

# Information Quality and the Cross-Section of Expected Returns\*

Chris Armstrong<sup>†</sup>      Snehal Banerjee<sup>‡</sup>      Carlos Corona<sup>§</sup>

November 2010

## Abstract

When investors learn about *both* systematic risk factors and firm-specific factor loadings, we show that information quality can generate cross-sectional variation in expected returns not captured by factor loadings. Firms with higher firm-specific information quality earn higher expected returns *after* controlling for average factor loadings, and this difference in expected returns decreases with aggregate information quality. Using proxies based on analyst forecasts, idiosyncratic volatility, and standard errors of beta estimates, we find that stocks in the highest information quality decile earn an average of 89 to 191 basis points per month more than stocks in the lowest decile after controlling for the Fama-French-Carhart risk factors and the Pastor-Stambaugh liquidity factor, and that these average returns are negatively related to aggregate information quality (as proxied for by the VIX).

---

\*We have benefited from comments from Andres Almazan, Ryan Ball, John Core, Lorenzo Garlappi, Mike Faulkender, Mike Fishman, Yaniv Konchitchki, Bob Korajczyk, Arvind Krishnamurthy, Christian Leuz, Dimitris Papanikolaou, Jonathan Parker, Lubos Pástor, Mitchell Petersen, Ernst Schaumburg, Viktor Todorov, Laura Veldkamp, Pietro Veronesi and seminar participants at the University of Toronto, Loyola University, the 2009 Financial Economics and Accounting Conference, and the 2010 Annual AFA Meetings. All errors are our own. A previous version of this paper was circulated under the title “Information Quality, Systematic Risk and the Cost of Capital.”

<sup>†</sup>The Wharton School, University of Pennsylvania, carms@wharton.upenn.edu

<sup>‡</sup>Kellogg School of Management, Northwestern University, snehal-banerjee@kellogg.northwestern.edu

<sup>§</sup>McCombs School of Business, University of Texas, Carlos.Corona@mcombs.utexas.edu

Does the quality of firm-specific information affect the cross-section of expected returns? In a model where investors are uncertain about *both* the aggregate risk-factors in the economy and firm-specific factor loadings, we show that firm-specific information quality can affect the cross-section of expected returns, even after controlling for the average level of factor loadings. Moreover, the effect of firm-specific information quality depends on the quality of the aggregate information about the systematic risk-factors. In particular, we show that after controlling for average loadings, firms with higher firm-specific information quality have higher expected returns and that the difference in expected returns decreases with aggregate information quality.

We test the model's predictions using hedge portfolios formed by sorting stocks on various proxies for firm-specific information quality based on analyst forecasts, idiosyncratic volatility, and the standard errors of beta estimates. In our sample from January 1986 through December 2008, we find that high information quality firms have higher returns on average than low information quality firms, and that this return difference is negatively related to aggregate information quality (measured using a proxy based on the VIX). These results persist even after controlling for standard equity risk-factors (i.e., the Fama-French-Carhart factors) and liquidity risk (using the Pastor-Stambaugh liquidity factor), and are economically significant. When sorted on our proxies for firm-specific information quality, the average return for the highest decile portfolio is 89 to 191 basis points per month higher than the average return for the lowest decile portfolio, even after controlling for the previously mentioned risk factors.

The setup of our model is fairly standard. We assume an exogenously specified stochastic discount factor, or pricing kernel, that is log-normally distributed. Each firm generates cash-flows that grow stochastically, and the growth rate is correlated with the risk-factor driving the pricing kernel. Hence, the expected return on a firm's stock depends on investors' beliefs about the risk-factor and the factor-loading of the firm's cash-flows. In contrast to much of the prior literature, however, we assume that investors are uncertain not only about future realizations of the aggregate risk-factor, but also about each firm's future factor loadings. As a result, investors use information to update their beliefs about the factor-loadings of cash-flows. This implies that firm-specific information is not idiosyncratic and cannot be diversified away.

The intuition for our results relies on the fact that the price of a stock's stream of cash-flows is a convex function of its growth rate adjusted for its covariance with the aggregate risk-factor. This implies that after controlling for the *average level* of factor loadings, prices are increasing in, and expected returns are decreasing in, the *uncertainty* about these loadings.

Since cash-flow growth is driven by the product of the firm's factor loading and the aggregate risk-factor, the effect of this firm-specific uncertainty depends on its interaction with the uncertainty about the risk-factor. Higher quality firm-specific information decreases the uncertainty in factor loadings and therefore increases the expected return, especially when aggregate information quality is low (i.e., aggregate uncertainty is high).

Our primary contribution is to emphasize that the *interaction* between firm-specific and aggregate information quality is important for understanding the cross-section of expected returns. In particular, aggregate information quality explains variation in the cross-section of expected returns, not through its own factor loading, but instead through its interaction with firm-specific information quality. This distinguishes our model from those in which aggregate information quality (or aggregate uncertainty) is a priced risk-factor. Second, we derive, and find evidence consistent with, a novel empirical prediction about the interaction between firm-specific and aggregate information quality that, to the best of our knowledge, has not been yet tested in the literature. Finally, our model provides a *common* theoretical basis for interpreting a number of empirical regularities between various firm characteristics (e.g. uncertainty in betas, idiosyncratic volatility, mean forecast errors across analysts) and expected returns that have been documented in prior literature. As we discuss in the next section, if one interprets analyst forecast dispersion and idiosyncratic volatility as being negatively related to firm-specific information quality, our model provides a possible explanation for the seemingly conflicting evidence about the cross-sectional relation between these variables and expected returns documented in the empirical literature.

The next section discusses some of the related theoretical and empirical literature. Section 2 presents the theoretical results of the paper and develops the empirical predictions of the model that we test. Section 3 presents the empirical analysis, with a description of the empirical proxies and the data in Section 3.1 and a discussion of the results in Section 3.2. Section 4 concludes the paper.

## 1 Related Literature

There are two standard approaches to modeling the cross-sectional relation between information quality and expected returns. The first approach considers this relation in single firm models, and generally concludes that increasing the quality of public information reduces informational asymmetry which, in turn, increases liquidity and therefore reduces the cost of capital (e.g., [Diamond and Verrecchia \(1991\)](#), [Baiman and Verrecchia \(1996\)](#), [Easley](#)

and O’Hara (2004)). However, the empirical evidence on this monotonic relation has been mixed.<sup>1</sup> Moreover, it is not clear that the intuition from these single-firm models extends to a large economy with multiple firms, since firm-specific information should be diversified away (e.g., Hughes, Liu, and Liu (2007)). The second approach considers the effect of learning about the aggregate risk-factors on the cross-section of expected returns (e.g., Veronesi (2000)).<sup>2</sup> In these models, the effect of information quality on the cross-section of expected returns is through the aggregate risk-premium, and therefore is largely captured by firm-specific factor loadings.

Our model provides novel implications for the cross-sectional relation between information quality and expected returns, since investors learn about both the aggregate risk-factor and firm-specific factor loadings. In contrast to single firm models, firm-specific information affects investors’ beliefs about systematic factor loadings and hence cannot be diversified away. Moreover, we show that the interaction between firm-specific and aggregate information quality has an important effect on the cross-section of expected returns. Therefore, in contrast to models in which investors only learn about the aggregate risk-factors, we show information quality can affect the cross-section of expected returns even after controlling for firm-specific betas. Finally, as we discuss in Section 2, there are two, potentially offsetting, effects of firm-specific information quality on expected returns in our model. This suggests a possible explanation for why empirical tests that only allow for a monotonic relation between expected returns and information quality in the prior literature have failed to provide consistent evidence.

Our model is most closely related to those in Pástor and Veronesi (2003) and Johnson (2004), but our results differ significantly from theirs. Pástor and Veronesi (2003) show that since a firm’s stock price is convex in its cash-flow growth, higher uncertainty about profitability leads to higher market-to-book ratios. Johnson (2004) argues that for unlevered firms, this has no effect on expected returns, but that in a levered firm, an increase in idiosyncratic volatility (which increases total volatility but keeps the risk premium constant)

---

<sup>1</sup>On the one hand, a number of papers including Easley and O’Hara (2004), Francis, LaFond, Olsson, and Schipper (2004), Francis, LaFond, Olsson, and Schipper (2005), Barth, Konchitchki, and Landsman (2007), and Francis, Nanda, and Olsson (2008) document that proxies of higher information quality or increased transparency are associated with lower expected returns. On the other hand, papers including Botosan (1997), Botosan and Plumlee (2002), Core, Guay, and Verdi (2008), and Duarte and Young (2009), find either limited or no evidence of a relation between information quality or disclosure and cost of capital.

<sup>2</sup>A number of recent papers, including Li (2005), Brevik and D’Addona (2005), Ai (2009), Gollier and Schlee (2009), and Croce, Lettau, and Ludvigson (2009) consider the effect of aggregate uncertainty on the aggregate equity risk premium, by extending the model in Veronesi (2000) to more general preferences and information environments.

decreases the expected return on levered equity.<sup>3</sup> In our model, since the firm-specific information is about factor loadings and not the cash-flows themselves, the effect of firm-specific information quality on expected returns does not rely on leverage. Moreover, unlike the effects in [Pástor and Veronesi \(2003\)](#) and [Johnson \(2004\)](#), we find that the relation between firm-specific information quality and expected returns is not monotonic but instead depends on the quality of aggregate information available to investors.

Our paper sheds light on the apparently conflicting empirical evidence about the relation between expected returns and firm-level proxies of uncertainty like idiosyncratic volatility and analyst forecast dispersion.<sup>4</sup> The empirical evidence for these relations is mixed. For instance, on the one hand, [Diether, Malloy, and Scherbina \(2002\)](#), [Johnson \(2004\)](#), [Goetzmann and Massa \(2005\)](#), and [Zhang \(2006\)](#) document a negative relation between forecast dispersion and expected returns, which is consistent with the [Miller \(1977\)](#) hypothesis. On the other hand, [Qu, Starks, and Yan \(2004\)](#) and [Banerjee \(2010\)](#) document a positive relationship between expected returns and dispersion, while [Anderson, Ghysels, and Juergens \(2005\)](#) find evidence of a negative relationship between expected returns and short-term dispersion, but a positive relationship between expected returns and long-term dispersion. Similarly, while [Lehmann \(1990\)](#), [Malkiel and Xu \(2002\)](#), [Fu \(2009\)](#), [Huang, Liu, Rhee, and Zhang \(2009\)](#), and [Spiegel and Wang \(2010\)](#) document a positive relation between idiosyncratic volatility and expected returns that is consistent with the [Merton \(1987\)](#) model, [Ang, Hodrick, Xing, and Zhang \(2006\)](#), [Ang, Hodrick, Xing, and Zhang \(2009\)](#) and others document a negative relationship. Our model provides a complementary mechanism through which uncertainty may affect expected returns, and suggests that this relation is non-monotonic and depends on the interaction between firm-specific and aggregate information. Empirically, we find that firms with higher information quality (low uncertainty) earn higher returns after controlling for firm-specific betas, but that these higher returns decrease non-linearly with aggregate information quality. Our analysis suggests that one must account for the variation in aggregate uncertainty when measuring the effect of firm-specific uncertainty on expected returns.

Our paper is also related to the literature on estimation risk. The early literature in this area (e.g., [Brown \(1979\)](#), [Bawa and Brown \(1979\)](#)) suggests that estimation risk should be diversifiable and therefore not priced. However, subsequent work has shown that the

---

<sup>3</sup>This is because the delta of the levered equity relative to the underlying assets of the firm is decreasing in the volatility of the assets of the firm.

<sup>4</sup>Note that while others have posited a relationship between analyst forecast dispersion and firm-specific uncertainty (e.g., [Johnson \(2004\)](#), [Qu, Starks, and Yan \(2004\)](#)), dispersion may also be affected by asymmetric information or heterogeneous beliefs (e.g., [Banerjee \(2010\)](#)). Instead of using the dispersion in analyst forecasts, we use measures of average prediction errors as proxies for firm-specific information quality.

availability of different amounts of information across securities may yield a non-diversifiable effect on equilibrium prices, even in a CAPM setting (e.g., [Barry and Brown \(1985\)](#), [Clarkson and Thompson \(1990\)](#)). More recently, [Kumar, Sorescu, Boehme, and Danielsen \(2008\)](#) extend this literature by proposing a conditional CAPM in which investors are uncertain about the higher moments of the distribution of returns and information signals. In another recent paper, [Adrian and Franzoni \(2008\)](#) extend the conditional CAPM by introducing unobservable long-run changes in conditional betas. Instead of developing predictions about specific models of estimation risk and learning about betas, we pursue a complementary approach by studying the effects of learning about betas and risk factors jointly, and show that the effect of estimation risk may depend on the extent to which investors learn about the risk factors.

## 2 Theory

### 2.1 Model Setup

We develop a standard model in which the expected return on a stream of dividends is determined by the co-movement of these dividends with the pricing kernel. The only exception to the standard setup we make is that investors face uncertainty about the factor loading of dividend growth on the pricing kernel. Dividend growth is assumed to be conditionally i.i.d. Although this assumption is made primarily for tractability, our main result seems qualitatively robust to allowing for persistence in dividend growth (see Subsection 2.3). Investors begin each period with an unconditional prior distribution over dividends and the risk-factor, and update their beliefs using publicly available information before calculating prices. We use these prices to determine the *unconditional* expected returns for assets, and show that uncertainty about firm-specific factor loadings generates cross-sectional variation in expected returns.

#### Pricing Kernel and Dividends

At the beginning of period  $t$ , investors' beliefs about the pricing kernel are given by

$$M_{t+1} = M_t \exp \left\{ -r_f - \frac{1}{2}V_m - m_{t+1} \right\} \text{ where } m_{t+1} \sim \mathcal{N}(0, V_m), \quad (1)$$

where the  $m_{t+1}$  are i.i.d, normally distributed variables with mean zero and variance  $V_m$ . The aggregate source of risk, or risk-factor, in the economy is driven by the random variable  $m_{t+1}$ . This implies that the unconditional (log) risk-free rate is given by  $r_f$ , since

$$-\log \left( E \left[ \frac{M_{t+1}}{M_t} \right] \right) = r_f + \frac{1}{2}V_m - \frac{1}{2}V_m = r_f. \quad (2)$$

Note that the existence of the pricing kernel relies only on the assumption that there is no arbitrage in the economy, which makes the current setup quite general. In particular, this representation can capture a variety of pricing models including consumption-based models and factor-based models, such as the CAPM.

At the beginning of each period, investors believe that firm  $i$ 's dividends at date  $t + 1$  are given by

$$D_{i,t+1} = D_{i,t} \exp \left\{ \bar{d}_i + \beta_{i,t+1} m_{t+1} + d_{i,t+1} \right\}, \quad (3)$$

which implies that the uncertainty in dividend growth can be decomposed into systematic and idiosyncratic components. The constant component of dividend growth is given by  $\bar{d}_i$ . The idiosyncratic component  $d_{i,t+1}$  is assumed to have an i.i.d. distribution given by

$$d_{i,t+1} \sim \mathcal{N}(0, V_{d,i}), \quad (4)$$

and is independent of  $m_{t+1}$  and  $\beta_{i,t+1}$ .

The systematic component of dividend growth depends on the *stochastic* factor loading  $\beta_{i,t+1}$  of the firm.<sup>5</sup> Unlike standard models, we assume that at time  $t$ , investors do not know the factor loading that drives dividend growth at time  $t + 1$ , but instead only have a probability distribution about it. We assume that investors' beliefs about  $\beta_{i,t+1}$  at the beginning of date  $t$  are given by

$$\beta_{i,t+1} \sim \mathcal{N}(b_i, V_{\beta,i}), \quad (5)$$

and that  $\beta_{i,t+1}$  is independent of  $m_{t+1}$ . In particular, factor loadings are assumed to be i.i.d. over time. This assumption is made for tractability, as it allows us to express expected returns explicitly in terms of the parameters. Numerical simulations in Subsection 2.3 suggest, however, that our results are qualitatively similar if we allow for persistence in factor loadings.

---

<sup>5</sup>This is an example of what is known as a random coefficients model (e.g. Cooley and Prescott (1976)). Ang and Chen (2007) use a similar setup to model the uncertainty faced by an econometrician when estimating time varying betas, and use this to argue that there is little evidence of a value premium in the long run.

Note that investors do not “eventually learn” the factor loading of a firm’s dividends. Given the distribution of  $\beta_{i,t+1}$  in (5), investors always face residual uncertainty about the future factor loadings of a firm’s dividends, and they use firm-specific information available in the current period to update their beliefs about these loadings. Finally, note that since factor loadings are stochastic, regressing past cash-flow growth on the aggregate risk-factor only provides investors with an estimate of past factor loadings, but does not perfectly reveal future factor loadings, which are relevant for setting current prices. Even in a model with persistent factor loadings, estimates of past factor loadings are only partially informative about future loadings and investors still face residual uncertainty about future loadings.<sup>6</sup>

### Aggregate and Firm-Specific Information Quality

At each date  $t$ , investors receive a public signal  $Y_{m,t}$  about  $m_{t+1}$  and a public signal  $Y_{i,t}$  about  $\beta_{i,t+1}$  of the form:

$$Y_{m,t} = m_{t+1} + e_{m,t} \text{ where } e_{m,t} \sim \mathcal{N}(0, V_{Y,m}) \quad (6)$$

$$Y_{i,t} = \beta_{i,t+1} + e_{i,t} \text{ where } e_{i,t} \sim \mathcal{N}(0, V_{Y,i}) \quad (7)$$

and where the  $e_{j,t}$  are independent of each other. While one could model the information available to investors using multiple, possibly correlated signals, the above specification allows us to distinguish the effects of aggregate and firm-specific information in a transparent and tractable manner. In particular, note that conditional on these signals, investors’ beliefs about  $m_{t+1}$  and  $\beta_{i,t+1}$  are given by Bayes Rule, as follows:

$$m_{t+1}|Y_{m,t} \sim \mathcal{N}(\lambda_m Y_{m,t}, V_m(1 - \lambda_m)) \text{ where } \lambda_m = \frac{V_m}{V_m + V_{Y,m}} \quad (8)$$

$$\beta_{i,t+1}|Y_{i,t} \sim \mathcal{N}(b_i + \lambda_{\beta,i}(Y_{i,t} - b_i), V_{\beta,i}(1 - \lambda_{\beta,i})) \text{ where } \lambda_{\beta,i} = \frac{V_{\beta,i}}{V_{\beta,i} + V_{Y,i}} \quad (9)$$

Under our assumptions, a higher value of  $\lambda$  implies that the signal available to investors is more precise and, as a result, their posterior variance (or uncertainty) is lower. Since  $\lambda_m \in [0, 1]$  parametrizes the quality of information available to investors about the aggregate risk-factor, we refer to it as aggregate information quality. Similarly,  $\lambda_{\beta,i} \in [0, 1]$  is a measure

---

<sup>6</sup>For instance, suppose investors believe that  $\beta_{i,t+1}$  followed an  $AR(1)$  process given by:

$$\beta_{i,t+1} = (1 - \rho) b_i + \rho \beta_{i,t} + e_{i,\beta,t+1} \text{ where } e_{i,\beta,t+1} \sim N(0, V_{\beta,i}).$$

Then, at date  $t$ , even if investors know  $\rho$  and  $b_i$ , and  $\beta_{i,t}$  were perfectly observable, investors would still face residual uncertainty about  $\beta_{i,t+1}$ .



of firm-specific information quality. To maintain tractability, we assume that the distribution of signals is also i.i.d., and so  $\lambda_m$  and  $\lambda_{\beta,i}$  are constant. The model's testable prediction will be based on a comparative statics exercise with respect to  $\lambda_m$  and  $\lambda_{\beta,i}$ , and our empirical analysis will exploit variation in firm-specific information quality (i.e.,  $\lambda_{\beta,i}$ ) across firms and variation in aggregate information quality (i.e.,  $\lambda_m$ ) over time.

## 2.2 The Pricing Equation and Expected Returns

By the definition of the pricing kernel, the price of firm  $i$ 's stream of dividends is given by

$$P_{i,t} = E_t \left[ \sum_{s=1}^{\infty} \frac{M_{t+s}}{M_t} D_{i,t+s} \right] = E_t \left[ \sum_{s=1}^{\infty} \frac{M_{t+s}}{M_t} D_{i,t+s} \middle| Y_{m,t}, Y_{i,t} \right], \quad (10)$$

and the unconditional expected return on the firm's dividend stream is then given by

$$E[R_{i,t+1}] = E \left[ \frac{P_{i,t+1} + D_{i,t+1}}{P_{i,t}} \right]. \quad (11)$$

Under an appropriate transversality condition and a condition that guarantees the expectation in (10) is well defined, we have our main result.

**Proposition 1** *Suppose the following transversality condition holds*

$$\lim_{T \rightarrow \infty} E_t \left[ \frac{M_T}{M_t} P_{i,T} \right] = 0, \quad (12)$$

*and suppose for the unconditional variances are small enough, such that*

$$1 - V_m V_{\beta,i} > 0. \quad (13)$$

*Then, firm  $i$ 's unconditional expected return is given by*

$$E[R_{i,t+1}] = \exp \left\{ r_f + \underbrace{b_i \frac{V_m(1-\lambda_m)}{1-V_{\beta,i}(1-\lambda_{\beta,i})V_m(1-\lambda_m)}}_{\text{beta effect}} + \underbrace{\frac{1}{2} V_m \left( 1 + \frac{\lambda_m - (1-\lambda_m)(1-V_{\beta,i}V_m(1-\lambda_m))}{(1-V_{\beta,i}(1-\lambda_{\beta,i})V_m(1-\lambda_m))^2} \right)}_{\text{convexity effect}} \right\}. \quad (14)$$

The proof is in the appendix. The necessary condition for the expectation in (10) to exist is given by the restriction (13). Intuitively, since the expectation involves the exponent

of the product of two normal random variables, one must ensure that the variance of these random variables is small enough for the integral to converge.<sup>7</sup>

While the expression in (14) appears quite complicated, note that when there is no information about  $m_{t+1}$  at time  $t$  and no uncertainty about  $\beta_{i,t+1}$  (i.e., if  $\lambda_m = 0$  and  $\beta_{i,t+1} = b_i$ ), the firm's expected return reduces to the familiar expression

$$E[R_{i,t+1}] = \exp\{r_f + b_i V_m\}. \quad (15)$$

To gain some intuition about the effect of information quality on expected returns, we can decompose the expression in (14) into two components: a *beta* effect and a *convexity* effect. The beta effect is the source of the cross-sectional variation in expected returns as a result of the correlation between cash-flow growth and the aggregate risk-factor. Note that an increase in either firm-specific or aggregate information quality leads to a decrease in  $\frac{V_m(1-\lambda_m)}{1-V_{\beta,i}(1-\lambda_{\beta,i})V_m(1-\lambda_m)}$ , which leads to an increase in expected returns when  $b_i$  is negative and a decrease in expected returns when  $b_i$  is positive. Intuitively, this is because an increase in information quality of either type attenuates the covariance of cash-flow growth with the aggregate risk-factor towards zero.

A second source of cross-sectional variation in expected returns is through the convexity effect. It is easy to verify that the convexity effect is always increasing in aggregate information quality (i.e.,  $\lambda_m$ ). However, the convexity effect of an increase in firm-specific information quality is not always positive, since it depends on the sign of  $\lambda_m - (1 - \lambda_m)(1 - V_{\beta,i}V_m(1 - \lambda_m))$ . In particular, the convexity effect is increasing in firm-specific information quality when aggregate information quality is small enough, i.e., when

$$\lambda_m \leq 1 - \frac{1 - \sqrt{1 - V_{\beta,i}V_m}}{V_{\beta,i}V_m}. \quad (16)$$

To see why, note that increasing firm-specific information quality reduces uncertainty about cash-flows, but that this has two potentially offsetting effects on the expected return:

- (1) A decrease in the uncertainty about cash-flow growth increases expected returns by decreasing the price-dividend ratio because the price-dividend ratio is a convex function of dividend growth. This is same intuition that drives the results in [Pástor and Veronesi](#)

---

<sup>7</sup>If we relax the assumption that  $\beta_{i,t+1}$  and  $m_{t+1}$  are independent, then the restriction in (13) is given by

$$(1 - V_{m,\beta_i})^2 - V_m V_{\beta,i} > 0,$$

where  $V_{m,\beta_i}$  is the covariance between  $\beta_{i,t+1}$  and  $m_{t+1}$ .

(2003) and Johnson (2004). However, since the reduction in uncertainty is about the systematic component of cash-flows and not about the idiosyncratic component, we do not need to rely on leverage for firm-specific uncertainty to affect expected returns (unlike Johnson (2004)).

- (2) Since higher firm-specific information quality decreases uncertainty about systematic factor loadings, it reduces systematic risk and hence decreases expected returns.

The overall convexity effect of an increase in firm-specific information quality,  $\lambda_{\beta,i}$ , depends on the relative sizes of these offsetting effects and is positive only when aggregate information quality,  $\lambda_m$ , is small enough (i.e., the first effect dominates the second).

The beta and convexity effects in our model appear to be related to similar results in a number of earlier papers, but there are important differences. As in our model, an increase in information quality leads to an attenuation in the covariance between cash-flows and the risk-factor in Pástor and Veronesi (2006) and Lambert, Leuz, and Verrecchia (2007). However, neither paper distinguishes between firm-specific and aggregate information quality — the former model only considers aggregate uncertainty and assumes investors know factor loadings with certainty, while the latter model assumes investors receive information about the payoffs of stocks as a whole. Similar to the convexity effect of  $\lambda_m$  in our model, an increase in aggregate information quality leads to an increase in expected returns across all stocks in Veronesi (2000). However, the mechanism through which this effect arises is different in the two models. In Veronesi (2000), more precise information about the aggregate dividend process increases the conditional covariance between consumption growth and aggregate stock returns and this leads to a higher equity risk premium. As a result, the cross-sectional effect of aggregate information quality is determined by the level of factor loadings. In our model, aggregate information quality affects the cross-section of expected returns not only through the level of factor loadings, but also through its interaction with the uncertainty about these loadings. Finally, our model suggests that if investors face uncertainty about the factor loadings of cash-flow growth, and not the mean growth rate (as in Pástor and Veronesi (2003) and Johnson (2004)), the convexity effect of firm-specific uncertainty is not monotonic and instead depends on the level of aggregate uncertainty.

Our model extends the intuition in a number of earlier papers that focus on the effect of either firm-specific or aggregate information in isolation. In our model, investors learn not only about the cash-flows of a specific firm, but also about aggregate risk factors in the economy. As a result, our model provides a novel prediction since the effect of learning about the cash-flows on expected returns depends crucially on what is learned about the risk

factor. In Section 3, we focus on this interaction when testing the following prediction from the model:

**Hypothesis 1** *After controlling for factor loadings, the difference in expected returns between high and low firm-specific information quality firms decreases in aggregate information quality (i.e.,  $\lambda_m$ ).*

Finally, note that for a stock with positive factor loadings, an increase in firm-specific information quality leads to a decrease in expected returns through the beta effect and an increase in expected returns through the convexity effect (when aggregate information quality is low enough). Hence, unlike a number of earlier empirical studies that test for a monotonic relationship between information quality and expected returns, we are careful to control for the beta effect when testing the model’s prediction about the convexity effect.

### 2.3 Persistence in factor loadings

In the previous analysis, we have assumed that investors’ beliefs about the distribution of factor loadings are i.i.d. and given by (5). As can be seen from the proof of Proposition 1, the independence assumption is important for tractability since it implies that the expectations that describe prices and returns can be calculated in closed form. This makes the intuition for the results clearer and the empirical predictions sharper.

However, given the empirical evidence of persistence in betas, it might be reasonable to instead assume that investors’ beliefs allow for this persistence. In particular, suppose investors believe that factor loadings evolve according to the following process:

$$\beta_{i,t+1} = (1 - \rho) b_i + \rho \beta_{i,t} + e_{i,\beta,t+1} \text{ where } e_{i,\beta,t+1} \sim \mathcal{N}(0, V_{\beta,i}) \quad (17)$$

where investors know the unconditional mean of beta,  $b_i$ , and the degree of persistence in beta,  $\rho$ . Furthermore, suppose at date  $t$ , investors observe  $\beta_{i,t}$  and receive noisy information about  $\beta_{i,t+1}$ , or alternatively,  $e_{i,\beta,t+1}$ , of the form:

$$Y_{i,t} = e_{i,\beta,t+1} + e_{i,t} \text{ where } e_{i,t} \sim \mathcal{N}(0, V_{Y,i}) \quad (18)$$

In this case, the evolution of factor loadings and the effect of firm-specific information quality are persistent, while still preserving conditional normality. Moreover, note that even though

learning about past factor loadings is informative about future factor loadings, investors still face residual uncertainty about future factor loadings in each period.

Although analytically deriving the results in closed form is intractable, numerical simulations suggest that the results of Proposition 1 extend to the case of persistent factor loadings. For each set of parameter values, we simulate 100,000 realizations of signals about  $m_{t+1}$  and  $\beta_{i,t+1}$ . For each of these signal realizations, we then simulate 25,000 sample paths for the dividend process and the pricing kernel to calculate estimates of the price of the asset. We then average across the realizations of the signals to estimate the unconditional expected returns for the given parameters.

Figure 1 plots the beta and convexity effects from these simulations of the model with and without persistent factor loadings. While the magnitude of the effects changes with the persistence in betas, the signs of the effects are consistent with the predictions of Proposition 1. Even with persistent factor loadings, the beta effect of information quality decreases expected returns and the convexity effect of aggregate information quality increases expected returns. Most importantly, the simulations suggest that interaction described in Hypothesis 1 extends to the case of persistent betas. In particular, the plots suggest that the convexity effect of firm-specific information quality decreases with aggregate information quality.

## 3 Empirical Analysis

### 3.1 Data Description and Empirical Proxies

In order to test whether firm-specific information quality affects expected returns even after controlling for factor loadings, we use a portfolio-based approach. Each month, we sort stocks into 10 portfolios based on various proxies of firm-specific information quality in the previous month. We calculate the difference in value-weighted return between the high- and low- information quality portfolios, which we refer to as the *HLIQ* (i.e., high-low information quality) portfolio. Hypothesis 1 implies that after controlling for betas, the expected return on the *HLIQ* portfolio should be decreasing in aggregate information quality. Relative to firm-level regressions, the portfolio approach is more robust to noise in the returns and mis-specification in the proxies for firm-specific information quality.

We rely on the following four complementary proxies of firm-specific information quality:

1. Standard error of beta estimates — Our first proxy is based on the standard errors

of beta estimates from regressions of daily returns on the Fama-French three factors. Each month, we sort firms based on the sum of standard errors of the beta estimates to calculate the returns on the  $HLIQ_{BSTE}$  portfolio. We interpret the standard errors of beta estimates as a noisy measure of the residual uncertainty that investors face about the risk-factor loadings of the firm.

2. Idiosyncratic volatility — Our second proxy is based on idiosyncratic return volatility. Following [Ang, Hodrick, Xing, and Zhang \(2006\)](#) and others, we calculate the realized variance of the residuals from a regression of a firm’s return on the Fama French factors. We then sort firms on this measure to calculate the returns on the  $HLIQ_{IV}$  portfolio. A number of papers show that higher uncertainty is associated with high volatility, both at the aggregate and the firm levels (e.g., [Pástor and Veronesi \(2003\)](#); also see [Pástor and Veronesi \(2009\)](#) for a recent survey). Hence, we argue that firms with higher idiosyncratic volatility are likely to have lower firm-specific information quality.
3. Mean forecast error — Our third proxy is based on the mean forecast error across analyst estimates of earnings per share calculated from the IBES Detail History file. This captures the notion that when the quality of public information about a firm is higher, analysts should make smaller forecast errors on average. Each month, we compute the mean absolute forecast error of annual earnings per share and scale it by the median estimate. We sort stocks on this measure of scaled mean forecast error to calculate returns on the  $HLIQ_{MFE}$  portfolio.
4. BKLS information quality — Our final proxy is based on the quality of information available to financial analysts, which we measure based on the approach developed in [Barron, Kim, Lim, and Stevens \(1998\)](#) (hereafter BKLS). The BKLS setup assumes that there are  $N$  financial analysts that issue earnings forecasts and that each analyst has two pieces of information: (i) a common signal with precision  $h$  that is available to all analysts, and (ii) a private signal with precision  $s$ . BKLS show that the quality of public information can be recovered from the following equation:

$$h = \frac{SE - \frac{D}{N}}{\left[\left(1 - \frac{1}{N}\right)D + SE\right]^2}, \quad (19)$$

where  $SE$  is the expected squared error of the mean forecast,  $D$  is the expected forecast dispersion, and  $N$  is the number of analysts issuing a forecast. We sort stocks based on the quality of public information (i.e.,  $h$ ) to calculate the returns on the  $HLIQ_{BKLS}$  portfolio.

Each of these proxies is imperfect and potentially reflects a different aspect of firm-specific information quality. In general, firm-specific information quality is difficult to measure and empirical proxies often confound information quality with other firm characteristics. From an *ex ante* perspective, we believe that the BKLS proxy provides the sharpest measure of firm-specific information quality. However, the BKLS measure limits the sample size of firms we can consider and is most sensitive to mis-specification (since it is derived within a specific model). On the other hand, while the standard error of beta estimates and idiosyncratic volatility are more noisy proxies of firm-specific information quality, they are also available for a much larger sample of firms and are more robust to mis-specification of the information structure available to investors. Hence, we do not want to focus attention on any single proxy, but instead consider the empirical evidence across all of them together. Assuming that these proxies capture some notion of firm-specific information quality, our model predicts a consistent cross-sectional relation between expected returns and these proxies.

We use the VIX Volatility Index provided by the Chicago Board Options Exchange (CBOE) to construct the proxy for aggregate information quality,  $AIQ = 1 - VIX$ .<sup>8</sup> The VIX is often referred to as a “fear index” and a number of papers in the learning literature relate higher aggregate uncertainty with higher stock market volatility and, consequently, higher implied volatility (e.g., [Timmermann \(1993\)](#), [David \(1997\)](#), [Veronesi \(1999\)](#), [Veronesi \(2000\)](#), [David and Veronesi \(2009\)](#)). We prefer using an implied volatility measure like the VIX instead of realized aggregate volatility since implied volatility is forward-looking and is therefore likely to be better suited to capture the uncertainty that investors face going forward. Since the firm-specific convexity effect depends non-linearly on the aggregate information quality, we use different specifications to test for this relationship in our regressions.

## 3.2 Empirical Specifications and Results

Our sample consists of monthly observations from January 1986 through December 2008. Each month, we trim the sample at the first and 99-th percentile of return observations so that our analysis is not driven by outliers. [Table 1](#) presents the summary statistics for the *HLIQ* portfolio returns based on the various firm-specific information quality proxies, and summary statistics on the monthly returns on the risk-factors we use to control for the effect of factor loadings (i.e., the excess return on the market (MKTRF), the Fama-French SMB

---

<sup>8</sup>The CBOE makes two versions of its Volatility Index available. The first, denoted by VXO, is based on the S&P 100 and is available from January 1986. The second, denoted by VIX, is based on the S&P 500 and is available from January 1990. We follow [Ang, Hodrick, Xing, and Zhang \(2006\)](#) in using the longer series — as they report, the two measures are very highly correlated (with correlation coefficients of 98%).

and HML portfolios, the Carhart momentum portfolio (UMD) and the Pastor-Stambaugh liquidity factor (PSVW)). The table also reports the summary statistics for our proxy of aggregate information quality ( $AIQ$ ). The average monthly returns range from 116 basis points for  $HLIQ_{BSTE}$  to 198 basis points for  $HLIQ_{BKLS}$ , suggesting that the cross-sectional variation in returns across high information quality and low information quality firms is economically significant.

The results for the  $HLIQ_{BSTE}$ ,  $HLIQ_{IV}$ ,  $HLIQ_{MFE}$ , and  $HLIQ_{BKLS}$  portfolios are reported in Tables 2 through 5, respectively. In each table, we first report the mean return on each  $HLIQ$  portfolio conditional on being the highest and lowest  $AIQ$  decile. Across all the information quality proxies, the average return on the  $HLIQ$  portfolio is significantly higher when aggregate information quality is low relative to when aggregate information quality is high. Moreover, the return on the  $HLIQ$  portfolios are always positive and statistically significant in the lowest  $AIQ$  decile but negative (though not always statistically significant) in the highest  $AIQ$  decile. This is consistent with the model’s prediction that the firm-specific convexity effect is positive when aggregate information quality is low, but negative when aggregate information quality is high.

Although suggestive, comparing the average  $HLIQ$  returns across low and high  $AIQ$  deciles is not a test of the firm-specific convexity effect since it does not control for the factor loadings of these portfolios. Therefore, we regress the return of each  $HLIQ$  portfolio on a set of aggregate risk-factors and test whether the residual return for the  $HLIQ$  portfolio is negatively related to aggregate information quality. The aggregate risk-factors used to control for betas are the excess return on the market (MKTRF), the Fama-French SMB and HML portfolios, the Carhart momentum portfolio (UMD) and the Pastor-Stambaugh liquidity factor (PSVW).

Since the model suggests a non-linear interaction between aggregate and firm-specific information quality, a limitation of the standard linear regression specification is that  $AIQ$  enters linearly. Hence, we report the estimates the following time-series regression:

$$HLIQ_t = \alpha_0 + \alpha_1 f(AIQ) + \beta_1 MKTRF_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 UMD_t + \beta_5 PSVW_t + \varepsilon_t, \quad (20)$$

where  $f(AIQ)$  is one of the following: (i)  $AIQ$  for the linear specification, (ii) the exponential of  $AIQ$ , (iii) an indicator variable for  $AIQ$  being in the 60<sup>th</sup> percentile or higher, and (iv) an indicator variable for  $AIQ$  being in the 80<sup>th</sup> percentile or higher. These specifications allow for a certain degree of non-linearity while keeping the analysis parsimonious. Hypothesis 1 predicts that the coefficient  $\alpha_1$  should be negative since the firm-specific convexity effect is negatively related to aggregate information quality. Also, note that since the return on



*HLIQ* is already an excess return (i.e., it is the difference in the returns of the two extreme decile portfolios), we do not subtract the risk-free rate.

The regression results in Tables 2 through 5 suggest that allowing for non-linearity is important when estimating the interaction between firm-specific and aggregate information quality. While the coefficient on *AIQ* is negative in all the specifications (across all information quality proxies), the coefficients are more statistically significant in the non-linear specifications. While not all the coefficients are statistically significant, the consistency across proxies in the sign of  $\alpha_1$  is reassuring and leads us to conclude that the evidence is consistent with the model’s predictions. Specifically, the estimates of  $\alpha_1$  across the various specifications imply that the difference in average returns between high information quality and low information quality firms decreases non-linearly with aggregate information quality.

The estimates of the factor loadings for the *HLIQ* portfolio returns are consistent across the various firm-specific IQ proxies. These estimates suggest that high information quality firms tend to have significantly lower market and SMB betas and significantly higher UMD betas. The *HLIQ<sub>IV</sub>* and *HLIQ<sub>BSTE</sub>* portfolios also have significantly positive HML betas in our sample. The coefficient on the Pastor-Stambaugh liquidity factor (PSVW) across all the specifications is small and not statistically significant, suggesting that the returns on the *HLIQ* portfolios are not driven by liquidity. Finally, although the model does not have a prediction for  $\alpha_0$ , the estimates for  $\alpha_0$  are always positive, and are often statistically significant, suggesting that even after controlling for risk-factor loadings and for variation in aggregate information quality, high information quality firms have higher expected returns. These effects are also economically significant — the alphas on the *HLIQ<sub>BSTE</sub>*, *HLIQ<sub>IV</sub>*, *HLIQ<sub>MFE</sub>*, and *HLIQ<sub>BKLS</sub>* portfolios relative to the risk-factors in (20) are 89, 98, 97, and 191 basis points per month, respectively. Moreover, these alphas tend to increase after controlling for *AIQ*, especially in the non-linear specifications. Moreover, the empirical evidence seems strongest for the BKLS proxy which is consistent with our *ex ante* view that the BKLS proxy provides the sharpest measure of firm-specific information quality in our analysis.

The results in Tables 2 through 5 are consistent with a negative cross-sectional relation between firm-specific uncertainty and expected returns. In particular, the evidence suggests that our model provides a possible explanation to the seemingly “puzzling” negative relation between expected returns and idiosyncratic volatility documented by Ang, Hodrick, Xing, and Zhang (2006) and others.<sup>9</sup> However, these tables also highlight the important effect of

---

<sup>9</sup>As Ang, Hodrick, Xing, and Zhang (2006) discuss, most models that incorporate market frictions or behavioral biases imply that idiosyncratic volatility should be positively related to expected returns. As such,

aggregate information quality on this relation. This suggests that accounting for aggregate uncertainty may be important in reconciling the apparently contradictory empirical evidence about the cross-sectional relation between uncertainty and expected returns documented in the literature so far.

## 4 Conclusions

We study the relation between information quality and expected returns in a standard asset pricing model where systematic risk determines expected returns. We allow investors to learn about both aggregate risk factors and about firm-specific factor loadings. We show that even after controlling for the effect of average risk-factor loadings, firm-specific information quality generates cross-sectional variation in expected stock returns. Moreover, the interaction between firm-specific and aggregate information quality is important. The excess return earned by high information quality firms (relative to low information quality firms) decreases non-linearly with aggregate information quality. We test these predictions using portfolios constructed using various proxies for information quality and find evidence consistent with the model that is both statistically and economically significant.

Our objective is to provide a first step in analyzing how both systematic and firm-specific information affects a firm's expected returns in a standard asset pricing framework. Our theoretical and empirical analysis suggests that the interaction between firm-specific and aggregate information has important implications for expected returns, and provides a benchmark for more sophisticated analysis that allows for endogenous disclosure or dynamics in learning.

---

they state that their “results on idiosyncratic volatility represent a substantive puzzle.”

## References

- Adrian, T., and F. Franzoni, 2008, Learning about Beta: Time-varying Factor Loading Expected Returns, and the Conditional CAPM, *Journal of Empirical Finance* forthcoming.
- Ai, H., 2009, Information Quality and Long-run Risks: Asset Pricing Implications, *Journal of Finance* forthcoming.
- Anderson, E.W., E. Ghysels, and J.L. Juergens, 2005, Do Heterogeneous Beliefs Matter for Asset Pricing?, *Review of Financial Studies* 18, 875–924.
- Ang, A., and J. Chen, 2007, CAPM over the long run: 1926–2001, *Journal of Empirical Finance* 14, 1–40.
- Ang, A., R.J. Hodrick, Y. Xing, and X. Zhang, 2006, The Cross-Section of Volatility and Expected Returns, *Journal of Finance* 61, 259.
- Ang, A., R.J. Hodrick, Y. Xing, and X. Zhang, 2009, High idiosyncratic volatility and low returns: International and further US evidence, *Journal of Financial Economics* 91, 1–22.
- Baiman, S., and R. Verrecchia, 1996, The relation among capital markets, financial disclosure, production efficiency, and insider trading, *Journal of Accounting Research* 34, 1–22.
- Banerjee, S., 2010, Learning from prices and the dispersion in beliefs, *Review of Financial Studies* forthcoming.
- Barron, O.E., O. Kim, S.C. Lim, and D.E. Stevens, 1998, Using analysts’ forecasts to measure properties of analysts’ information environment, *Accounting Review* pp. 421–433.
- Barry, C.B., and S.J. Brown, 1985, Differential information and security market equilibrium, *Journal of Financial and Quantitative Analysis* pp. 407–422.
- Barth, M.E., Y. Konchitchki, and W.R. Landsman, 2007, Cost of capital and financial statements transparency, *Working Paper, Stanford University*.
- Bawa, V., and S. Brown, 1979, Capital market equilibrium: Does estimation risk really matter, *Estimation Risk and Optimal Portfolio Choice*.
- Botosan, C.A., 1997, Disclosure level and the cost of equity capital, *The Accounting Review* 72, 323–349.
- Botosan, C.A., and M.A. Plumlee, 2002, A re-examination of disclosure level and the expected cost of equity capital, *Journal of Accounting Research* 40, 21–40.
- Brevik, F., and S. D’Addona, 2005, Information quality and stock returns revisited, Working paper, Univ. of St. Gallen, Dep. of Economics.
- Brown, S., 1979, The effect of estimation risk on capital market equilibrium, *Journal of Financial and Quantitative Analysis* pp. 215–220.

- Clarkson, P.M., and R. Thompson, 1990, Empirical estimates of beta when investors face estimation risk, *Journal of Finance* pp. 431–453.
- Cooley, T.F., and E.C. Prescott, 1976, Estimation in the presence of stochastic parameter variation, *Econometrica* pp. 167–184.
- Core, J.E., W.R. Guay, and R. Verdi, 2008, Is accruals quality a priced risk factor?, *Journal of Accounting and Economics* 46, 2–22.
- Croce, M., M. Lettau, and S. Ludvigson, 2009, Investor information, long-run risk, and the duration of risky cash flows, Working Paper, NBER.
- David, A., 1997, Fluctuating confidence in stock markets: Implications for returns and volatility, *Journal of Financial and Quantitative Analysis* 32, 427–462.
- David, Alexander, and Pietro Veronesi, 2009, Macroeconomic Uncertainty and Fear Measures Extracted from Index Options, *SSRN eLibrary*.
- Diamond, D.W., and R.E. Verrecchia, 1991, Disclosure, liquidity, and the cost of capital, *Journal of Finance* pp. 1325–1359.
- Diether, K.B., C.J. Malloy, and A. Scherbina, 2002, Differences of opinion and the cross section of stock returns, *Journal of Finance* 57, 2113–2141.
- Duarte, J., and L. Young, 2009, Why is PIN priced?, *Journal of Financial Economics* 91, 119–138.
- Easley, D., and M. O’Hara, 2004, Information and the cost of capital, *Journal of Finance* pp. 1553–1583.
- Francis, J., R. LaFond, P.M. Olsson, and K. Schipper, 2004, Costs of equity and earnings attributes, *Accounting Review* 79, 967–1010.
- Francis, J., R. LaFond, P. Olsson, and K. Schipper, 2005, The market pricing of accruals quality, *Journal of Accounting and Economics* 39, 295–327.
- Francis, J., D. Nanda, and P. Olsson, 2008, Voluntary disclosure, information quality, and costs of capital, Discussion paper, Duke University Working Paper.
- Fu, F., 2009, Idiosyncratic risk and the cross-section of expected stock returns, *Journal of Financial Economics* 91, 24–37.
- Goetzmann, W.N., and M. Massa, 2005, Dispersion of opinion and stock returns, *Journal of Financial Markets* 8, 324–349.
- Gollier, C., and E. Schlee, 2009, Information and the equity premium, *Journal of the European Economic Association* forthcoming.
- Huang, W., Q. Liu, S.G. Rhee, and L. Zhang, 2009, Return reversals, idiosyncratic risk, and expected returns, *Review of Financial Studies*.

- Hughes, J.S., J. Liu, and J. Liu, 2007, Information Asymmetry, Diversification, and Cost of Capital, *The Accounting Review* 82, 705–729.
- Johnson, T.C., 2004, Forecast Dispersion and the Cross Section of Expected Returns, *The Journal of Finance* 59, 1957–1978.
- Kumar, P., S.M. Sorescu, R.D. Boehme, and B.R. Danielsen, 2008, Estimation Risk, Information, and the Conditional CAPM: Theory and Evidence, *Review of Financial Studies* 21, 1037.
- Lambert, R., C. Leuz, and R. Verrecchia, 2007, Accounting information, disclosure, and the cost of capital, *Journal of Accounting Research* 45, 385–420.
- Lehmann, B.N., 1990, Residual risk revisited, *Journal of Econometrics* 45, 71–97.
- Li, G., 2005, Information quality, learning, and stock market returns, *Journal of Financial and Quantitative Analysis* 40, 595–620.
- Malkiel, B.G., and Y. Xu, 2002, Idiosyncratic risk and security returns, *Working Paper, University of Texas at Dallas*.
- Merton, R.C., 1987, A simple model of capital market equilibrium with incomplete information, *Journal of Finance* 42, 483–510.
- Miller, E., 1977, Risk, uncertainty, and divergence of opinion, *Journal of Finance* 32, 1151–1168.
- Pástor, L., and P. Veronesi, 2003, Stock valuation and learning about profitability, *Journal of Finance* pp. 1749–1789.
- Pástor, L., and P. Veronesi, 2006, Was there a Nasdaq bubble in the late 1990s?, *Journal of Financial Economics* 81, 61–100.
- Pástor, L., and P. Veronesi, 2009, Learning in Financial Markets, *Annual Review of Financial Economics* 1.
- Qu, S., L. Starks, and H. Yan, 2004, Risk, dispersion of analyst forecasts and stock returns, Working paper, University of Texas at Austin.
- Spiegel, M., and X. Wang, 2010, Cross-sectional variation in stock returns: Liquidity and idiosyncratic risk, *Working Paper, Yale University*.
- Timmermann, A.G., 1993, How learning in financial markets generates excess volatility and predictability in stock prices, *The Quarterly Journal of Economics* 108, 1135–1145.
- Veronesi, P., 1999, Stock market overreactions to bad news in good times: a rational expectations equilibrium model, *Review of Financial Studies* 12, 975.
- Veronesi, P., 2000, How does information quality affect stock returns?, *The Journal of Finance* 55, 807–837.

Zhang, X.F., 2006, Information Uncertainty and Stock Returns, *The Journal of Finance* 61, 105–137.

## 5 Appendix

**Proof of Proposition 1.** Let  $Z_{t+\tau}$  denote

$$Z_{i,t+s+1} = \frac{M_{t+s+1}}{M_{t+s}} \frac{D_{t+s+1}}{D_{t+s}} = \exp \left\{ \bar{d}_i - r_f - \frac{1}{2} V_{m,0} + (\beta_{i,t+s+1} - 1) m_{t+s+1} + d_{i,t+s+1} \right\} \quad (21)$$

Then the price is given by

$$P_t = E_t \left[ \sum_{s=1}^{\infty} \frac{M_{t+s}}{M_t} D_{i,t+s} \right] = D_{i,t} \sum_{s=1}^{\infty} E_t \left[ \prod_{\tau=1}^s Z_{i,t+\tau} \right] \quad (22)$$

Expected returns are given by

$$E[R_{i,t+1}] = E \left[ \frac{P_{i,t+1} + D_{i,t+1}}{P_{i,t}} \right] = E \left[ \frac{D_{i,t+1}}{D_{i,t}} \frac{\left(1 + \frac{P_{i,t+1}}{D_{i,t+1}}\right)}{\frac{P_{i,t}}{D_{i,t}}} \right] \quad (23)$$

$$= E \left[ \frac{D_{i,t+1}}{D_{i,t}} \frac{\left(1 + \frac{P_{i,t+1}}{D_{i,t+1}}\right)}{E_t \left( Z_{t+1} \left(1 + \frac{P_{i,t+1}}{D_{i,t+1}}\right) \right)} \right] \quad (24)$$

In general,  $Z_{i,t+1}$  and  $P_{i,t+1}/D_{i,t+1}$  are correlated, and so the expression for expected returns involves evaluating the infinite sum of products of  $Z_{i,t+s}$ . However, we assume that all the random variables are i.i.d., and so we can express the unconditional expected return as

$$E[R_{i,t+1}] = E \left[ E_t \left[ \frac{D_{i,t+1}}{D_{i,t}} \right] \frac{E_t \left(1 + \frac{P_{i,t+1}}{D_{i,t+1}}\right)}{E_t(Z_{t+1}) E_t \left(1 + \frac{P_{i,t+1}}{D_{i,t+1}}\right)} \right] = E \left[ \frac{E_t \left[ \frac{D_{i,t+1}}{D_{i,t}} \right]}{E_t[Z_{i,t+1}]} \right] \quad (25)$$

To evaluate this expectation, denote  $x_{t+1} \equiv \begin{pmatrix} m_{t+1} \\ \beta_{i,t+1} \end{pmatrix}$ ,  $G = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$  and  $a = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$  and  $H_t = I - 2GV_{x,t}$ , where the unconditional distribution of  $x$  is given by

$$x_{t+1} \sim N(\hat{x}_0, V_{x,0}), \quad (26)$$

and the distribution of  $x$  conditional on date  $t$  information is

$$x_{t+1} | Y_{m,t}, Y_{i,t} \sim N(\hat{x}_t, V_{x,t}). \quad (27)$$

The law of iterated expectations and the law of total variance imply that the unconditional distribution of  $\hat{x}_t$  is given by

$$\hat{x}_t \sim N(\hat{x}_0, V_{x,0} - V_{x,t}) \quad (28)$$

Also, note that  $x'_{t+1} G x_{t+1} = \beta_{i,t+1} m_{t+1}$  and  $a x_{t+1} = -m_{t+1}$ . Since  $\beta_{i,t+1}$  and  $m_{t+1}$  are uncorrelated,  $\det(H_t) = \det(I - V_{\beta,i} V_m (1 - \lambda_m) (1 - \lambda_{\beta,i}))$ . Under the assumption that the

unconditional variances are small enough (i.e., restriction (13) holds), we know that the conditional variances are small enough and so this determinant is positive. This implies that the following expectations exist and are given by:

$$\begin{aligned}
E_t \left[ \frac{D_{i,t+1}}{D_{i,t}} \right] &= E_t \left[ \exp \left\{ \bar{d}_i + d_{i,t+1} + x'_{t+1} G x_{t+1} \right\} \right] \\
&= \exp \left\{ \bar{d}_i + \frac{1}{2} V_d \right\} |H_t|^{-1/2} \exp \left\{ -\frac{1}{2} \hat{x}_t (I - H_t^{-1}) V_{x,t}^{-1} \hat{x}_t \right\}, \text{ and} \quad (29) \\
E_t [Z_{i,t+1}] &= E_t \left[ \exp \left\{ \bar{d}_i - r_f - \frac{1}{2} V_{m,0} + d_{i,t+1} + x'_{t+1} G x_{t+1} + a x_{t+1} \right\} \right] \\
&= \exp \left\{ \bar{d}_i + \frac{1}{2} V_d - r_f - \frac{1}{2} V_{m,0} \right\} \\
&\quad \times |H_t|^{-1/2} \exp \left\{ -\frac{1}{2} \hat{x}_t V_{x,t}^{-1} \hat{x}_t + \frac{1}{2} (\hat{x}_t + V_{x,t} a)' H_t^{-1} V_{x,t}^{-1} (\hat{x}_t + V_{x,t} a) \right\}, \quad (30)
\end{aligned}$$

which implies,

$$\begin{aligned}
E [R_{i,t+1}] &= E \left[ \exp \left\{ r_f + \frac{1}{2} V_{m,0} - \hat{x}'_t H_t^{-1} a - \frac{1}{2} a' V_{x,t} H_t^{-1} a \right\} \right] \\
&= \exp \left\{ r_f + \frac{1}{2} V_{m,0} - \hat{x}'_0 H_0^{-1} a + \frac{1}{2} (H_0^{-1} a)' (V_{x,0} - V_{x,t}) H_0^{-1} a - \frac{1}{2} a' V_{x,t} H_t^{-1} a \right\} \\
&= \exp \left\{ r_f + \frac{b_i V_m (1 - \lambda_m)}{1 - V_{\beta,i} V_m (1 - \lambda_m) (1 - \lambda_{\beta,i})} + \frac{1}{2} V_{m,0} \left( 1 + \frac{\lambda_m - (1 - \lambda_m) (1 - V_{\beta,i} V_m (1 - \lambda_m))}{(1 - V_{\beta,i} V_m (1 - \lambda_m) (1 - \lambda_{\beta,i}))^2} \right) \right\}
\end{aligned}$$

which is the expression for the expected return. ■



## 6 Figures and Tables

Figure 1: Beta and convexity effects with persistent factor loadings

This figure plots the beta and convexity effects from numerical simulations of the benchmark model with i.i.d. factor loadings and a model with persistent factor loadings. The factor loading, or beta, is assumed to follow an AR(1) process:

$$\beta_{i,t+1} = (1 - \rho) b_i + \rho \beta_{i,t} + e_{i,\beta,t+1} \text{ where } e_{i,\beta,t+1} \sim \mathcal{N}(0, V_{\beta,i}),$$

and investors receive systematic information with quality  $\lambda_m$  and firm-specific information about  $e_{i,\beta,t+1}$  with quality  $\lambda_\beta$ . For each level of  $\rho = \{0, 0.75\}$ , the model is simulated for two levels of  $b_i$  and different levels of  $\lambda_m$  and  $\lambda_\beta$  in order to calculate the beta and convexity effects. The other relevant parameters of the model are given by  $r_f = 0.05$ ,  $V_m = 0.5$  and  $V_{\beta,i} = 0.5$ .

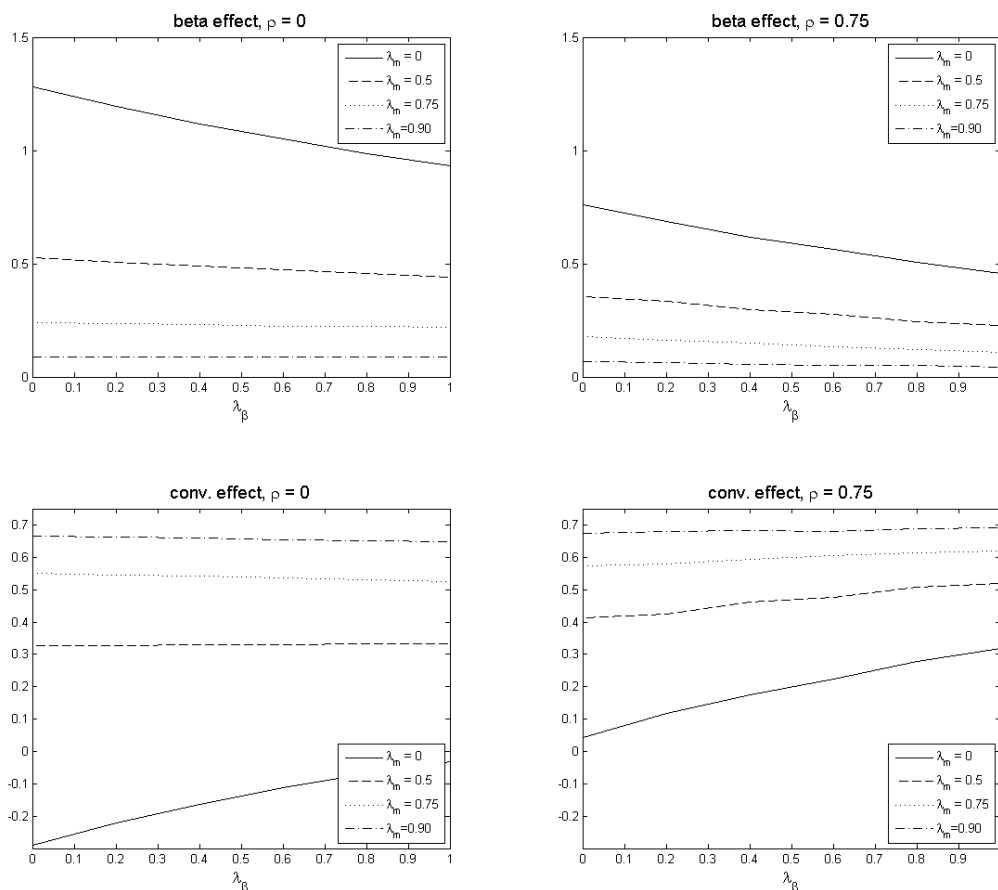


Table 1: Summary statistics for relevant variables.

This table reports the summary statistics for the portfolio returns formed by sorting stocks on lagged firm-specific information quality proxies. The four portfolio returns correspond to the following firm-specific information quality proxies: scaled mean forecast error ( $HLIQ_{MFE}$ ), BKLS uncertainty ( $HLIQ_{MFE}$ ), the idiosyncratic volatility relative to the Fama French three factor model ( $HLIQ_{IV}$ ), and the sum of standard errors of the Fama French three factor beta estimates ( $HLIQ_{BSTE}$ ). The table reports the average number of stocks in each decile portfolio for each of the proxies (Avg. #). The table also reports summary statistics on the Fama French factors (i.e., MKTRF, SMB, HML), the Carhart momentum factor (i.e., UMD), the Pastor-Stambaugh liquidity factor (i.e., PSVW), the risk-free rate and the proxy for aggregate information quality (i.e.,  $AIQ = 1 - VIX$ ). The sample consists of 276 monthly observations, starting from January 1986 through December 2008.

Variable	Mean	Median	Min	Max	Std. Dev	Avg. #
$HLIQ_{BSTE}$	0.0116	0.0165	-0.3538	0.3335	0.0881	715
$HLIQ_{IV}$	0.0127	0.0159	-0.3664	0.3597	0.0905	716
$HLIQ_{MFE}$	0.0117	0.0102	-0.2374	0.2245	0.0631	229
$HLIQ_{BKLS}$	0.0198	0.0182	-0.2274	0.1894	0.0495	317
MKTRF	0.0044	0.0103	-0.2314	0.1243	0.0452	
SMB	0.0006	-0.0015	-0.1685	0.2199	0.0339	
HML	0.0032	0.0027	-0.1237	0.1387	0.0309	
UMD	0.0085	0.0079	-0.2504	0.1835	0.0445	
PSVW	0.0045	0.0044	-0.1248	0.1099	0.0358	
Risk-free	0.0037	0.0039	0.0002	0.0079	0.0016	
$AIQ$	0.7908	0.8054	0.3859	0.9018	0.0832	

Table 2: Characteristics of  $HLIQ_{BSTE}$  Returns

This table presents the return characteristics of the high-low information quality portfolio based on standard error of beta estimates in the Fama French 3-factor model ( $HLIQ_{BSTE}$ ). The first panel reports mean return conditional on the lowest and highest aggregate information quality decile, and the z-statistic for whether the difference in returns is statistically significant. The second panel reports results from time-series regressions of the information quality portfolio returns on the Fama French factors (i.e., MKTRF, SMB, HML), the Carhart momentum factor (i.e., UMD), the Pastor-Stambaugh liquidity factor (i.e., PSVW), and on variants of aggregate information quality. The t-statistics are based on Newey-West standard errors (with 1 lag).

	Mean Ret	Std. Dev	N	t-stat	z-stat
Low $AIQ$	0.0643	0.1323	27	2.53	2.95
High $AIQ$	-0.0128	0.0309	27	-2.15	

	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
Intercept	0.0089	2.39	0.0622	1.60	0.0202	2.64	0.0142	2.77	0.0121	2.85
MKTRF	-0.4413	-4.99	-0.3744	-4.05	-0.3907	-4.34	-0.4143	-4.71	-0.4088	-4.64
SMB	-1.4875	-11.01	-1.4727	-10.87	-1.4800	-10.98	-1.4775	-11.11	-1.4735	-10.76
HML	0.6718	3.26	0.7205	3.55	0.7129	3.55	0.7057	3.54	0.7152	3.49
UMD	0.4737	3.89	0.4824	3.97	0.4816	3.96	0.4755	3.89	0.4838	4.02
PSVW	-0.1573	-1.68	-0.1495	-1.60	-0.1554	-1.69	-0.1728	-1.88	-0.1600	-1.70
$AIQ$			-0.0681	-1.43						
$e^{\{10 \times AIQ\}}$					-0.0003	-2.16				
$AIQ \geq 60^{th}$							-0.0138	-2.22		
$AIQ \geq 80^{th}$									-0.0180	-2.83
Adj. $R^2$	66.89		67.09		67.33		67.35		67.41	

Table 3: Characteristics of  $HLIQ_{IV}$  Returns

This table presents the return characteristics of the high-low information quality portfolio based on idiosyncratic volatility relative to the Fama French 3-factor model ( $HLIQ_{IV}$ ). The first panel reports mean return conditional on the lowest and highest aggregate information quality decile, and the z-statistic for whether the difference in returns is statistically significant. The second panel reports results from time-series regressions of the information quality portfolio returns on the Fama French factors (i.e., MKTRF, SMB, HML), the Carhart momentum factor (i.e., UMD), the Pastor-Stambaugh liquidity factor (i.e., PSVW), and on variants of aggregate information quality. The t-statistics are based on Newey-West standard errors (with 1 lag).

	Mean Ret	Std. Dev	N	t-stat	z-stat
Low $AIQ$	0.0729	0.1355	27	2.80	3.12
High $AIQ$	-0.0109	0.0337	27	-1.68	

	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
Intercept	0.0098	2.60	0.0739	1.93	0.0222	2.82	0.0154	2.89	0.0129	2.99
MKTRF	-0.4803	-5.21	-0.3998	-3.89	-0.4252	-4.47	-0.4522	-4.90	-0.4482	-4.86
SMB	-1.4262	-10.29	-1.4085	-10.11	-1.4181	-10.26	-1.4158	-10.38	-1.4124	-10.10
HML	0.7177	3.47	0.7762	3.78	0.7625	3.78	0.7531	3.76	0.7606	3.70
UMD	0.4884	3.85	0.4989	3.92	0.4971	3.91	0.4903	3.84	0.4984	3.96
PSVW	-0.1489	-1.55	-0.1395	-1.47	-0.1469	-1.56	-0.1651	-1.74	-0.1515	-1.58
$AIQ$			-0.0819	-1.75						
$e^{\{10 \times AIQ\}}$					-0.0004	-2.26				
$AIQ \geq 60^{th}$							-0.0144	-2.21		
$AIQ \geq 80^{th}$									-0.0178	-2.61
Adj. $R^2$	64.08		64.38		64.58		64.54		64.54	

Table 4: Characteristics of  $HLIQ_{MFE}$  Returns

This table presents the return characteristics of the high-low information quality portfolio based on mean forecast error ( $HLIQ_{MFE}$ ). The first panel reports mean return conditional on the lowest and highest aggregate information quality decile, and the z-statistic for whether the difference in returns is statistically significant. The second panel reports results from time-series regressions of the information quality portfolio returns on the Fama French factors (i.e., MKTRF, SMB, HML), the Carhart momentum factor (i.e., UMD), the Pastor-Stambaugh liquidity factor (i.e., PSVW), and on variants of aggregate information quality. The t-statistics are based on Newey-West standard errors (with 3 lags).

	Mean Ret	Std. Dev	N	t-stat	z-stat
Low $AIQ$	0.0516	0.0964	27	2.78	3.36
High $AIQ$	-0.0128	0.0249	27	-2.68	

	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
Intercept	0.0097	3.28	0.0426	1.50	0.0192	3.02	0.0151	3.62	0.0128	3.85
MKTRF	-0.3993	-5.20	-0.3580	-4.38	-0.3568	-4.64	-0.3718	-4.94	-0.3673	-4.87
SMB	-0.8913	-7.96	-0.8822	-7.81	-0.8850	-7.91	-0.8811	-8.00	-0.8775	-7.83
HML	0.1750	1.02	0.2050	1.24	0.2094	1.29	0.2095	1.29	0.2177	1.32
UMD	0.4689	4.96	0.4743	4.93	0.4756	4.95	0.4708	4.95	0.4789	5.09
PSVW	-0.0717	-1.10	-0.0669	-1.03	-0.0701	-1.09	-0.0875	-1.34	-0.0743	-1.13
$AIQ$			-0.0420	-1.20						
$e^{\{10 \times AIQ\}}$					-0.0003	-2.10				
$AIQ \geq 60^{th}$							-0.0140	-2.57		
$AIQ \geq 80^{th}$									-0.0177	-3.18
Adj. $R^2$	51.83		51.89		52.42		52.82		52.87	

Table 5: Characteristics of  $HLIQ_{BKLS}$  Returns

This table presents the return characteristics of the high-low information quality portfolio based on BKLS uncertainty ( $HLIQ_{BKLS}$ ). The first panel reports mean return conditional on the lowest and highest aggregate information quality decile, and the z-statistic for whether the difference in returns is statistically significant. The second panel reports results from time-series regressions of the information quality portfolio returns on the Fama French factors (i.e., MKTRF, SMB, HML), the Carhart momentum factor (i.e., UMD), the Pastor-Stambaugh liquidity factor (i.e., PSVW), and on variants of aggregate information quality. The t-statistics are based on Newey-West standard errors (with 3 lags).

	Mean Ret	Std. Dev	N	t-stat	z-stat
Low $AIQ$	0.0607	0.0631	27	5.00	5.26
High $AIQ$	-0.0096	0.0289	27	-1.72	

	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
Intercept	0.0191	7.56	0.0938	3.42	0.0332	5.66	0.0256	7.12	0.0228	7.91
MKTRF	-0.2728	-6.04	-0.1789	-3.34	-0.2098	-4.78	-0.2397	-5.70	-0.2342	-5.48
SMB	-0.7493	-6.87	-0.7286	-6.59	-0.7400	-6.82	-0.7369	-6.86	-0.7326	-6.70
HML	-0.0434	-0.26	0.0249	0.16	0.0078	0.05	-0.0016	-0.01	0.0081	0.05
UMD	0.3336	3.82	0.3458	3.88	0.3435	3.86	0.3358	3.82	0.3456	3.98
PSVW	-0.0839	-1.56	-0.0729	-1.40	-0.0815	-1.57	-0.1029	-1.93	-0.0870	-1.60
$AIQ$			-0.0955	-2.81						
$e^{\{10 \times AIQ\}}$					-0.0004	-3.17				
$AIQ \geq 60^{th}$							-0.0169	-3.29		
$AIQ \geq 80^{th}$									-0.0214	-3.98
Adj. $R^2$	43.73		45.51		46.27		46.29		46.40	