2/3 = 21\frac{1}{3}$ ,  $2(b^2 - 2\sigma_q^2) = 24$ ,  $2b^2 = 32$  and  $\frac{b^2}{3} - 2\sigma_q^2 = 1\frac{1}{3}$ .

On the right side of Figure 3, management's information is sufficiently important that, if shareholders are in control and do not delegate, management will tell shareholders whether  $\tilde{a}$  is "low" or "high" (N = 2). In this case, shareholder control is optimal when shareholders' misperception bias is close to management's bias as defined by (17), and is shown by the right yellow (or lightly shaded) triangle. This is due to the compensation effect. If it were not for the compensation effect, management control would be optimal for all values of  $\overline{e}$  when N = 2. Again the upper bound on the distance

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between shareholders' misperception bias and management's bias for shareholder control to be optimal decreases to zero as the importance of management's information increases over the range in which N = 2. Also, the maximum distance between shareholders' misperception bias and management's bias for shareholder control to be optimal increases with increases in management's bias and with decreases in the variance of shareholders' signal. An increase in management's bias makes management control less attractive while a reduction in uncertainty about shareholders' decision if they do not delegate makes shareholder control more attractive.

As the importance of management's information increases, optimal control can switch from shareholders to management and back again, as is clear from Figure 3. This contrasts with the base case in which increases in the importance of management's information cause optimal control to switch from shareholders to management but not the reverse. The reason for the reversal in this case is that increases in the importance of management information can trigger the compensation effect. This effect is not present in the base case.

Intuitively, it seems obvious that, as agency costs increase, for any given value of the importance of management's information, shareholder control should become more attractive. Indeed, this is generally the case, but if the increase in agency costs is not too large, it can result in the counter-intuitive result that optimal control switches from shareholders to management. This is due to the fact that a small increase in agency costs can reduce communication from management and eliminate the compensation effect. That is, an increase in agency costs may trigger a reduction in communication from management that increases the cost of shareholder misperception by more than it increases the cost of management control. This is shown in Figure 4.

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#### Figure 4

This figure shows how a small increase in agency costs can cause a switch in control from shareholders to management. In the blue (or darkly shaded) area, shareholder control is optimal for  $b^2 = 16$ , but management control is optimal for  $b^2 = 17$ . For this figure,  $\sigma_q^2 = 2$ .

We summarize the results of this section in the following proposition.

Proposition 4. When shareholders are misinformed, their misperception bias satisfies (13), and

for all  $\sigma_a \in \left(\frac{2b}{\sqrt{3}}, b\sqrt{2}\right)$ ,  $\tilde{q} < p^*$  with probability one,

(i) Shareholder-control is strictly optimal in either of the following two cases: (a)  $\sigma_a < b$ , and the direct cost of shareholders' misperception,  $\sigma_q^2 + \overline{e}^2$ , is sufficiently small relative to  $\sigma_a$  as specified in (16) or (b)  $\sigma_a$  and *b* are such that management will reveal whether  $\tilde{a}$  is "low" or "high," and shareholders' misperception bias is sufficiently close to management's bias, as specified by (17).

Management control is strictly optimal in all other cases.

- (ii) Comparative statics for the importance of management's information: Increases in the importance of management's information may cause optimal control to switch from shareholders to management and back again.
- (iii) Comparative statics for agency cost: For case (a) in the above paragraph, any increase in agency costs increases the range of values of the direct cost of shareholders' misperception for which shareholder control is optimal. For case (b), suppose agency costs increase from b to b', where

$$b < b' < 2b/\sqrt{3}$$
. Then, for  $\sigma_a^2 \in (4b^2/3, 4b'^2/3)$  and  $(\overline{e} - b)^2 \le b^2 - \frac{\sigma_a^2}{2} - 2\sigma_q^2$ , optimal control

switches from shareholders to management (The proof of this part follows trivially from (16) and (17) and is, therefore, omitted.).

The main contributions of this section are the two, counterintuitive comparative statics results in Proposition 4. Without the formal model and analysis, one's intuition is likely to lead to the conclusion that increases in the importance of management's information would always increase the advantage of management control, while increases in agency costs would have the opposite effect. This intuition is flawed because it ignores the compensation effect.

## 5 Non-value-maximizing Shareholders

In this section we consider whether shareholder control may still maximize firm value even when controlling shareholders have goals other than value maximization, e.g., preservation of the environment, support of a political agenda, private benefits of control, etc. The presence of non-value-maximizing (NVM) shareholders raises the issue of whether value maximization is still an appropriate goal. There are two obvious possibilities. One is to assume that the goal is to maximize the objective of the NVM shareholders. This simply repeats the base case analysis of section 3 if we reinterpret management's bias as being relative to the objective of the NVM shareholders. The results of that section carry over to this case. The other possibility, which we adopt in this section, is to assume the objective is value-

maximization and ask whether the value-maximizing (VM) shareholders are better off with their nonvalue-maximizing (NVM) co-investors in control than with management in control.

We model NVM shareholders as being biased, like management, but with a potentially different bias,  $\beta$ . Formally, NVM shareholders choose a decision *s* that minimizes the loss function

 $E(s - (\tilde{p} + \tilde{a} + \beta))^2$ . NVM shareholders' optimally choose  $s = E(\tilde{a} + \tilde{p}) + \beta$ , where the expectation is conditional on whatever information they have about  $\tilde{a}$  and  $\tilde{p}$ . The parameter  $\beta$  measures the extent to which these shareholders will deviate from the optimal decision to further their social agenda. Note that  $\beta$  could be either positive or negative. If, for example, *s* is a minimum acceptable bid for selling the firm and NVM shareholders fear losing control, they may prefer a higher-than-optimal minimum acceptable bid (positive  $\beta$ ). On the other hand, if the decision is the size of a new plant, NVM shareholders may prefer a smaller-than-optimal plant if this will reduce emissions (negative  $\beta$ ). Management minimizes  $E(s - (\tilde{p} + \tilde{a} + b))^2$ , given their information, as before.

The difference between management's bias, *b*, and shareholders' bias,  $\beta$ , denoted  $B = b - \beta$  and referred to as the *net bias*, plays an important role in the analysis of this section.<sup>25</sup> If B > 0, all the results of section 3 apply, except that *B* replaces *b* in all calculations. In particular,  $p^*$  and  $d = P - p^*$  are as described in Proposition 1 with *b* replaced by *B*. If B < 0, then NVM shareholders delegate when  $\tilde{p} \in [0, p^*]$ , and  $p^*$  and d are calculated as in section 3 except that *b* is replaced by |B| and  $p^* = d$ . Also, for B < 0, management never delegates to shareholders if and only if  $\sigma_p \leq |B|$ . Thus the counterpart of Assumption 2 in this case is  $\sigma_p < |B|$ , which also implies that NVM shareholders do not

<sup>&</sup>lt;sup>25</sup> The one-dimensional nature of the decision implies that the difference in preferences between NVM shareholders and management can be measured by a single parameter. If, for example, the decision (and the private information) had two components, say size and ecological friendliness, the two groups' preferences might differ on both dimensions, but it would not necessarily be possible to characterize congruence of preferences by a single parameter. Presumably, how the preferences differ on the two dimensions would affect communication between the parties. Since this would take us out of the CS framework, we leave it for future work. We are grateful to a referee for pointing out this issue.

communicate any private information to management (other than what may be communicated by their delegation decision). We therefore replace Assumption 2 with

Assumption 3:  $\sigma_p < |B|$ , or, equivalently, either  $\beta \le b - \sigma_p$  or  $\beta \ge b + \sigma_p$ .

Since 
$$\sigma_p < |B|$$
,  $L(B,P) = \sigma_p^2$  and  $L(B,d) = \sigma(d)^2$ . Assuming that  $\sigma_p > 0$ , if NVM

shareholders are in control, the expected loss in firm value is

$$\frac{d}{P}\left(b^{2}+\sigma\left(d\right)^{2}\right)+\left(1-\frac{d}{P}\right)\left(\beta^{2}+L\left(B,A\right)\right).$$
(18)

The expression in (18) is the same as in the base case (equation (6)), except for three effects that are similar to those discussed in section 4.2. First, the size of the delegation region, *d*, is determined by the net bias *B* rather than management's bias *b*. Second, the loss when shareholders do not delegate is increased by the cost of the NVM shareholders' bias,  $\beta^2$ . This effect obviously reduces the attractiveness of shareholder control and is the effect on which opponents of shareholder control focus. Third, the loss due to imperfect communication from management is determined by the net bias instead of management's bias. Since the net bias can be smaller than management's bias, communication of management's information can be more precise than in the base case, resulting in smaller loss. As is shown in Proposition 5 below, this effect causes shareholder control to be optimal in some cases.

If management is in control, the expected loss in firm value is  $b^2 + \sigma_p^2$ , as before. Therefore, shareholders should control if and only if

$$\frac{d}{P}\left(b^{2}+\sigma\left(d\right)^{2}\right)+\left(1-\frac{d}{P}\right)\left(\beta^{2}+L\left(B,A\right)\right)\leq b^{2}+\sigma_{p}^{2}.$$
(19)

The main result of this section is to characterize optimal control of decisions for various values of the NVM shareholders' bias,  $\beta$ , and the importance of management's information,  $\sigma_a^2$ , for fixed values of management's bias, *b*, and the importance of shareholders' information,  $\sigma_p$ . A more formal statement of this result and the proof are given in the appendix.

**Proposition 5.** Assume  $b - \overline{p} > \sigma_p$ .<sup>26</sup> For any given values of management's bias, *b*, and the importance of shareholders' information,  $\sigma_p$ , the possible combinations of the NVM shareholders' bias,  $\beta$ , and the importance of management's information,  $\sigma_a^2$ , can be divided into three regions as depicted in Figure 5. In particular:

- It is strictly optimal for NVM shareholders to control decisions if NVM shareholders' bias is smaller than b σ<sub>p</sub>, and the importance of management's information is below a threshold. This threshold is given by a function H(β) for β≤ p̄ and by a function G(β) for p̄ ≤ β < b σ<sub>p</sub>, as shown in Figure 5.
- It is strictly optimal for management to control decisions if the importance of management's information is below the threshold given by  $G(\beta)$  and either
  - NVM shareholders' bias is greater than  $b + \sigma_p$ , or
  - NVM shareholders' bias is smaller than  $\overline{p}$ , and the importance of management's information is above the threshold given by  $H(\beta)$ .
- Control is irrelevant for any combination of NVM shareholders' bias, β, and the importance of management's information, σ<sub>a</sub><sup>2</sup>, such that the importance of management's information exceeds the threshold given by G(β), since, in this case, shareholders always delegate if in control.

<sup>&</sup>lt;sup>26</sup> This is the richest case. When this inequality fails, the result is qualitatively similar.





This figure shows, for various combinations of values of NVM shareholders' bias,  $\beta$ , and the importance of management's information,  $\sigma_a^2$ , which party optimally controls the decision. The values of the other parameters are b = 4 and  $\sigma_p = 1$ . These values imply that  $\overline{p} = \sqrt{3} \approx 1.73$  and  $H(\beta) = 0$  for  $\beta \le -\sqrt{b^2 + \sigma_p^2} \approx -4.12$ . Note that Assumption 3 implies that values of NVM shareholders' bias between  $b - \sigma_p = 3$  and  $b + \sigma_p = 5$  are not considered. This accounts for the white space in the middle of the figure.

The proposition is depicted in Figure 5 which shows that, indeed, shareholder control is optimal for some decisions. When NVM shareholders are either less biased in the same direction as management  $(0 < \beta < b - \sigma_p)$  or are biased in the opposite direction ( $\beta < 0$ ), there is a tradeoff. When NVM

shareholders' bias is similar to that of management but smaller ( $\overline{p} < \beta < b - \sigma_p$ ), the net bias is small. In this case, management is willing to communicate much of their information to shareholders if shareholders are in control and decide not to delegate. Since NVM shareholders' bias is smaller than that of management, and little of management's information is lost if shareholders make the decision, the cost of letting shareholders decide is small, so it is optimal for them to control such decisions. As NVM shareholders' bias decreases, holding management's bias fixed, the net bias increases. This reduces communication from management, but, if shareholders' bias is positive, like management's, the reduction in shareholders' bias also reduces the inefficiency of the shareholders' decision, cet. par. The net effect on control could go either way. If shareholders' bias is opposite that of management, i.e.,  $\beta < 0$ , however, further reductions in NVM shareholders' bias *increase* the inefficiency of the shareholders' decision. Now, both the communication effect and the direct effect on the decision of shareholders of the reduction in NVM shareholders' bias work against shareholder control of the decision. Thus when NVM shareholders' bias is negative (opposite that of management), as NVM shareholders' bias decreases (and the net bias increases), the importance of management's information must decrease in order for shareholder control to be optimal. That is, the threshold H in Figure 5 decreases as  $\beta$  becomes more negative. For sufficiently negative  $\beta$ , NVM shareholders' bias is so large (in absolute value) that it is optimal for management to control even if they have no private information.

Consider decisions for which NVM shareholders' bias is in the same direction as management's bias but larger, i.e.,  $\beta > b + \sigma_p$ . In this case, if shareholders actually make the decision, it's even more biased than management's decision would be for the same information. On the other hand, shareholders have information about  $\tilde{p}$  that management doesn't have and won't learn from shareholders, while shareholders may learn at least some information about  $\tilde{a}$  from management. Moreover, shareholders, if in control, may delegate to management based on their (shareholders') private information. Consequently, it is not intuitively obvious who should control in this situation. It turns out that the fact that NVM shareholders' bias "outweighs" the combination of management's bias and the importance of

their own information, i.e.,  $\beta > b + \sigma_p$ , management control is always optimal in this region.

When management's information is sufficiently important ( $\sigma_a^2 > G(\beta)$ ), shareholders, if in control, always delegate to management. As a result, the delegation decision conveys no information. Also, shareholders convey no information directly. Consequently control is irrelevant: regardless of who controls the decision, management will always actually make the decision with no information from shareholders.

This section shows that, even when controlling shareholders have biases that prevent them from choosing the value-maximizing decision, it may still be value-maximizing for them to control some decisions. In particular, they should control decisions for which the shareholders' bias is similar to that of management (i.e., the net bias is small), and the importance of management's information is also small.

## 6 Discussion and Conclusions

In this paper, we address the issue of which corporate decisions are best controlled directly by shareholders. Using a model that accounts for private information, delegation, communication and agency considerations, we show that popular arguments both for and against direct shareholder control are flawed. For example, a strong intuitive argument has been advanced by several commentators that shareholders should not control major corporate decisions because, unlike management, they do not possess the relevant information. We show, however, that shareholders should control decisions for which they have none of the information possessed by management and have no private information of their own, provided these shareholders are aware of their ignorance and the extent of management's private information. This result follows, in part, from the failure of the simple argument to take account of the fact that shareholders can delegate the decision to management. On the other hand, others have argued that, because shareholders can delegate and want to maximize value (i.e., have no agency problem), they should control every major decision. We show that this argument is incorrect, because, if shareholders have private information, they will fail to delegate optimally.

Others have argued against shareholder control on the grounds that either shareholders

overestimate the extent of their information or that shareholders have agendas other than value maximization. We show that in both cases there are still some decisions for which shareholder control is optimal. This is due, in part, to the fact that shareholder biases, due either to misperception or non-value maximizing agendas, may improve communication from management to shareholders.

We view the main contribution of our analysis as improving our intuition about shareholder control of decisions by highlighting some less-than-obvious considerations involving strategic communication and delegation. Nevertheless, it is interesting to consider how the results of the basic model may be applied to some actual decisions. One example is how much cash to distribute to shareholders, a decision about which management is likely to have important information, while shareholders are likely to have little or no important private information. In this case one might be tempted to conclude that it is obvious that management should control the cash distribution decision. Our results imply just the opposite conclusion. This particular decision seems to be often contested by activist shareholders, generally without much success. Our results suggest that perhaps the governance rules should be changed to make it easier for such shareholders to exercise control over payout policy.

Another example is replacement of management. Both parties are likely to have private information about the distribution of talent in the population of potential replacements. It is reasonable to assume that management and shareholders have information of comparable importance in assessing the availability of replacements of various levels of ability. In this case, the model implies that shareholder control is optimal, regardless of the level of private benefits of control or how important the parties' private information is (as long as they are of similar importance).

As a final example, consider the optimal proportion of performance-based compensation. For this decision, management's information is likely to be more important than that of shareholders with regard to low level executives. On the other hand, for top executives, management's information about the optimal compensation scheme may be of roughly comparable importance to that of shareholders' information. The model then implies that, assuming similar agency costs for the two decisions, shareholder control is more likely to be optimal for top level compensation decisions than for lower level compensation. Moreover, it seems reasonable to suppose that agency costs for decisions involving one's own compensation are likely to be larger than for decisions involving the compensation of others. This would reinforce the previous conclusion.

Obviously, we have neglected a number of important issues regarding the optimality of direct shareholder control of decisions. The most glaring of these omissions is our assumption that there are no differences of opinion, information, or preferences among shareholders (or at least among the controlling group of shareholders). When such differences exist, the issue arises as to how they are resolved in making decisions (both delegation decisions and "substantive" decisions). Obviously, this involves voting in some form or another. The same can be said about differences among managers. We have also assumed that all the parameters of the information structure and preferences are common knowledge. Relaxing these assumptions will, we believe, lead to interesting results. This, of course, is left for future work.

Another avenue for future work involves broadening the set of mechanisms for communicating information. Using an intermediary with preferences between those of management and those of shareholders, such as the board of directors or a group of shareholders sympathetic to management, could also improve outcomes and affect optimal control.

Another, counter-intuitive device that might improve communication is to introduce noise into the transmission of signals between shareholders and management. Blume, Board and Kawamura (2007) shows that the introduction of such noise can result in a Pareto improvement relative to the best Crawford-Sobel equilibrium. Therefore, improvements may be possible by adding this type of noise when shareholders are in control. If so, this would strengthen the case for shareholder control.

A third possibility for improving communication involves multiple stages of communication as considered in Krishna and Morgan (2004). Two of the results of Krishna and Morgan (2004) can be applied to the current model if delegation is ruled out. The first is that multi-stage communication cannot improve the outcome without exogenous randomization as part of the mechanism. The second result, shown by example, is that with exogenous randomization, multi-stage communication can, indeed,

improve the outcome. This is discussed in more detail in Harris and Raviv (2005). Since, in our model, management never delegates to shareholders, this result suggests that, when management is in control, multi-stage communication may improve the outcome. This would strengthen the case for management control.

If there are multiple decisions to be made at the same time, and shareholders and management each had private information about each decision, they could communicate a ranking of their private information across the various decisions, either in addition to or instead of information about the values of each decision's private information. For example, suppose the decisions were the size of two plants, management is in control, and both parties have private information about the optimal size of each plant. If we extend our assumptions about the size of management's bias relative to the importance of shareholder information to this setting, shareholders would communicate none of their information about the individual optimal plant sizes. Chakraborty and Harbaugh (2007) show, however, that under some symmetry assumptions in addition to the assumptions made here, it is an equilibrium for shareholders to reveal the ranking of the optimal plant sizes.

Other extensions include allowing the "sale" of control rights at the interim date, allowing the initial allocation of control to be contingent on verifiable information that arrives at the interim date, or an analysis of dynamic considerations. In a dynamic model, one could allow control in one period to be contingent on observable past performance. Also, in such a model, one might usefully distinguish between temporary delegation of control in a given period contingent on the realization of private information and permanent transfer of control contingent on such information.

Finally, consider the question of who should control the decision of who controls substantive decisions. This is what Bebchuk (2005) refers to as controlling the "rules of the game." Suppose that such rules-of-the-game decisions can be made contingent on the parameters b,  $\sigma_a$ , and  $\sigma_p$  describing the substantive decisions, shareholders want to maximize firm value, and shareholders are not misinformed. In this case, shareholders should control rules-of-the-game decisions and, contingent on the parameters,

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allocate control as described in Proposition 2. Even if the rules-of-the-game decisions cannot be contingent on the relevant parameters, it seems clear that value maximizing shareholders should make them, provided there is no *private* information about their likely values. Matters become more complicated if there is private information about the likely values of parameters, shareholders are misinformed, or shareholders have other agendas. This topic is also left for future work.

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# Control of Corporate Decisions: Shareholders vs. Management

# Appendix

The expressions, for N(b,A),  $\{a_i\}_{i=1}^{N(b,A)}$ , and L(b,A) mentioned in footnote 18 and derived in Harris and Raviv (2008a) are

$$N(b,A) = \left\langle \frac{1}{2} \left( \sqrt{1 + 2A/b} - 1 \right) \right\rangle,$$
$$a_i = \frac{iA}{N(b,A)} - 2i \left( N(b,A) - i \right) b, \text{ for } i = 0, \dots, N(b,A)$$

where, for any real number x,  $\langle x \rangle$  is the smallest integer greater than or equal to x, and

$$L(b,A) = E\left[\overline{a}(r(\tilde{a})) + p - (\tilde{a} + p)\right]^2 = E\left[\overline{a}(r(\tilde{a})) - \tilde{a}\right]^2$$

## **Proof of Proposition 1**

**Proposition 1:** Suppose shareholders are in control of a decision. The unique pure-strategy Perfect Bayes' Equilibrium of the resulting game is as follows:

- If shareholders have no private information ( $\sigma_p = 0$ ) they will delegate the decision if and only if  $\sigma_a \ge b$ , i.e., if and only if management's information is more important than agency costs, regardless of the realization of  $\tilde{p}$ . In this case, management infers nothing from the delegation decision, and, if shareholders do delegate, management chooses s = a + b, where *a* is the realization of  $\tilde{a}$ . [This is shown in Dessein (2002).]
- If shareholders have private information (σ<sub>p</sub> > 0), shareholders will delegate the decision if and only if the realized value of their private information exceeds a threshold p\*∈[0, P]. This threshold and the equilibrium strategies and beliefs of management depend on the values of the parameters, σ<sub>a</sub>, σ<sub>p</sub>, and b. There are three cases to consider.
  - Case 1:  $\sigma_a \leq b$ . In this case, shareholders never delegate, unless  $\tilde{p} = P$  and  $\sigma_a = b$ , i.e.,

$$p^* = P \text{ and } d = 0.$$
 (20)

If shareholders do not delegate (this is their equilibrium move, unless  $\sigma_a = b$  and  $\tilde{p} = P$ ), management's inference and strategy are irrelevant, since it has no moves. If shareholders delegate (this is an off-equilibrium-path move, unless  $\sigma_a = b$  and  $\tilde{p} = P$ ), management infers that  $\tilde{p} = P$  with probability one and chooses s = a + P + b, where *a* is the realization of  $\tilde{a}$ .

• Case 2:  $P \le 2\left[\sqrt{L(b,A)} - b\right]$ . In this case, shareholders always delegate, i.e.,

$$p^* = 0 \quad \text{and} \quad d = P \,. \tag{21}$$

Management infers nothing if shareholders delegate (this is their equilibrium move) and chooses  $s = a + \frac{P}{2} + b$ , where *a* is the realization of  $\tilde{a}$ . If shareholders do not delegate (this is an off-equilibrium-path move), management's inference and strategy are irrelevant, since it has no moves.

• Case 3: 
$$\sigma_a > b$$
 and  $P > 2\left[\sqrt{L(b,A)} - b\right]$ . In this case,  $p^* \in (0,P)$ ,  $d \in (0,P)$  and satisfy

$$P - p^* = d = 2\left[\sqrt{L(b,A)} - b\right].$$
(22)

If shareholders delegate, management's posterior belief about  $\tilde{p}$  is uniform on  $[p^*, P]$ ,

and it chooses  $s = a + \frac{P + p^*}{2} + b$ , where *a* is the realization of  $\tilde{a}$ . If shareholders do not delegate, management's inference and strategy are irrelevant, since it has no moves.

There are no off-equilibrium-path moves in this case.

**Proof.** We first show that the delegation region must be an upper interval, assuming that shareholders, when in control, always delegate when they are indifferent between delegating and not delegating. Let  $D \subseteq [0, P]$  denote the delegation region. We want to show that, if  $D \neq \emptyset$ , then

 $D = [p^*, P]$ , for some  $p^* \in [0, P]$ .

To show this, let  $\lambda(p,D)$  denote the shareholders' expected loss from delegating when  $\tilde{p} = p$ ,  $D \neq \emptyset$  is the delegation region, and management chooses  $s = \hat{p} + b + a$ , where *a* is the realization of  $\tilde{a}$ and  $\hat{p} = E(\tilde{p} | \tilde{p} \in D)$ . Then

$$\lambda(p,D) = E\left(\left(\hat{p}+b+\tilde{a}-(p+\tilde{a})\right)^2\right) = \left(\hat{p}+b-p\right)^2.$$
(23)

Recall L(b, A) is the shareholders' expected loss from not delegating. Note that L(b, A) is independent of D and the realization of  $\tilde{p}$ .

By definition of D,

$$D = \left\{ p \in [0, P] \middle| \lambda(p, D) \le L(b, A) \right\}.$$
(24)

It follows from (23) and (24), that

$$D = \begin{bmatrix} 0, P \end{bmatrix} \cap \begin{bmatrix} \hat{p} + b - \sqrt{L(b, A)}, \, \hat{p} + b + \sqrt{L(b, A)} \end{bmatrix}.$$
(25)

Therefore *D* is a closed interval in [0, P]. Let  $d_1, d_2 \in [0, P]$  be such that  $D = [d_1, d_2]$ . Then, the definition of  $\hat{p}$  and the fact that  $\tilde{p}$  is uniformly distributed imply that

$$\hat{p} = \frac{d_1 + d_2}{2} \,.$$

Suppose  $d_2 < P$ . Then, it follows from (25) that

$$d_2 = \hat{p} + b + \sqrt{L(b,A)}$$

and

$$d_1 \ge \hat{p} + b - \sqrt{L(b, A)}$$

Therefore,

$$\hat{p} = \frac{d_1 + d_2}{2} \ge \frac{2(\hat{p} + b) - \sqrt{L(b, A)} + \sqrt{L(b, A)}}{2} = \hat{p} + b.$$

This contradicts b > 0. Consequently,  $d_2 = P$ , which completes the proof of our claim that the equilibrium delegation region is of the form  $[p^*, P]$ .

To characterize  $p^*$ , define a function f on [0, P] as follows: for any  $x \in [0, P]$ , f(b, P - x) is the loss to shareholders of delegating, given that management believes the threshold is x, and given that the actual realization of  $\tilde{p}$  is exactly x. If management believes that shareholders delegate if and only if

$$\tilde{p} \in [x, P]$$
,  $\hat{p} = \frac{P+x}{2}$ , so management which observes  $\tilde{a} = a$  chooses  $s = a + \frac{P+x}{2} + b$  (recall that

shareholders communicate no information about  $\tilde{p}$  other than what can be inferred from the fact of delegation). Thus

$$f(b, P-x) = \left(a + \frac{P+x}{2} + b - (a+x)\right)^2 = \left(\frac{P-x}{2} + b\right)^2.$$
 (26)

Since *x* is the farthest point in the delegation region from management's choice of *s* (remember, b > 0, so  $\hat{p}$  is more than halfway between *x* and *P*), f(b, P - x) represents the worst-case loss from delegating. In order for *x* to be an equilibrium threshold for delegating, this worst-case loss from delegating must be just equal to the loss from not delegating, L(b, A), provided that  $x \in (0, P)$ . If L(b, A) < f(b, P - x), the loss from delegating will be greater than the loss from not delegating for some values of  $\tilde{p} > x$ . If L(b, A) > f(b, P - x), the loss from delegating will be less than the loss from not delegating for some values of  $\tilde{p} < x$ . Thus, if  $p^* \in (0, P)$ ,  $p^*$  must satisfy

$$f(b, P-p^*) = \left(\frac{P-p^*}{2} + b\right)^2 = \left(\frac{d}{2} + b\right)^2 = L(b, A).$$
(27)

Solving (27) for *d* gives

$$d = 2\left[\sqrt{L(b,A)} - b\right].$$
(28)

Clearly, the formula for d in (28) is valid if and only if it results in a value between 0 and P. This is the case in Case 3 of the proposition. In Case 3, it is clear that the inference of management regarding

 $\tilde{p}$  claimed in the proposition satisfies Bayes' rule, given the delegation strategy of shareholders, and the strategy of management claimed in the proposition is optimal for management given their beliefs.

Now consider Case 1. If  $L(b, A) < b^2$ , then, by Lemma 1 of Harris and Raviv (2008b),

 $L(b, A) = \sigma_a^2 < b^2$ , and (28) results in d < 0. In this case, if shareholders delegate, and management chooses s = a + P + b, shareholders lose  $(a + P + b - a - p)^2 \ge b^2$  for all  $p \in [0, P]$ . If shareholders do not delegate, they lose  $L(b, A) = \sigma_a^2 < b^2$ . Consequently, it is optimal for shareholders not to delegate when  $L(b, A) < b^2$ , regardless of the realization of  $\tilde{p}$ . If  $L(b, A) = b^2$ , then  $L(b, A) = \sigma_a^2$ . The above argument for shareholders goes through for all  $p \in [0, P)$ . If  $\tilde{p} = P$ , then shareholders are indifferent between delegating and not delegating, so it is optimal for them to delegate (and is consistent with our assumption above that shareholders always delegate when indifferent). As mentioned above, however,  $L(b, A) \le b^2$  if and only if  $\sigma_a \le b$ , so we have that d = 0 and  $p^* = P$  if and only if  $\sigma_a \le b$ . Since, when shareholders are in control, management moves only if shareholders delegate, management's beliefs and strategy are irrelevant if shareholders delegate. If  $L(b, A) = b^2$ , management's beliefs satisfy Bayes' rule when shareholders delegate. If  $L(b, A) < b^2$ , delegation is not on the equilibrium path, so management's beliefs need not be justified in this event. Given their assumed beliefs, if shareholders delegate in this case, the assumed strategy of management (s = a + P + b) is optimal.

If  $L(b,A) \ge f(b,P)$ , or, using (26),  $P \le 2\left[\sqrt{L(b,A)} - b\right]$ , the loss from not delegating exceeds the loss from delegating for all realizations of  $\tilde{p} \in [0,P]$ , and (28) results in  $d \ge P$ . In this case, management has sufficient information to warrant delegating to them regardless of the realization of  $\tilde{p}$ , so it is optimal for shareholders to delegate for all realizations of  $\tilde{p}$ , i.e., d = P and  $p^* = 0$ . Clearly, the assumed beliefs of management follows Bayes' rule if shareholders delegate, and management's strategy is optimal given their beliefs. Of course, if shareholders do not delegate, management's beliefs and strategy are irrelevant. With respect to uniqueness of the equilibrium, it is clear that there are no other pure-strategy equilibria when  $\sigma_a > b$ . When  $\sigma_a < b$ , one must consider the possibility that there are additional equilibria supported by different off-equilibrium-path beliefs. In particular, suppose that  $\sigma_a < b$  and management's expectation of  $\tilde{p}$  conditional on delegation is  $\hat{p} < P$ . In that case, the optimal delegation strategy for shareholders is to delegate whenever  $p \in [\min \{\hat{p} + b - \sigma_a, P\}, \min \{\hat{p} + b + \sigma_a, P\}]$ . If  $\hat{p} + b - \sigma_a \ge P$ , then  $\hat{p} > P$ , since  $\sigma_a < b$ . But this contradicts  $\hat{p} < P$ . If, on the other hand,  $\hat{p} + b - \sigma_a < P$ , then shareholders only delegate when  $p > \hat{p}$ , again since  $\sigma_a < b$ . But then  $\hat{p}$  cannot be the expectation of  $\tilde{p}$  conditional on delegation. Consequently, in any equilibrium when  $\sigma_a < b$ , management's expectation of  $\tilde{p}$  conditional on delegation must be at least P. Since P is the upper bound of the support of  $\tilde{p}$ , this implies that management must have the claimed beliefs in equilibrium and, hence, this is the only equilibrium.

Q.E.D.

### Proof of Lemma 1

**Lemma 1.** (a) Let  $d^*$  minimize

$$\frac{x}{P}\left(b^{2}+\sigma\left(x\right)^{2}\right)+\left(1-\frac{x}{P}\right)L\left(b,A\right).$$
(29)

with respect to x, subject to  $x \in [0, P]$ , and let  $p^{**} = P - d^*$ . Then  $p^{**}$  is an *ex-ante*-optimal delegation threshold, and  $d^*$  and  $p^{**}$  are given by

If 
$$\sigma_a \le b$$
,  $d^* = 0$ ,  $p^{**} = P$ ; (30)

if 
$$P \le 2\left[\sqrt{L(b,A) - b^2}\right], d^* = P, p^{**} = 0;$$
 (31)

otherwise, 
$$d^* = P - p^{**} = 2\left[\sqrt{L(b,A) - b^2}\right] \in (0,P).$$
 (32)

Moreover,  $d^* \ge d$  and  $p^{**} \le p^*$ , with strict inequalities whenever  $\sigma_a > b$  and d < P.

(b) The loss from shareholder control if shareholders can commit to delegate if and only if

 $\tilde{p} \in [p^{**}, P]$  is smaller than the loss from management control, strictly if  $P > 2\left[\sqrt{L(b, A) - b^2}\right]$ .

**Proof.** (a) The problem has a solution, since the objective function is continuous and the constraint set is compact. Obviously, the objective function in (29) is the *ex ante* expected loss if shareholders delegate if and only if  $\tilde{p} \ge P - x$ . Consequently,  $p^{**}$  is an *ex-ante*-optimal delegation threshold.

For  $\sigma_a < b$ ,  $L(b, A) < b^2$  (see Harris and Raviv (2008b), Lemma 1), so the objective function in (29) is strictly decreasing in *x*. Therefore, for  $\sigma_a < b$ ,  $d^* = 0$  and  $p^{**} = P$ .

Using  $\sigma(x)^2 = x^2/12$ , the first-order condition for minimizing (29) is

$$\frac{x^2}{4} + b^2 - L(b, A) = 0.$$
(33)

For  $P < 2\left[\sqrt{L(b,A)-b^2}\right]$ , the left hand side of (33) is strictly negative at x = P. Therefore, if  $P < 2\left[\sqrt{L(b,A)-b^2}\right]$ ,  $d^* = P$  and  $p^{**} = 0$ . For  $P \ge 2\left[\sqrt{L(b,A)-b^2}\right]$  and  $\sigma_a \ge b$ , solving (33) for x, yields  $d^* = 2\left[\sqrt{L(b,A)-b^2}\right] \in [0,P]$ .

Clearly, the second order condition is satisfied, since the left hand side of (33) is increasing in x.

Finally, if 
$$\sigma_a > b$$
, and  $d < P$ ,  $d^* = \min\left\{2\left[\sqrt{L(b,A) - b^2}\right], P\right\}$  and  $d = 2\left[\sqrt{L(b,A)} - b\right]$ . It is

easy to check that  $\sqrt{L(b,A)} - b < \sqrt{L(b,A) - b^2}$  for  $L(b,A) > b^2$ , which is equivalent to  $\sigma_a > b$  (see Harris and Raviv (2008b), Lemma 1). If  $\sigma_a \le b$ , then  $d^* = d = 0$ , and if d = P, then  $d^* = d = P$ .

(b) The loss from shareholder control in this case is given by the expression in (29) with  $x = d^*$ . Since x = P is a feasible solution to the problem in part (a), we must have

$$\frac{P}{P}\left(b^{2} + \sigma\left(P\right)^{2}\right) + \left(1 - \frac{P}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{2} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{*} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{2} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{*} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{*} + \sigma_{p}^{2} = L_{M} \ge \frac{d^{*}}{P}\left(b^{*} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = b^{*} + \sigma_{p}^{2} = L_{M} = \frac{d^{*}}{P}\left(b^{*} + \sigma\left(d^{*}\right)^{2}\right) + \left(1 - \frac{d^{*}}{P}\right)L(b, A) = \frac{d^{*}}{P}\left(b^{*} + \sigma\left(d^{*}\right)^{2}\right)$$

Part (a) shows, however, that  $d^* < P$  if  $P > 2\left[\sqrt{L(b,A) - b^2}\right]$ . Consequently, for this case, the

## **Proof of Proposition 2**

#### **Proposition 2.**

- (i) If the information of management is less important than agency costs ( $\sigma_a < b$ ), then for any  $\sigma_p \in [0,b]$ , shareholder control is optimal ( $\Delta > 0$ ).
- (ii) For every  $\sigma_a \ge b$ , there are two boundaries for  $\sigma_p$ ,  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$ , such that when the importance of the shareholders' information  $\sigma_p \in (\sigma_p^L(\sigma_a), \sigma_p^U(\sigma_a))$ , management control is strictly optimal ( $\Delta < 0$ ). When the importance of the shareholders' information  $\sigma_p > \sigma_p^U(\sigma_a)$ , shareholder control is strictly optimal ( $\Delta > 0$ ). When  $\sigma_p \le \sigma_p^L(\sigma_a)$  or  $\sigma_p = \sigma_p^U(\sigma_a)$ , control is irrelevant ( $\Delta = 0$ ).
- (iii) The functions  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$  satisfy the following properties:  $\sigma_p^L(b) = \sigma_p^U(b) = 0$ , for every  $\sigma_a > b$ ,  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$  are continuous and strictly increasing in  $\sigma_a$ , and  $\sigma_a > \sigma_p^U(\sigma_a) > \sigma_p^L(\sigma_a) > 0$ .

**Proof.** First suppose  $\sigma_a < b$ . In this case, as shown in Proposition 1, d = 0, so

$$\Delta = b^2 + \sigma_p^2 - L(b, A) > \sigma_p^2 > 0,$$

since, from Lemma 1 of Harris and Raviv (2008b),  $\sigma_a < b$  implies that  $L(b, A) < b^2$ . This proves part (i). Henceforth, we assume  $\sigma_a \ge b$ .

Define  $\sigma_p^L$  by

$$f\left(b,\sigma_{p}^{L}\sqrt{12}\right) = \left(\frac{\sigma_{p}^{L}\sqrt{12}}{2} + b\right)^{2} = L\left(b,\sigma_{a}\sqrt{12}\right),$$
(34)

or

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$$\sigma_p^L(\sigma_a) = \frac{2}{\sqrt{12}} \left( \sqrt{L(b, \sigma_a \sqrt{12})} - b \right).$$
(35)

From Lemma 1 of Harris and Raviv (2008b),  $\sigma_a \ge b$  implies that  $\sigma_p^L$  as given in (35) is non-negative.

Since 
$$\sigma_a \le b$$
 implies that  $L(b, A) = L(b, \sigma_a \sqrt{12}) = \sigma_a^2$ , it is obvious from (35) that  $\sigma_p^L(b) = 0$ .

Since *L* is strictly increasing and continuous in its second argument (Lemma 1 of Harris and Raviv (2008b)), it is also obvious from (35) that  $\sigma_p^L(\sigma_a)$  is increasing in  $\sigma_a$ ,  $\forall \sigma_a > b$ , as claimed in part (iii).

Since *f* is clearly increasing in its second argument, (34) implies that  $f(b, \sigma_p \sqrt{12}) \leq L(b, \sigma_a \sqrt{12})$ if and only if  $\sigma_p \leq \sigma_p^L(\sigma_a)$ . Hence, from (20)–(22) and (34),  $d = \min\{\sigma_p^L(\sigma_a)\sqrt{12}, P\}$ . It follows immediately that, for  $\sigma_p \leq \sigma_p^L(\sigma_a)$ , d = P and  $\Delta = 0$ , as claimed in part (ii).

The next step is to develop a formula for  $\sigma_p^U(\sigma_a)$ . Assuming for the time being that

 $\sigma_p^U(\sigma_a) > \sigma_p^L(\sigma_a)$  and using  $d = \min\left\{\sigma_p^L(\sigma_a)\sqrt{12}, P\right\}$ , we can write the condition defining  $\sigma_p^U(\sigma_a)$  as

$$\sigma_p^U \left( R - \left( \sigma_p^U \right)^2 \right) = \sigma_p^L \left( R - \left( \sigma_p^L \right)^2 \right), \tag{36}$$

where  $R = L(b, \sigma_a \sqrt{12}) - b^2$ . Rewrite (36) as

$$\left(\sigma_{p}^{U}\right)^{3}-\left(\sigma_{p}^{L}\right)^{3}-R\left(\sigma_{p}^{U}-\sigma_{p}^{L}\right)=0.$$
(37)

Since we are assuming that  $\sigma_p^U > \sigma_p^L$ , we can divide (37) by  $\sigma_p^U - \sigma_p^L$  to obtain

$$\left(\sigma_p^U\right)^2 + \sigma_p^U \sigma_p^L + \left(\sigma_p^L\right)^2 - R = 0.$$

The solution of this equation of interest to us is given by

$$\sigma_{p}^{U} = \frac{-\sigma_{p}^{L} + \sqrt{4R - 3(\sigma_{p}^{L})^{2}}}{2}.$$
(38)

For  $\sigma_p^U$  to be given by equation (38), we need only show that this value exceeds  $\sigma_p^L$ . For this, it suffices to show that  $R > 3(\sigma_p^L)^2$ . But from (35) and the definition of *R*, we have that  $R > 3(\sigma_p^L)^2$  if and

only if  $\sqrt{L(b,\sigma_a\sqrt{12})} + b > \sqrt{L(b,\sigma_a\sqrt{12})} - b$ , which is clearly true, since b > 0. Consequently, we have shown that  $\sigma_p^U$  is indeed given by equation (38) and that  $\sigma_p^U(\sigma_a) > \sigma_p^L(\sigma_a)$ , as claimed in (iii). For  $\sigma_a = b$ ,  $R = \sigma_a^2 - b^2 = 0$ , and, as shown previously,  $\sigma_p^L = 0$ . Consequently,  $\sigma_p^U(b) = 0$ , as claimed in (iii).

To show that  $\sigma_p^U(\sigma_a) < \sigma_a$ , from (38), it suffices to show that

$$R = L\left(b, \sigma_a \sqrt{12}\right) - b^2 < \sigma_a^2 + \sigma_a \sigma_p^L + \left(\sigma_p^L\right)^2.$$
(39)

It is easy to check that  $L(b, \sigma_a \sqrt{12}) \le \sigma_a^2$ .<sup>1</sup> Therefore (39) is clearly satisfied since *b*,  $\sigma_a$ , and  $\sigma_p^L$  are all positive.

To complete the proof of part (iii), it remains to show that  $\sigma_p^U(\sigma_a)$  is continuous and strictly increasing in  $\sigma_a$ . For this, it suffices to show that  $\sigma_p^U(\sigma_a)$  is continuous and strictly increasing in  $\sqrt{L(b,\sigma_a\sqrt{12})}$ . To make the formulas easier to read, let  $z = \sqrt{L(b,\sigma_a\sqrt{12})}$ . Then, substituting for *R* and  $\sigma_p^L$  in (38), we have

$$\sigma_{p}^{U} = \frac{1}{2} \left[ -\frac{2}{\sqrt{12}} (z-b) + \sqrt{3(z^{2}-b^{2}) + 2b(z-b)} \right].$$
(40)

It is clear from (40) that  $\sigma_p^U$  is continuous in z. It is easy to check that the derivative of the right hand side of (40) with respect to z is positive if and only if  $3z^2 + 2bz + b^2 > 0$ , which is clearly true. This completes the proof of part (iii).

<sup>1</sup> If  $N(b, \sigma_a \sqrt{12}) = 1$ , then  $L(b, \sigma_a \sqrt{12}) = \sigma_a^2$ , and we are done. Suppose  $N(b, \sigma_a \sqrt{12}) = n \ge 2$ . Then  $L(b, \sigma_a \sqrt{12}) = \frac{\sigma_a^2}{n^2} + \frac{b^2(n^2 - 1)}{3} < \sigma_a^2$  if and only if  $\sigma_a^2 > \frac{(bn)^2}{3}$ . But, as shown in Lemma 1 of Harris and Raviv (2008b),  $N(b, \sigma_a \sqrt{12}) = n$  implies that A > 2bn(n-1) or  $\sigma_a^2 > \frac{(bn(n-1))^2}{3} \ge \frac{(bn)^2}{3}$ , for  $n \ge 2$ . To complete the proof of part (ii), we must show that  $\Delta < 0$  for  $\sigma_p^L(\sigma_a) < \sigma_p < \sigma_p^U(\sigma_a)$ , and  $\Delta > 0$  for  $\sigma_p > \sigma_p^U(\sigma_a)$ . It is easy to check that  $\Delta$  is convex in  $\sigma_p$  for  $\sigma_p \ge \sigma_p^L$ . Since  $\Delta \equiv 0$  for  $\sigma_p \le \sigma_p^L$ ,  $\Delta$  can cross zero at most once at some  $\sigma_p > \sigma_p^L$  and only from below (see Figure 6). Consequently, this must occur at  $\sigma_p^U$ , and  $\Delta < 0$  for  $\sigma_p^L(\sigma_a) < \sigma_p < \sigma_p^U(\sigma_a)$ ,  $\Delta > 0$  for  $\sigma_p > \sigma_p^U(\sigma_a)$ , and  $\Delta = 0$  for  $\sigma_p = \sigma_p^U(\sigma_a)$  as claimed. Q.E.D.



Importance of Shareholders' Information

#### Figure 6

This graph shows the net gain to shareholder control,  $\Delta$ , as a function of the importance of shareholders' information. Shareholder control is optimal whenever the importance of shareholders' information,  $\sigma_p$ , exceeds  $\sigma_p^U \approx 3.45$ . For this figure, b = 4,  $\sigma_a = 8$ , and  $\sigma_p^L = 0.95658525 < b$ .

### **Proof of Proposition 3**

#### **Proposition 3.**

- An increase in the importance of shareholders' information can result in a shift from management control being strictly optimal to shareholder control being strictly optimal but not the reverse. Similarly, an increase in the importance of management's information can result in a shift from shareholder control being strictly optimal to management control being strictly optimal but not the reverse.
- Suppose agency costs increase from b = b<sub>L</sub> to b = b<sub>H</sub> with b<sub>H</sub> > b<sub>L</sub> > 0. There exists a largest value of σ<sub>a</sub>, denoted b̄, with b̄ > b<sub>H</sub> (b̄ may be infinite), such that whenever (σ<sub>a</sub>, σ<sub>p</sub>) ∈ [b<sub>L</sub>, b̄]×[σ<sup>U</sup><sub>p</sub>(σ<sub>a</sub>, b<sub>H</sub>), σ<sup>U</sup><sub>p</sub>(σ<sub>a</sub>, b<sub>L</sub>)], the increase in agency costs results in a shift from management control being optimal to shareholder control being strictly optimal. If b̄ is finite, then there is also a region of values of (σ<sub>a</sub>, σ<sub>p</sub>) in which the increase in agency costs results in a shift from shareholder control being strictly optimal to management control being strictly optimal to management control being strictly optimal. If b̄ is results in a shift from shareholder control being strictly optimal to management control being strictly optimal. This case does occur for some values of b<sub>L</sub> and b<sub>H</sub>.

**Proof.** The first part of the proposition follows immediately from Proposition 2.

For the second part, we define  $\sigma_p^U$  to be zero for  $\sigma_a \leq b$ . Since  $\sigma_p^U$  is strictly increasing in  $\sigma_a$ for  $\sigma_a \geq b$  (see Proposition 2),  $\sigma_p^U(\sigma_a, b_L) > \sigma_p^U(\sigma_a, b_H)$  for  $\sigma_a \in (b_L, b_H]$ . Consequently, since  $\sigma_p^U$  is continuous in  $\sigma_a$  (see Proposition 2),  $\sigma_p^U(\sigma_a, b_L) > \sigma_p^U(\sigma_a, b_H)$  for some interval of values of  $\sigma_a \geq b_H$ . If, for some values of  $\sigma_a > b_H$ ,  $\sigma_p^U(\sigma_a, b_L) \leq \sigma_p^U(\sigma_a, b_H)$ , define  $\overline{b}$  to be the minimum such value (the minimum exists, since, by continuity of  $\sigma_p^U$  in  $\sigma_a$ , the set of values of  $\sigma_a > b_H$  such that  $\sigma_p^U(\sigma_a, b_L) \leq \sigma_p^U(\sigma_a, b_H)$  is closed and bounded below). If  $\sigma_p^U(\sigma_a, b_L) > \sigma_p^U(\sigma_a, b_H)$  for all  $\sigma_a > b_L$ , define  $\overline{b} = \infty$ . Consequently,  $[b_L, \overline{b}] \times [\sigma_p^U(\sigma_a, b_H), \sigma_p^U(\sigma_a, b_L)] \neq \emptyset$ . The example in Figure 2 shows that there are values of  $b_L$  and  $b_H$  for which  $\overline{b}$  is finite. The result now follows from Proposition 2.

Q.E.D.

## **Proof of Lemma 3**

Lemma 3. If shareholders control the decision, the ex ante expected loss is given by

$$L_{s} = \varphi\left(\sigma_{p}^{2} + b^{2}\right) + \left(1 - \varphi\right) \left[\sigma_{p}^{2} + L(b, A) + \sigma_{q}^{2} + \overline{e}^{2} - \frac{N(b, A) - 1}{N(b, A)}\overline{e}\left(\overline{e} + 2b\right)\right],\tag{41}$$

where  $\varphi = \Pr(\tilde{q} \ge p^*)$  is the probability of delegation.

**Proof.** Management is indifferent among all reports in a given partition cell, as in the base case of section 3. Thus, given a realization *a* of  $\tilde{a}$ , management chooses a report in  $[a_{i-1}, a_i]$  if and only if *i* solves

$$\min_{j \in \{1,...,N\}} E\left\{ \left[ \overline{a}_j + \tilde{q} - \left(a + b + \tilde{p}\right) \right]^2 \middle| \tilde{q} \le p^* \right\},\$$

where N = N(b, A). It is easy to check that solving this problem is equivalent to solving

$$\min_{j \in \{1,\dots,N\}} \left[ \overline{a}_j - \left( a + b - \overline{e} \right) \right]^2.$$
(42)

Suppose  $a + b - \overline{e}$  is equidistant from  $\overline{a}_i$  and  $\overline{a}_{i+1}$ . By construction of  $\{a_i\}$ , however,  $a_i + b$  is equidistant from  $\overline{a}_i$  and  $\overline{a}_{i+1}$ . Therefore, we must have  $a + b - \overline{e} = a_i + b$  or  $a = a_i + \overline{e}$ . It follows that the solution of (42) is the value of *i* such that  $a \in [\alpha_{i-1}, \alpha_i]$ , where, if  $\overline{e} \ge 0$ ,

$$\alpha_0 = a_0 = 0,$$

$$\alpha_i = \min\{a_i + \overline{e}, A\}, i \in \{1, \dots, N\},$$
(43)

and if  $\overline{e} < 0$ ,

$$\alpha_N = a_N = A,$$
  

$$\alpha_i = \max\left\{a_i + \overline{e}, 0\right\}, i \in \{0, \dots, N-1\}.$$
(44)

The expected loss in value if shareholders are in control, given  $\tilde{q} = q < p^*$  (so they do not delegate), is then

$$\frac{1}{AP} \int_{0}^{P} \left[ \sum_{i=1}^{N} \int_{\alpha_{i-1}}^{\alpha_{i}} \left[ \overline{a}_{i} + q - (a+p) \right]^{2} da \right] dp \qquad (45)$$

$$= \sigma_{p}^{2} + \left(q - \overline{p}\right)^{2} + \frac{1}{A} \sum_{i=1}^{N} \left(\alpha_{i} - \alpha_{i-1}\right) \left[ 2\left(q - \overline{p}\right)\left(\overline{a}_{i} - \overline{\alpha}_{i}\right) + \left(\overline{a}_{i} - \overline{\alpha}_{i}\right)^{2} + \sigma\left(\alpha_{i} - \alpha_{i-1}\right)^{2} \right],$$

where  $\overline{\alpha}_i = (\alpha_{i-1} + \alpha_i)/2$ . Taking the expectation of the right-hand side of (45) with respect to q, given that  $q < p^*$ , we see that the expected loss in value if shareholders are in control and do not delegate is

$$\sigma_p^2 + \sigma_q^2 + \overline{e}^2 + \frac{1}{A} \sum_{i=1}^N (\alpha_i - \alpha_{i-1}) \Big[ 2\overline{e} (\overline{a}_i - \overline{\alpha}_i) + (\overline{a}_i - \overline{\alpha}_i)^2 + \sigma (\alpha_i - \alpha_{i-1})^2 \Big].$$
(46)

Condition (13) implies that  $\alpha_i = a_i + \overline{e}$  for  $i \in \{1, ..., N-1\}$ . It then follows from (46) that shareholders' expected loss in value if they are in control and do not delegate is

$$\sigma_p^2 + L(b,A) + \sigma_q^2 + \overline{e}^2 - \frac{N(b,A) - 1}{N(b,A)} \overline{e} \left(\overline{e} + 2b\right).$$

$$\tag{47}$$

This results in the expression for the *ex ante* loss to shareholders as stated in the lemma.

## **Proof of Proposition 5**

**Lemma 2.** Define 
$$\beta_0 = \frac{1}{2} \left( b - \sqrt{b^2 + 2\sigma_p^2} \right)$$
. Then  $\beta_0 < 0$  and if  $b > \sigma_p/2$ , then  $b - \beta_0 < 2b$ .

**Proof.** It is obvious that  $\beta_0 < 0$  and that  $-b < \beta_0$  if and only if  $b > \sigma_p/2$ . Clearly,  $b - \beta_0 < 2b$  if and only if  $-b < \beta_0$ .

**Proposition 5** (see Figure 5). Assume  $b - \overline{p} > \sigma_p$ . Then there exist continuous functions,

 $G(\beta)$ , with G symmetric with respect to  $\beta = b$ , and  $H(\beta)$  such that  $H(\overline{p}) = G(\overline{p})$ ,  $H(\beta) = 0$ 

for  $\beta \leq \beta_s$ ,  $H(\beta)$  is increasing in  $\beta$  for  $\beta \leq b/2$ , and

- shareholders, when in control, always delegate to management and control is irrelevant whenever σ<sub>a</sub><sup>2</sup> ≥ G(β);
- for decisions for which  $\beta > b + \sigma_p$ , management control is strictly optimal if

 $\sigma_a^2 < G(\beta);$ 

- for decisions for which  $b \sigma_p > \beta > \overline{p}$ , shareholder control is strictly optimal if  $\sigma_a^2 < G(\beta)$ ;
- for decisions for which p̄ > β, shareholder-control is strictly optimal if σ<sub>a</sub><sup>2</sup> < H(β) and management control is strictly optimal if H(β) < σ<sub>a</sub><sup>2</sup> < G(β); management control is also strictly optimal for σ<sub>a</sub><sup>2</sup> = H(β) = 0 when β < β<sub>s</sub>.

**Proof.** Define g(B) as the value of  $\sigma_a^2$  such that  $L(B, \sigma_a \sqrt{12}) = \left(\frac{P}{2} + |B|\right)^2$ . It is easy to check, using the facts that  $L(B, \sigma_a \sqrt{12}) = \sigma_a^2$  for  $\sigma_a^2 \le B^2$  and  $L(B, \sigma_a \sqrt{12}) \to \infty$  as  $\sigma_a \to \infty$ , and L is continuous in its second argument (see Lemma 1 in Harris and Raviv (2008b)), that such a value of  $\sigma_a^2$ exists and is larger than  $B^2$ . Thus  $g(B) > B^2$  for all B. Since L depends on B only through  $B^2$ , and  $\left(\frac{P}{2}+|B|\right)^2$  depends on *B* only through |B|, *g* is symmetric with respect to B=0. Finally, since *L* and  $\left(\frac{P}{2}+|B|\right)^2$  are continuous in B (for  $B \neq 0$ ), so is g. From the definition of g, the fact that L is increasing in its second argument (see Lemma 1 in Harris and Raviv (2008b)), and Proposition 1, for  $\sigma_a^2 \ge g(B)$ , d = P, i.e., shareholders, if in control, always delegate to management. In this case, control is irrelevant, since management always makes the decision with no information from shareholders, regardless of control, i.e., (19) is satisfied as an equality. Define  $G(\beta) = g(b - \beta)$ . Therefore, for  $\sigma_a^2 \ge G(\beta)$ , shareholders, if in control, always delegate to management and, control is irrelevant, as claimed in the first bullet. Moreover, G is continuous and symmetric with respect to  $\beta = b$ . For the remainder of the proof, we consider only the case in which  $\sigma_a^2 < G(\beta)$  or, equivalently,  $\sigma_a^2 < g(B)$ .

We split the remainder of the proof into two cases,  $\sigma_a^2 \le B^2$  and  $\sigma_a^2 > B^2$ .

**Case 1:**  $\sigma_a^2 \leq B^2$ . In this case, neither party will delegate to the other nor will they communicate any of their private information. Therefore, condition (19) becomes  $\beta^2 + \sigma_a^2 \leq b^2 + \sigma_p^2$ , which can be rewritten in terms of the net bias, *B* as

$$\sigma_a^2 \le \sigma_p^2 - \left(B^2 - 2bB\right). \tag{48}$$

Define  $h_1(B) = \sigma_p^2 - (B^2 - 2bB)$ . Clearly, (48) is satisfied if and only if  $\sigma_a^2 \le h_1(B)$ . If  $B < -\sigma_p$ , or, equivalently,  $\beta > b + \sigma_p$ , then  $B^2 - 2bB > B^2 \ge \sigma_p^2$ , since B < 0, so  $h_1(B) < 0$  for *B* in this range. Consequently, for  $\beta > b + \sigma_p$ , (48) cannot be satisfied, and management control is optimal.

At this point, it is convenient to redefine  $h_1(B) = \max \{\sigma_p^2 - (B^2 - 2bB), 0\}$  If  $B > \sigma_p$ , it is easy to check that  $h_1(B) \le B^2$  if and only if  $B \ge b - \beta_0$  with equality if and only if  $B = b - \beta_0$ . Note that the assumption that  $b - \overline{p} > \sigma_p$  implies that  $b > \sigma_p/2$  which implies that  $0 < b - \beta_0 < 2b$  by Lemma 2. Moreover,  $h_1$  is strictly decreasing in *B* for  $B_0 > B \ge b - \beta_0$ , where  $B_0 = b + \sqrt{b^2 + \sigma_p^2}$ , and  $h_1(B) \equiv 0$ , for all  $B \ge B_0$ . Thus, for  $\sigma_a^2 \le B^2$ , management control is strictly optimal for  $B < -\sigma_p$  (as shown above) and for  $B > b - \beta_0$  if and only if  $\sigma_a^2 > h_1(B)$ . For  $\sigma_p \le B < b - \beta_0$ , shareholder control is optimal for all  $\sigma_a^2 \le B^2 \le h_1(B)$ . This completes the characterization of optimal control for  $\sigma_a^2 \le B^2$ . Define  $H_1(\beta) = h_1(b - \beta)$ . Thus  $H_1$  is strictly increasing in  $\beta$  for  $\beta_s < \beta \le \beta_0$  and is identically zero for  $\beta \le \beta_s = b - B_0 = -\sqrt{b^2 + \sigma_p^2}$ .

**Case 2:**  $\sigma_a^2 > B^2$ . In this case  $d \in (0, P)$  (recall we assume  $\sigma_a^2 < g(B)$ ), so we have

$$L(B,A) = f(B,d) = \left(\frac{d}{2} + |B|\right)^2.$$
 Substituting  $\left(\frac{d}{2} + |B|\right)^2$  for *L*, we can write the left hand side of (19) as  
$$F(d) = \frac{d}{P}b^2 + \left(1 - \frac{d}{P}\right)\beta^2 + \frac{d}{P}\sigma(d)^2 + \left(1 - \frac{d}{P}\right)\left(\frac{d}{2} + |B|\right)^2.$$

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We claim that the function F is strictly concave in d on (0, P). To see this, note that

$$F''(d) = \frac{1}{2} - \frac{2}{P} \left( \frac{d}{2} + |B| \right) < \frac{1}{2} - \frac{2|B|}{P}.$$

But  $P = \sqrt{12}\sigma_p \le \sqrt{12}|B| < 4|B|$ . Consequently, F''(d) < 0 and *F* is strictly concave as claimed. Also  $F(0) = B^2 + \beta^2 = b^2 + 2B(B-b)$ , and  $F(P) = \sigma_p^2 + b^2 = \text{right hand side of (19)}$ .

Since *F* is strictly concave and  $F(P) = \sigma_p^2 + b^2$ , if  $F(0) \ge \sigma_p^2 + b^2$ , then  $F(d) > \sigma_p^2 + b^2$  for all  $d \in (0, P)$ . Therefore, in this case (19) is false, i.e., management control is strictly optimal, for all  $\sigma_a^2 < g(B)$ , or, equivalently, for all  $\sigma_a^2 < G(\beta)$ .

Now suppose  $\beta > b + \sigma_p$ , or, equivalently,  $B < -\sigma_p$ . Then,  $2B(B-b) > B^2 \ge \sigma_p^2$ , so

 $F(0) > \sigma_p^2 + b^2$ . Consequently, management control is strictly optimal for all  $B^2 < \sigma_a^2 < g(B) = G(\beta)$ . Together with the previous result that management control is strictly optimal when  $\beta > b + \sigma_p$  for all  $\sigma_a^2 \le B^2$ , we have completed the proof of the second bullet.

Next, suppose  $B > \sigma_p$ , or, equivalently,  $\beta < b - \sigma_p$ . Then it is easy to check that  $F(0) < \sigma_p^2 + b^2$ if and only if  $B < b - \beta_0$ . Consequently, if  $B \ge b - \beta_0$ , or, equivalently,  $\beta \le \beta_0$ ,  $F(0) \ge \sigma_p^2 + b^2$ , so management control is strictly optimal. Now consider  $\sigma_p < B < b - \beta_0$  (equivalently  $b - \sigma_p > \beta > \beta_0$ ), so that  $F(0) < \sigma_p^2 + b^2$ . There are two possible cases. Since *F* is strictly concave, if  $F'(P) \ge 0$ , then  $F(d) < \sigma_p^2 + b^2$  for all  $d \in (0, P)$  which implies that it is strictly optimal for shareholders to control regardless of the value of  $B^2 < \sigma_a^2 < g(B)$ . If F'(P) < 0, then there exists a unique  $d_0 \in (0, P)$  such that  $F(d) \le \sigma_p^2 + b^2$  for all  $d \le d_0$ ,  $F(d) > \sigma_p^2 + b^2$  for all  $d > d_0$ , and  $F'(d_0) > 0$ . That is, it is optimal for shareholders to control if and only if  $d \le d_0$ . But *d* is a continuous, increasing function of  $\sigma_a^2$ , d = 0 for  $\sigma_a^2 \le B^2$ , and d = P for  $\sigma_a^2 \ge g(B)$ , so, for each *B* such that F'(P) < 0, there is a unique value of  $\sigma_a^2 \in (B^2, g(B))$  such that  $d = d_0$  for that value of  $\sigma_a^2$ . Define  $h_0(B)$  to be the value of  $\sigma_a^2$  for which  $d = d_0$  for *B* such that F'(P) < 0. For such *B*, shareholder control is strictly optimal for  $\sigma_a^2 < h_0(B)$ , and management control is optimal for  $\sigma_a^2 > h_0(B)$ . It is clear from the construction that  $g(B) > h_0(B) > B^2$ . Since *F*, *L*, and *f* are continuous in *B*, so is  $h_0$ .

It is easy to check that

$$F'(P) = \frac{1}{\overline{p}}(b - \overline{p} - B)B$$
.

Consequently, F'(P) < 0 for  $b - \overline{p} < B < b - \beta_0$ , and  $F'(P) \ge 0$  for  $\sigma_p \le B \le b - \overline{p}$ . It follows that shareholder control is strictly optimal for  $\sigma_p \le B \le b - \overline{p}$  and  $B^2 < \sigma_a^2 < g(B)$  and for  $b - \overline{p} < B < b - \beta_0$ with  $B^2 < \sigma_a^2 < h_0(B)$ , while management control is strictly optimal for  $b - \overline{p} < B < b - \beta_0$  for  $h_0(B) < \sigma_a^2 < g(B)$ . Define  $H_0(\beta) = h_0(b - \beta)$ . Restated in terms of  $\beta$ , we have shown that shareholder control is strictly optimal for  $b - \sigma_p \ge \beta \ge \overline{p}$  and  $B^2 < \sigma_a^2 < G(\beta)$  and for  $\overline{p} > \beta > \beta_0$  with  $B^2 < \sigma_a^2 < H_0(\beta)$ , while management control is strictly optimal for  $\overline{p} > \beta > \beta_0$  for  $H_0(\beta) < \sigma_a^2 < G(\beta)$ .

Now, since F' is continuous and F'(P) = 0 for  $B = b - \overline{p}$ ,  $d_0 \uparrow P$  as  $B \downarrow b - \overline{p}$ . But, for any *B*, g(B) is the smallest value of  $\sigma_a^2$  such that d = P. Consequently,  $h_0(B) \rightarrow g(b - \overline{p})$  as  $B \downarrow b - \overline{p}$ . Define  $h_0(b - \overline{p}) = g(b - \overline{p})$ , so  $h_0$  is continuous at  $b - \overline{p}$ . Moreover, as  $B \uparrow b - \beta_0$ ,  $d_0 \downarrow 0$ , so  $h_0(B) \rightarrow B^2 = h_1(b - \beta_0)$ . Therefore, define

$$h(B) = \begin{cases} h_0(B) \text{ for } B \in [b - \overline{p}, b - \beta_0], \\ h_1(B) \text{ for } B \ge b - \beta_0. \end{cases}$$

Then h is continuous in B,  $h(b-\overline{p}) = g(b-\overline{p})$ , and h(B) = 0 for all  $B \ge B_0$ .

Finally, note that, for B > 0,

$$\frac{\partial F(d)}{\partial B} = 2\left(1 - \frac{d}{P}\right)\left(2B - b + \frac{d}{2}\right).$$

Consequently,  $\frac{\partial F(d)}{\partial B} > 0$  for all  $d \in (0, P)$  if  $B \ge b/2$ . Since  $F'(d_0) > 0$ , it follows that, for  $B \ge b/2$ ,

 $d_0$  is strictly decreasing in *B*. Therefore, so is  $h_0(B)$ . Since we have already shown that  $h_1(B)$  is strictly decreasing in *B* for  $B_0 > B \ge b - \beta_0$ , h(B) is strictly decreasing in *B* for  $B_0 > B \ge b/2$ .

Finally, define  $H(\beta) = h(b - \beta)$ . Then *H* is continuous in  $\beta$ ,  $H(\overline{p}) = G(\overline{p})$ ,  $H(\beta) = 0$  for all  $\beta \le \beta_s$ , and  $H(\beta)$  is strictly increasing in  $\beta$  for  $\beta_s < \beta \le b/2$ . Q.E.D.

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