Market Run-Ups, Market Freezes, and Leverage*

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June 20, 2011

*This draft is in process.

Abstract

We study trade between a buyer and a seller when both may have existing inventories of assets similar to those being traded. We analyze how these inventories affect trade, information dissemination, and price formation. We show that when the buyer's and seller's initial leverage is moderate, inventories increase price and trade volume, but when leverage is high, trade may become impossible (a "market freeze"). Our analysis predicts a pattern of trade in which prices and trade volume first increase, and then markets break down. We use our model to discuss implications for regulatory intervention in illiquid markets. We also analyze the effects of competition between multiple buyers.

*We thank Jeremy Berkowitz, Mitchell Berlin, Alexander Bleck, Michal Kowalik, James Thompson, and Alexei Tchistyi, for their helpful comments. We also thank conference participants at the AFA meeting, FIRS meeting, European Economic Association meetings, Eastern Finance Association meetings, and the Federal Reserve System Committee on Financial Structure and Regulation. Any remaining errors are our own. The views expressed here are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System.

1 Introduction

Consider the sale of mortgages by a loan originator to a buyer. As widely noted, the originator has a natural information advantage and knows more about the quality of the underlying assets than other market participants. One consequence, which has been much discussed, is that he will attempt to sell only the worst mortgages.[1] However, a second important feature of this transaction has received much less attention. Both the buyer and the seller may hold significant inventories of mortgages similar to those being sold, and they may care about the market valuation of these inventories, which affects how much leverage they can take. Consequently, they may care about the dissemination of any information that affects market valuations of their inventories. In this paper, we analyze how inventories affect trade, information dissemination, and price formation. Our setting applies to the sale of mortgage-related products, but more broadly, to situations in which the seller has more information about the value of the asset.

One important setting in which these issues arise is when the buyer is a broker-dealer, effectively acting as a market-maker. We use a variant of the seminal Glosten and Milgrom (1985) model of price-setting in markets with asymmetric information, in which the uninformed party (the market-maker in their model, here the buyer) posts a "bid" price at which he is prepared to buy.[2] We depart from Glosten and Milgrom by first analyzing the benchmark case in which the buyer is a monopolist, and then modelling the effects of competition when more buyers are present.

Our main result is that the effect of inventories on trade depends on the buyer's and seller's initial leverage. When leverage is moderate, inventories increase price and trade volume (a "market run up"), but when leverage is high, trade may become impossible (a "market freeze"). Our model predicts a pattern of trade in which prices and trade volume first increase, and then markets break down.

The intuition is as follows. Since the seller has an information advantage, a sale reveals information about the value of the traded asset. This information may be used to reassess the value of inventoried assets and the amount of leverage that the buyer and seller can take. To ensure neither agent violates his leverage constraint (i.e., that

[1] See, for example, Ashcraft and Schuermann (2008); and Downing, Jaffee and Wallace (2009).

[2] In our setting, it is natural to assume that the direction of trade is always from the informed seller to the uninformed buyer, and so we do not consider ask prices.

the market value of each agent's assets is greater than the value of his liabilities), the buyer may need to offer a higher price, which increases the market's posterior about the value of the asset, and hence of inventories. The higher price also increases the probability that the seller will accept the offer. However, when the leverage constraint is too tight, the buyer can no longer increase the price without losing money; thus, trade collapses and market participants learn nothing about asset values.[3]

Because asset fundamentals remain unchanged, the model's prediction that a run-up in prices may precede a market breakdown may seem like a "bubble." Instead, the run-up in prices reflects the fact that increasing inventories force the buyer to increase his bid. Specifically, when the buyer adds assets to his balance sheet, he reduces the market value of his existing assets and increases his leverage. This forces him to bid a higher price in the next trade, or else not bid at all.

As noted above, we use our framework to analyze the effects of competiton between multiple buyers. In broad terms, the results from the benchmark single-buyer case continue to hold. In addition, we obtain two further interesting results. First, there is sometimes a "spillover" of capital constraints, whereby one buyer is forced to raise his bid price to a level that satisfies a *competing* buyer's capital constraint. Second, in other instances a buyer is forced to acquire assets at a *loss-making* price solely to ensure that a competing buyer does not acquire them at a lower price. In each of these cases, competition and capital constraints combine to push the price strictly higher than would be the case under either competition alone, or a single buyer with capital constraint.

We use our model to discuss implications for regulatory intervention in illiquid markets (See Section 4). On the buyer side, our analysis highlights the potential role of a large investor unencumbered by existing inventories (the government, for example). On the seller side, our analysis suggests limitations to the standard prescription that sellers should retain a stake in the assets they sell.

Related literature. Numerous papers explore the link between leverage and trade. For example, Shleifer and Vishny (1992) show that high leverage may force firms to sell assets at fire-sale prices.[4] Other papers explore feedback effects between asset

[3]In contrast, in a standard lemons problem with common knowledge of the gains from trade but without inventories, trade never completely breaks down.

[4]Two recent papers build on Shleifer and Vishny's intuition to explain market freezes. Diamond and Rajan (forthcoming) show that the prospect of fire sales may drive down current prices and induce banks not to sell their assets because the gains from selling are captured by the bank's

prices and leverage: Low prices reduce borrowing capacity, and hence asset holdings and prices also.[5] In contrast, we model a situation in which firms can meet their financial needs by staying with the status quo. Therefore, there is no need for fire sales or "cash in the market" pricing, as in Allen and Gale (1994). The only motive for trade in our paper is that the buyer values the asset more than the seller does, and both agents know this.

In a contemporaneous paper, Milbradt (2010) studies a trader subject to a mark-to-market leverage constraint, i.e., one based on the last trade price rather than on the actual expected value of the asset. The trader may suspend trade so that losses are not revealed. The main difference between our paper and his is that in his paper the price is exogenous, whereas in our paper the price is endogenous. In addition, our results do not depend on a specific accounting or regulatory regime. Instead, the market value of existing assets is derived from Bayes' rule.[6] Our main results continue to hold, however, even if we assume marking to market.

The idea that inventory holdings affect trade is found in the market microstructure literature. For example, Amihud and Mendelson (1980) study a market-maker whose total inventory must satisfy an exogenously imposed constraint, and Ho and Stoll (1981, 1983) study market-makers who care about the risk of holding inventories. These papers assume symmetric information and therefore are silent with respect to our main results. More importantly, these papers predict that as inventories increase, prices fall—a prediction that seems inconsistent with the empirical findings in Manaster and Mann (1996). We show instead that when market makers care about the market value of their inventories, higher inventories may lead to higher prices.

Finally, our paper relates to the literature on equity issuance, in which the issuing firm cares about the market valuation of its remaining equity.[7] However, we do not focus on signaling. Instead, we show how leverage affects the buyer's bidding strategy and the probability of trade.

creditors rather than the bank's equity holders. Acharya, Gale, and Yorulmazer (forthcoming) explain why banks may not be able to roll over short-term loans, even though they post collateral whose value is expected to be high in the long term.

[5] See, for example, Kiyotaki and Moore (1997), Kyle and Xiong (2001), Brunnermeier and Pedersen (2009), and Acharya and Viswanathan (forthcoming). Adrian and Shin (2010) provide empirical evidence.

[6] In this respect, our paper differs from the literature on the effects of market-value accounting, e.g., Allen and Carletti (2008); Heaton, Lucas, and McDonald (2010); and Plantin, Sapra, and Shin (2008).

[7] See, for example, Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989).

Paper outline. The paper proceeds as follows. Section 2 describes the model. In Section 3, we show our main result in a static model, both for the case in which the seller is concerned about the value of his inventories and the case in which the buyer is concerned. In Section 4, we discuss policy implications, and in Section 5 we extend the results to a two-period framework, in which leverage is endogenous. In Section 6, we anlayze the effects of competiton between multiple buyers, and in Section 7, we discuss extensions and robustness (e.g., the effect of marking to market). We conclude in Section 8. The appendix contains proofs.

2 The Model

There is a risk-neutral buyer and a risk-neutral seller. The value of an asset is v to the seller and $v + \Delta$ to the buyer, where $\Delta > 0$ denotes the gains from trade. It is common knowledge that v is drawn from a uniform distribution on $[0, 1]$. The seller knows v. Everyone else is uncertain about the value of v; consequently, trade affects posterior beliefs about v. Since $\Delta > 0$, trade is always efficient.

In the monopolist case, we assume the following trading process: The buyer makes a take-it-or-leave-it offer to buy q units of the asset at a price per unit p. The seller can either accept or reject the offer.[8] If the seller accepts, the seller's profit is $\pi_s = q(p - v)$, and the buyer's profit is $\pi_b = q(v + \Delta - p)$. If the seller rejects the offer, each ends up with a zero profit. The outcome of the bargaining game is publicly observable and is denoted by $\psi = (p, q, \phi)$, where ϕ denotes whether the seller accepted the offer ($\phi = 1$) or rejected it ($\phi = 0$).[9]

At the point when the buyer trades with the seller, he has already acquired an inventory of M_b units of the asset (see Section 5 below). The seller owns x units of the asset for sale and $M_s - x$ units of a second asset for which there are no gains from trade. The values of the two assets are correlated, and for simplicity, we assume perfect correlation; i.e., the value of the second asset is v to both the buyer and the seller.[10]

[8] From Samuelson (1984), this is the buyer's most preferred trading mechanism in the benchmark case in which the buyer and seller do not care about the market valuation of existing inventories.

[9] In Section 7, we show that the results remain even if market participants do not observe the terms of rejected offers.

[10] Giving the buyer an additional inventory of the second asset would slightly complicate the algebra but would have no qualitative effect on our analysis.

Both the buyer and the seller are subject to capital constraints, which require that the market value of their assets be high enough relative to the value of their liabilities. For $i = b, s$, we write $h_i(\psi)$ for the market value of one unit of asset, conditional on the trade outcome ψ. The functions $h_i(\psi)$ are derived using Bayes' rule, as explained in the next section. Let L_i denote an agent's debt, and Z_i an agent's cash holdings, both prior to trade.

The seller's capital constraint is

$$h_s(\psi)(M_s - \phi q) + \phi pq + Z_s \geq L_s, \tag{1}$$

where $M_s - \phi q$ is the seller's total inventory of assets net of trade, and $\phi pq + Z_s$ is the seller's cash, net of trade.

Similar to the seller's, the buyer's capital constraint is

$$h_b(\psi)(M_b + \phi q) - \phi pq + Z_b \geq L_b. \tag{2}$$

In Section 7, we discuss a more general case in which the value of the buyer's and seller's asset holdings is subject to a "haircut"; that is, only a fraction of the market value contributes toward the capital constraint. Formally, the functions $h_i(\psi)$ in the two equations above are replaced with $\alpha_i h_i(\psi)$, where $\alpha_i \in (0, 1]$ is a constant.

The seller's utility depends on profits and on whether the capital constraint holds, as follows:

$$U_s = \begin{cases} \phi\pi_s & \text{if } h_s(\psi)(M_s - \phi q) + \phi pq \geq L_s - Z_s \\ \phi\pi_s - F_s & \text{otherwise.} \end{cases} \tag{3}$$

The buyer's utility is obtained in a similar way:

$$U_b = \begin{cases} \phi\pi_b & \text{if } h_b(\psi)(M_b + \phi q) - \phi pq \geq L_b - Z_b \\ \phi\pi_b - F_b & \text{otherwise.} \end{cases} \tag{4}$$

The buyer's and seller's capital constraints may be imposed by a regulator or by potential lenders. One can interpret F_s and F_b as the loss of growth opportunities due to closure by the regulator or due to the fact that the lender seizes assets if the capital constraint is violated.

We focus on the case in which the capital constraints are satisfied before trading begins. This assumption allows us to focus on the question of how the buyer and seller change their behavior to avoid violating the capital constraint, rather than on the much-studied fire sales that follow when the constraints are violated. We also assume that F_s and F_b are sufficiently high so that the buyer's and seller's first priority is to satisfy their capital constraints. Specifically, we assume that $F_s > x$ and $F_b > x(1+\Delta)$.[11]

Finally, we assume that the quantity of the asset available for trade is small relative to the buyer's existing asset holdings; that is,

Assumption 1 $x < M_b$

This assumption ensures that increasing the bid loosens the buyer's capital constraint (see subsection 3.3). Increasing the bid always loosens the seller's capital constraint, even without Assumption 1 (subsection 3.2).

3 Trade, Volume, and Prices with a Monopolist Buyer

We focus on three cases: (1) the benchmark case in which neither the buyer nor the seller cares about the market valuation of their existing inventories; (2) the case in which only the seller cares about inventories;[12] and (3) the case in which only the buyer cares about inventories.[13]

3.1 Benchmark Case (No One Cares About Inventories)

In the benchmark case, the buyer offers a pair (p, q) to maximize his expected profits subject to $q \leq x$. To ensure that the seller's acceptance decision is nontrivial, we assume that the gains from trade are not too high, $\Delta < \frac{1}{2}$, so that the buyer always offers to pay $p \leq 1$. The seller accepts the offer if and only if $v \leq p$, which happens with probability p (since v is uniform on $[0, 1]$). Conditional on the seller accepting the offer, the expected value of the asset is $\frac{1}{2}p$ to the seller and $\frac{1}{2}p + \Delta$ to the buyer, and since the buyer pays p, his expected profit per unit bought is $\Delta - \frac{1}{2}p$. Taking into account the probability of trade and the quantity traded, the buyer's expected profit is $\pi(p, q) = q\pi(p)$, where $\pi(p) \equiv p(\Delta - \frac{1}{2}p)$.

[11] Since $\pi_s \leq x$ and $\pi_b \leq x(1+\Delta)$, the assumption ensures that the disutility from violating the capital constraint (F_i) is always larger than the potential profit.

[12] For instance, $Z_b - x \geq L_b$, so the buyer's capital constraint is satisfied even if he pays the highest possible price for the asset ($1) and the asset's value turns out to be zero.

[13] For instance, $Z_s \geq L_s$, so the seller's capital constraint is satisfied even if the value of his assets is zero.

The buyer's profit-maximizing bid is to buy everything, $q = x$, for a price $p = \Delta$. Thus, the probability of trade, p, increases when the gain from trade is higher. The gains from trade are split equally between the buyer and seller. The seller obtains rents because of his private information. The buyer obtains rents because he is the one making the offer.

Proposition 1 *In the benchmark case, the buyer offers to buy x units at a price per unit Δ. The seller accepts this offer if and only if $v \leq \Delta$.*

For use below, observe that any $p \in [0, 2\Delta]$ provides the buyer with nonnegative profits. Thus, starting from the optimal bid, the buyer can increase the price, while still maintaining positive profits.

3.2 Only the Seller Cares About the Value of His Inventory

In this subsection, we consider the case in which the seller cares about the value of his inventory but the buyer does not. A necessary condition for this case is that the seller's liabilities exceed his cash, $L_s > Z_s$.

Since the seller's capital constraint is initially satisfied and the cost of violating it is high, it is optimal for the seller to accept an offer with $q > 0$ if and only if (i) he makes nonnegative profits; that is, $v \leq p$; and (ii) conditional on accepting the offer, the capital constraint is not violated.[14] A standard "lemons" argument implies that if the capital constraint is satisfied after an offer is accepted, it is also satisfied after the same offer is rejected.

Since v is uniform on $[0, 1]$, we obtain from Bayes' rule that if the seller accepts an offer with $q > 0$, the market's expected value of the asset is $h_s(\psi) = \frac{1}{2}p$; if the seller rejects, $h_s(\psi) = \frac{1}{2}(1 + p)$, which is the expected value of v given that $v \in [p, 1]$. If the buyer does not make an offer (i.e., $q = 0$), the marker's expected value of the asset simply matches the prior, $h_s(\psi) = \frac{1}{2}$.

Substituting h_s into the seller's capital constraint delivers

$$\frac{1}{2}p(M_s - q) + pq \geq L_s - Z_s. \tag{5}$$

Define $\delta_s \equiv \frac{L_s - Z_s}{\frac{1}{2}M_s}$, a measure of the seller's initial leverage (i.e., the ratio of his net liabilities to the initial market valuation of his assets). The seller's capital constraint

[14] Note that the seller does not gain from ex post loosening his capital constraint. Hence, rejecting a profitable offer in order to increase the market value of his asset is suboptimal.

can be rewritten as

$$p \geq \frac{\delta_s}{1 + \frac{q}{M_s}}. \tag{6}$$

Holding the offer price p fixed, it is easier to satisfy the seller's capital constraint when q is higher. Intuitively, since the market valuation of the asset is less than the sale price (for standard adverse selection reasons), replacing assets with cash relaxes the capital constraint. Consequently, if the buyer finds it worthwhile to bid at all, he bids for the entire quantity available, $q = x$. Bidding for a lower quantity not only lowers the buyer's profits, but it also makes it harder to satisfy the seller's capital constraint and have him accept the offer.

The buyer's problem reduces to choosing p to maximize his expected profits $\pi(p, x)$, such that $p \geq \frac{\delta_s}{1+\frac{x}{M_s}}$ so that the seller's capital constraint is satisfied. Since the buyer loses money from bids $p > 2\Delta$, trade is impossible if $\frac{\delta_s}{1+\frac{x}{M_s}} > 2\Delta$. If instead $\frac{\delta}{1+\frac{x}{M_s}} \leq 2\Delta$, the buyer bids as close to his benchmark bid of Δ as possible; that is, $p = \max(\Delta, \frac{\delta_s}{1+\frac{x}{M_s}})$.

Proposition 2 *When only the seller cares about the value of his inventory, trade can occur if and only if $\delta_s \leq 2\Delta(1 + \frac{x}{M_s})$. In this case, the buyer offers to buy x units at a price per unit $\max(\Delta, \frac{\delta_s}{1+\frac{x}{M_s}})$, and the seller accepts if and only if $v \leq p$.*

When the seller's initial leverage is low, the price and the probability of trade are the same as in the benchmark case because the seller has enough slack to satisfy his capital constraint even though trade reduces the perceived value of his remaining assets. When leverage increases, so that the seller has less slack, the buyer must increase his bid to ensure that the seller's capital constraint holds if he accepts the offer. Since a higher bid increases the probability that the seller will accept the offer, the probability of trade increases. Finally, if leverage is too high, the market breaks down because any bid that is high enough to satisfy the seller's capital constraint yields negative expected profits to the buyer.

The seller's leverage shifts bargaining power from the buyer to the seller. With no leverage, the buyer's bargaining power allows him to reduce the price and probability of trade so that the buyer's profits are maximized. However, when leverage increases, the buyer is forced to increase the price so that the seller's capital constraint is satisfied. We focus on an extreme case in which the buyer has all of the bargaining power, but the nature of the result remains even if the buyer has only some of the bargaining power.

Increasing x (the maximum amount that can be sold) increases the region in which trade can happen. Intuitively, when x increases, the nontraded asset becomes a smaller fraction of the seller's balance sheet, and it does not matter that its perceived value falls. Increasing Δ (the gains from trade) also increases the region in which trade can happen because the buyer can bid a higher price without losing money.

An immediate corollary to Proposition 2 concerns the effect of high leverage and the corresponding market breakdown on the revelation of the seller's information about asset values:

Corollary 1 *If initial leverage is high, $\delta_s > 2\Delta(1 + \frac{x}{M_s})$, market participants learn nothing about the value v of the asset.*

3.3 Only the Buyer Cares About the Value of His Inventory

In this subsection, we consider the case in which the buyer cares about the value of his inventory but the seller does not. A sufficient condition for this case is that the buyer's liabilities exceed his cash, $L_b > Z_b$. We show that the effect of leverage on the probability of trade is similar to the seller's case. However, now it may be optimal for the buyer to purchase *less* than the full amount; that is, we may have $q \in (0, x)$. In addition, in the seller's case, expected volume, $q \Pr(v \leq p)$, was discontinuous in leverage, whereas here expected volume is typically continuous in leverage; it first rises and then drops gradually to zero.

The buyer's capital constraint could be based either on the value of his assets to others, i.e., on the conditional expectation of v; or it could be based on the value of his assets to himself, in which case the gains from trade Δ should also be included. To handle both cases, let $\gamma \in [0, 1]$ and define $h_b(\psi) = \frac{1}{2}p + \gamma\Delta$ for cases in which the seller accepts an offer, and $h_b(\psi) = \frac{1}{2}(1 + p) + \gamma\Delta$ for those in which he rejects the offer. As before, for the case of no offer ($q = 0$), define $h_b(\psi) = \frac{1}{2} + \gamma\Delta$.

Substituting h_b into the buyer's capital constraint yields

$$(\frac{1}{2}p + \gamma\Delta)(M_b + q) - pq \geq L_b - Z_b. \tag{7}$$

Define $\delta_b \equiv \frac{L_b - Z_b}{(\frac{1}{2} + \gamma\Delta)M_b}$, a measure of the buyer's initial leverage. Since $q \leq x < M_b$

(Assumption 1), the buyer's capital constraint can be rewritten as

$$p \geq \frac{\delta_b + 2\gamma\Delta(\delta_b - 1 - \frac{q}{M_b})}{1 - \frac{q}{M_b}} \equiv p(q), \tag{8}$$

where $p(q)$ is the minimum price that the buyer can offer so that his capital constraint remains satisfied if the seller accepts the offer.

Equation (8) implies that increasing p loosens the capital constraint. Increasing the price increases the perceived value of existing inventories, which helps loosen the capital constraint, but it also increases the amount the buyer pays for the additional units he purchases, which tightens the capital constraint. When the amount of inventories is large relative to the amount for sale (Assumption 1), the first effect dominates.

The buyer's problem is to choose a bid $(p, q) \in [0, 1] \times [0, x]$ to maximize $\pi(p, q)$, subject to his capital constraint. In cases of indifference, we assume that the buyer makes the bid associated with the highest quantity q, thereby maximizing social welfare. As in the seller's case, whenever there is trade, $p \leq 2\Delta$, so that the buyer does not lose money.

Start with the special case $\gamma = 1$. In this case, the coefficient of q in equation (7) is nonnegative, and it is optimal to offer either $q = 0$ or $q = x$, as in the seller's case. Intuitively, if $\gamma = 1$, the capital constraint captures the full asset value to the buyer, and since the buyer makes nonnegative profits, buying more assets relaxes the capital constraint. As in the seller's case, trade can happen only if the buyer's initial leverage δ_b is sufficiently low so that $p(x) \leq 2\Delta$; that is, if $\delta_b \leq \frac{4\Delta}{1+2\Delta}$. If trade happens, the buyer chooses $p = \max(\Delta, p(x))$, and the seller accepts if and only if $v \leq p$.

Proposition 3 *When only the buyer cares about the value of his inventory and* $\gamma = 1$, *trade can happen if and only if* $\delta_b \leq \frac{4\Delta}{1+2\Delta}$. *In this case, the buyer offers to buy* x *units at a price per unit* $\max(\Delta, p(x))$.

In contrast, when $\gamma < 1$, an interior solution $q \in (0, x)$ may be optimal. When the buyer is highly levered, he must bid a very high price to satisfy his capital constraint. However, with such a high price, the buyer ends up paying more than the borrowing capacity of the assets he purchases, and this tightens his capital constraint; for example, if $\gamma = 0$, the buyer pays pq, but the borrowing capacity increases by only $\frac{1}{2}pq$. The buyer can mitigate this problem by reducing the quantity. Reducing the quantity not only helps the buyer relax his capital constraint, but it also allows him to reduce the price so that he can increase his profits.

Proposition 4 *When only the buyer cares about the value of his inventory and $\gamma < 1$, trade can happen if and only if the buyer's initial leverage satisfies $\delta_b < \frac{2(1+\gamma)\Delta}{1+2\gamma\Delta}$. If leverage is sufficiently low, the capital constraint does not bind, and the buyer offers to buy the entire quantity x for a price Δ. As leverage increases, the buyer increases the price; and as leverage increases further, the buyer also reduces the quantity he offers to buy. Both price and quantity are continuous in leverage. As leverage approaches $\frac{2(1+\gamma)\Delta}{1+2\gamma\Delta}$, the price approaches 2Δ and the quantity approaches zero. Expected volume is also continuous in leverage; it first increases and then drops to zero.*

As in the seller's case, the probability of trade first increases in leverage, but then trade collapses. When $\gamma = 1$, expected volume is discontinuous in leverage, as in the seller's case. However, when $\gamma < 1$, expected volume is continuous in leverage; it first increases and then falls to zero, as the buyer reduces the quantity. The initial increase in expected volume occurs because at moderate levels of leverage, the buyer increases the price but keeps the quantity unchanged, at $q = x$. The buyer does so because reducing the quantity either tightens the capital constraint (and reduces profits), or else it relaxes the constraint but reduces profits too much; however, in the latter case, it is more profitable to relax the capital constraint only by increasing the price.

4 Policy Implications

Our model implies that socially efficient trade can completely break down ("freeze") if the seller has an information advantage and if either the buyer or the seller is both highly leveraged and holds significant inventories of similar assets. This implication is consistent with the freeze in the markets for mortgage-backed securities during the recent financial crisis. Adrian and Shin (2010) document a sharp increase in dealers' leverage, while many market observers expressed the view that concerns about the value of inventories induced firms not to sell their assets. For example, an analyst was quoted in *American Banker*[15] as saying that "Other [companies] may be wary of selling assets for fear of establishing a market-clearing price that could force them to mark down the carrying value of their nonperforming portfolio." Also related is

[15] "Nonperformance Space: Risky Assets Find Market" (*American Banker*, August 19, 2009).

the view expressed in Lewis' book (2010) that dealers who sold credit default swaps on subprime mortgage bonds did not make a market in these securities so that the bad information is not revealed and their positions do not lose money. Moreover, and consistent with our results in Section 5, Lewis suggests that prior to the crisis, prices increased in a way not supported by fundamentals.[16]

Our analysis has implications for government attempts to defrost markets and for regulatory proposals aimed at improving market functioning.

4.1 Defrosting Frozen Markets

Consider the case in which only the buyer cares about inventory values and in which trade has completely broken down; that is, $\delta_b > \frac{2(1+\gamma)\Delta}{1+2\gamma\Delta}$ (Propositions 3 and 4).

Purchasing assets from the seller. One option open to a government is to offer to buy the seller's assets. Formally, suppose the government's valuation of the asset is $v + \Delta_g$, where $\Delta_g < \Delta$, and as before v is private information to the seller. In line with commonly voiced concerns, a general problem with voluntary government purchase schemes is that sellers part with only their worst assets: If the government offers to pay p, the seller sells only if $v \leq p$ (the same as with a private buyer) and makes an expected profit of $\frac{p^2}{2}$.

A central question is whether the government purchase scheme can succeed without taxpayer subsidies (in expectation). Our model has two implications in this respect. First, observe that because of the rent the seller makes from his informational advantage, a subsidy-free purchase scheme is possible *only if* the asset is worth more to the government than to the seller ($\Delta_g > 0$); for example, if the government is a more efficient holder of risk than the asset seller. Second, even if this condition is satisfied, a subsidy-free purchase scheme imposes a cost on the original potential buyer. Recall that this buyer does not purchase the asset himself because doing so violates his capital constraint. However, the same is true when the government buys

[16] For example, on page 184, Lewis writes that "Burry [an investor who bought credit default swaps on subprime mortgage bonds] sent his list of credit default swaps to Goldman and Bank of America and Morgan Stanley with the idea that they would show it to possible buyers, so he might get some idea of the market price. That, after all, was the dealer's stated function: middleman. Market-makers. That is not the function they served, however. 'It seemed the dealers were just sitting on my lists and bidding extremely opportunistically themselves,' said Burry. The data from the mortgage servicers was worse every month...and yet the price of insuring those loans, they said, was falling." On page 185, he adds that "The firms always claimed that they had no position themselves...but their behavior told him otherwise."

the asset at unsubsidized terms.[17] A similar issue arises if the government subsidizes a second private buyer to purchase the asset. Consequently, if either the asset is worth less to the government than to the seller, $\Delta_g \leq 0$, or if the government wishes to avoid hurting the original buyer, a taxpayer subsidy is required to defrost the frozen market.[18]

Purchasing assets from the buyer. If the government is a more efficient holder of assets than the seller, i.e., $\Delta_g > 0$, then the government might be able to unfreeze the market without taxpayer subsidies and without hurting the original buyer by purchasing assets from the buyer rather than the seller. For this to succeed, the government valuation of the asset must not be too low relative to the buyer's. Removing assets from the buyer's balance sheet reduces the buyer's leverage and allows him to purchase the seller's assets without violating the capital constraint. This is true if purchasing assets from the buyer does not expose the government to adverse selection (as in our model), but it is also true if the buyer has more information than the government.

To see that, suppose the government plans to spend Z_g dollars. If there is no adverse selection, the value to the government of each unit purchased from the buyer is $\frac{1}{2}+\Delta_g$, and the government can buy $Z_g/(\frac{1}{2}+\Delta_g)$ units without a taxpayer subsidy. Since each of these units allows the buyer to borrow $\frac{1}{2}+\gamma\Delta$ dollars, the government purchase loosens the buyer's capital constraint only if the sale proceeds exceed the assets' borrowing capacity; that is, if $Z_g > (\frac{1}{2}+\gamma\Delta)\frac{Z_g}{\frac{1}{2}+\Delta_g}$. This reduces to $\Delta_g > \gamma\Delta$.[19] If purchasing assets from the buyer involves adverse selection, the government valuation will need to be even higher.

4.2 Should Regulation Mandate Some Retention of the Asset by the Seller?

A commonly voiced regulatory proposal is that sellers of assets subject to asymmetric information problems, such as issuers of asset-backed securities, should be required to

[17] Formally, since the market has broken down, we know that $p(q) > 2\Delta$, for every $q \in (0,x]$; and since $\Delta_g < \Delta$, it follows that any unsubsidized offer $p \leq 2\Delta_g$ satisfies $p < 2\Delta$. Therefore, $p < p(q)$ for every $q \in (0,x]$, and any unsubsidized offer violates the buyer's capital constraint.

[18] The idea that a purchase by one buyer (or the government) can hurt another buyer is analyzed in more detail in Section 6, in which we discuss competition among multiple buyers.

[19] If the buyer can borrow against only a fraction $\alpha \in (0,1)$ of his assets (see Section 7), the condition above becomes $Z_g > \alpha(\frac{1}{2}+\gamma\Delta)\frac{Z_g}{\frac{1}{2}+\Delta_g}$, which reduces to $\Delta_g > \alpha\gamma\Delta - \frac{1}{2}(1-\alpha)$.

retain some stake in the assets they sell.[20] Our analysis identifies a potential cost to this proposal, namely, that under some circumstances it leads to a market breakdown. To see this, reinterpret the parameter x in our model as stemming from a regulation mandating that the seller retain a fraction $\frac{M_s - x}{M_s}$ of the asset he is selling. From Proposition 2, whenever x is sufficiently low, trade is impossible because the seller cares too much about the market's perception of the value of the assets he is forced to retain. Moreover, notice that this case arises more easily when the seller is highly leveraged (measured by δ_s). And, of course, in addition to the possibility of a market breakdown, restricting the amount the seller can trade reduces the expected volume of trade.

The goal that regulators appear to have in mind with this regulation is to reduce moral hazard on the part of asset sellers; for example, to discourage loan originators from making bad loans and/or shirking on monitoring later on. Our analysis does not speak to this issue, and it seems likely that the regulation will have its intended effect in this regard. Our point here is instead to draw attention to a potentially significant cost of this regulation, namely, that it can lead to the breakdown of socially efficient trade.

5 Endogenous Leverage

So far, we have taken traders' leverage, and hence the tightness of their capital constraints, as given. In practice, both emerge endogenously from prior decisions. We model this by extending our single-period model to a two-period model in which one buyer trades sequentially with two potential sellers. We focus on the case in which the buyer is capital constrained but the sellers are not. Leverage is endogenous because the outcome of trade with the first seller affects the value of the buyer's assets before the second trade. We characterize the buyer's optimal bidding strategy. One of the results is that a market freeze may be preceded by a run-up in prices and increased trade volume.

Each seller sells a different asset; seller i ($i = 1, 2$) sells asset i. The value (per unit) of asset i is v_i to the seller and $v_i + \Delta$ to the buyer, where v_1, v_2 are independent random variables drawn from a uniform distribution on [0,1]. Each seller can sell at most x units. Before trading begins, the buyer has inventories of M units of asset

[20] See, for example, section 15G of the Investor Protection and Securities Reform Act of 2010.

1 and M units of asset 2. Since the values of the two assets are independent, one cannot infer anything about the value of one asset by observing trade in the other asset. This allows us to focus only on the effect of leverage. As before, assume $x < M$ and $\Delta \in (0, 1/2)$.

In the first period, the buyer makes a take-it-or-leave-it-offer (p_1, q_1) to the first seller, who can either accept or reject the offer. In the second period, the buyer makes a take-it-or-leave-it-offer (p_2, q_2) to the second seller, who can also either accept or reject it. Assume that $q_i \in \{0\} \cup [\underline{q}, x]$; that is, if the buyer offers to buy something, he must buy at least $\underline{q} > 0$ units. The parameter $\underline{q}$ can be made arbitrarily small; as we explain below, this assumption is made to avoid an open set problem. Note that adding this assumption has no substantive effect on the results in the previous sections.

Let $\phi_i = 1$ if seller i accepts the offer (i.e., $v_i \leq p_i$), and $\phi_i = 0$ otherwise; and denote by $h(\psi_i)$ the perceived value of asset i given the trading outcome $\psi_i = (p_i, q_i, \phi_i)$. The buyer's capital constraint is

$$\sum_{i=1}^{2} [h(\psi_i)(M + \phi_i q_i) - \phi_i p_i q_i] \geq L_b - Z_b. \tag{9}$$

As in Subsection 3.3, if seller i accepts, $h(\psi_i) = \frac{1}{2}p_i + \gamma\Delta$; if seller i rejects, $h(\psi_i) = \frac{1}{2}(1 + p_i) + \gamma\Delta$; if the buyer makes no offer in period i, the value remains at its ex ante level, $h(\psi_i) = \frac{1}{2} + \gamma\Delta$.

If the buyer makes an offer in the first period but does not make an offer in the second period, equation (9) becomes

$$[h(\psi_1)(M + \phi_1 q_1) - \phi_1 p_1 q_1] + (\frac{1}{2} + \gamma\Delta)(M) \geq L_b - Z_b. \tag{10}$$

We refer to inequality (10) as the first-period capital constraint, since it corresponds to the case in which the game ends exogenously after the first period. We refer to inequality (9) as the second-period constraint.

For simplicity, we assume that the discount rate equals zero. If the capital constraint is satisfied in both periods (i.e., if equations (9) and (10) are satisfied), the buyer's utility is $\pi(p_1, q_1) + \pi(p_2, q_2)$. Otherwise, the buyer's utility is $\pi(p_1, q_1) + \pi(p_2, q_2) - F$. The assumption that the buyer incurs a cost if he violates the first-period constraint can be motivated by assuming that with a sufficiently high probability the game ends after the first period and the buyer cannot make a second offer.

The buyer's problem is to choose a sequence of offers $(p_i, q_i)_{i=1,2}$ to maximize his expected utility. As before, in cases of indifference, we assume the buyer makes the bid associated with the highest quantity, thereby maximizing social welfare. We also assume that initially the capital constraint is satisfied and that F is sufficiently large so that satisfying the capital constraint in each period is the first priority.

Since the parameters in each round are the same, it is suboptimal to delay offers; if it is suboptimal to make an offer in the first round, it is also suboptimal to make an offer in the second round. Thus, a bidding strategy can be summarized by $(p_1, q_1; p_a, q_a; p_r, q_r)$, where (p_1, q_1) denotes the offer to the first seller, and $(p_a, q_q), (p_r, q_r)$ denote the offer to the second seller given that the first seller accepted or rejected the offer, respectively.

Since the first seller accepts the offer with probability p_1, the buyer's expected utility is

$$\pi(p_1, q_1) + p_1 \pi(p_a, q_a) + (1 - p_1)\pi(p_r, q_r). \tag{11}$$

The problem reduces to finding a bidding strategy that maximizes the buyer's expected utility such that equations (9) and (10) are satisfied — both after acceptance and after rejection — when the sellers respond optimally.

For expositional ease, we focus on the case $\gamma = 1$. Then it follows from Proposition 3 that in the second period the buyer offers to buy either everything or nothing and makes nonnegative profits. In contrast, in the first period, the buyer's profits may be negative, and it may be optimal for him to offer neither 0 nor x, but instead the lowest amount possible, $\underline{q}$. Offering $\underline{q}$ (together with a very high price) might be optimal because if the offer is rejected, the market valuation of the buyer's inventory is high, and so the buyer enters the second period with a very slack capital constraint; this allows him to make a profitable trade. Since the buyer loses money if the first offer is accepted, it is optimal to reduce the quantity to the lowest amount possible that is greater than zero.

Lemma 1 *The second-period offer satisfies $p_a, p_r \leq 2\Delta$ and $q_a, q_r \in \{0, x\}$; that is, the buyer offers to buy either everything or nothing, and he makes nonnegative profits. The first-period offer satisfies $q_1 \in \{0, \underline{q}, x\}$. If $q_1 = \underline{q}$, expected profits in the first period are negative; that is, $p_1 > 2\Delta$. If $q_1 = x$, expected profits in the first period are nonnegative; that is, $p_1 \leq 2\Delta$.*

Trade affects the buyer's leverage, as follows:

Lemma 2 *The acceptance of an offer tightens the capital constraint.*

Accepted offers reduce the market value of existing assets. However, the purchase of new assets may generate a profit. On net, these two forces tighten the capital constraint, since, by assumption, inventories are large relative to new trades ($M > x$).

In contrast, rejected offers relax the capital constraint. It turns out that whenever the buyer's first-period bid is rejected, his capital constraint becomes sufficiently slack that he can make his unconstrained optimal bid of Δ in the second period. The intuition is that an offer p_1 satisfies the capital constraint only if either the bid price p_1 is high — in which case a rejected bid results in a substantial slackening of the capital constraint; or the capital constraint is very slack to begin with.

Lemma 3 *If $p_1 > 0$, then $(p_r, q_r) = (\Delta, x)$.*

Our main result in this section is:

Proposition 5 *Trade can happen (i.e., $p_1 > 0$) if and only if the buyer's initial leverage is not too high.*

(i) When leverage is low, the buyer makes the benchmark bid (Δ, x) in both periods.

(ii) When leverage is intermediate, the buyer offers to pay strictly more than the benchmark in the first period, and if the first offer is accepted, the buyer offers to pay even more (i.e., $p_a > p_1 > \Delta$); in both periods the buyer bids for the maximum amount x.

(iii) When leverage is high, the buyer withdraws from the market in the second period if his first offer is accepted. The buyer's initial bid (p_1) is increasing in leverage. In particular, when initial leverage is sufficiently high, the buyer initially bids more than the benchmark; that is, the market freeze is preceded by high prices. The quantity the buyer bids for in the first period is decreasing in leverage.

Proposition 5 captures a few aspects of a dynamic behavior. If initial leverage is relatively moderate, the buyer has enough slack in his capital constraint to make two rounds of offers. But unless leverage is very low, the buyer still needs to consider his capital constraint, and this leads him to bid more than the benchmark price in both periods. If his first bid is accepted, his capital constraint is tightened, forcing him to bid even more in the second period. In other words, the price at which trade occurs rises with successful trades.

If instead initial leverage is high, the buyer has insufficient slack to have two bids accepted. Thus, there must be a period in which trade does not occur. In particular, if the buyer's first period offer is accepted, his capital constraint is too tight to make a bid in the second period and the market freezes. The proposition also sheds light on the price path leading up to this market freeze. When initial leverage is very high, the capital constraint is binding, forcing the buyer to make a high bid. Thus, the market freeze may be preceded by a run-up in prices.

6 Competition Among Buyers

Suppose there are two potential buyers, who are subject to capital constraints. Buyer i has an inventory of M_i units of asset, and net liabilities $L_i - Z_i$. The gain from trade with buyer i is Δ_i. The seller has x units for sale, and he is not subject to a capital constraint. Everything is common knowledge, except for the true value of the asset (v), which is private information to the seller. As before, $\Delta_i < \frac{1}{2}$, $x < M_i$, and the cost for violating the capital constraint is large.

We model competition by assuming that both buyers make offers simultaneously. Buyer i offers a price and quantity (p_i, q_i). If $v > \max(p_1, p_2)$, the seller rejects both offers and trade does not take place. Otherwise, the seller accepts the offer with the highest price. If both buyers post the same price, the seller chooses one of them randomly. If the buyer who posted the highest price, or the buyer who was randomly selected by the seller, does not offer to buy the full amount, the seller may also trade with the other buyer, but only if the price posted by the other buyer is above the seller's valuation. Hence, when the seller trades with both buyers, one can conclude the v is uniform on $[0, \min(p_1, p_2)]$. Posting a price of zero is equivalent to not making an offer, and we assume, without loss of generality, that $p_i > 0$ if and only if $q_i > 0$.

With competition, one should take into account the fact that trade may cause to a violation not only of the capital constraint of the buyer who trades, but also of the capital constraint of the other buyer, who does not trade. Denote by $p_i(x)$ the minimum price that buyer i must offer so that his capital constraint remains satisfied if the seller accepts only his offer. The expression for $p_i(x)$ follows from equation (8), and is given by

$$p_i(x) = \frac{\delta_i + 2\gamma_i\Delta_i(\delta_i - 1 - \frac{x}{M_i})}{1 - \frac{x}{M_i}}, \tag{12}$$

where $\delta_i \equiv \frac{L_i - Z_i}{(\frac{1}{2}+\gamma_i\Delta_i)M_i}$ is buyer i's initial leverage. Note that $p_i(0)$ is the minimum price that buyer i can afford for the seller to accept from buyer $-i$; if the seller accepts any lower price, buyer i's capital constraint is violated. Observe that $p_i(x)$ is a monotone increasing transformation of our leverage measure δ_i.

From Proposition 3, a monopolist buyer offers $\max(p_i(x), \Delta)$, provided $p_i(x) \leq 2\Delta$: this is the price that maximizes his profits, subject to satisfying his capital constraint. For use below, it is convenient to define $r_i \equiv \max(p_i(x), \Delta)$, and analogously, $r_i' = \max(p_i(0), \Delta)$. We focus on the case $\gamma_1 = \gamma_2 = 1$, so that the capital constraint for each buyer captures the full asset value to him.

Lemma 4 *For every $i \in \{1, 2\}$, one of the following is true: (i) $r_i = r_i' = 2\Delta_i$; (ii) $r_i > r_i' > 2\Delta_i$; or (iii) $r_i \leq r_i' < 2\Delta_i$.*

Intuitively, if $r_i < 2\Delta$, the buyer makes positive profits if his offer at price r_i is accepted, and since $\gamma_i = 1$, this helps to loosen his capital constraint. If the seller accepted the other buyer's offer, the buyer whose offer is not accepted would not make profits, and so it would be harder to satisfy his constraint. Hence, $r_i \leq r_i'$ (with an equality when $r_i = r_i' = \Delta$). In contrast, if $r_i > 2\Delta$, the buyer makes negative profits, which tightens his capital constraint. If the other buyer were the one who trades, the buyer would not lose money and it would be easier to satisfy his constraint. Hence, $r_i > r_i'$.

We characterize Nash equilibria of the bidding game. We focus on equilibria that survive iterated elimination of weakly dominated strategies. In our setting, this has the benefit of delivering a unique equilibrium for all parameter values (and this equilibrium does not depend on the order of elimination). We also note cases in which there exist other equilibria in which neither buyer plays a weakly dominated strategy (i.e., equilibria which survive only the first round of elimination).

To avoid technical issues, as in Section 5, we assume that the minimum quantity a buyer can offer to buy is $\underline{q}$, i.e., $q_i \in \{0\} \cap [\underline{q}, x]$, and we also assume that the price space is finite, and the values $\{r_i, r_i - \varepsilon, r_i + \varepsilon, 2\Delta_i, 2\Delta_i - \varepsilon, 2\Delta_i + \varepsilon\}_{i\in\{1,2\}}$ lie within this space. The tick size ε is assumed to be close to zero and for clarity, we exclude it from the statements of the results.

The next lemma characterizes the set of offers that are not weakly dominated.

Lemma 5 *(A) If* $r_i < 2\Delta_i$*, the only offers that are not weakly dominated satisfy* $p_i \in [r_i, 2\Delta_i)$ *and* $q_i = x$*.*

(B) If $r_i = 2\Delta_i$*, the only offer that is not weakly dominated is* $(p_i, q_i) = (r_i, x)$*.*

(C) If $r_i > 2\Delta_i$*, the only offers that are not weakly dominated are* $p_i = 0$*,* $(p_i, q_i) = (r_i, \underline{q})$ *and* $(p_i, q_i) = (r_i, x)$*.*

Part (A) says that when a buyer has a profitable trade, he always makes an offer that yields positive profits and that does not violate his capital constraint, just as in the single-buyer case (Subsection 3.3). Part (C) reflects the fact that, with competition, the buyer may also offer to buy the asset when it is not profitable to him. The buyer makes this "preemptive" bid to ensure that his capital constraint is not violated should the other buyer make an offer at a low-price.

6.1 Benchmark Case (No One Cares About Inventories)

In the benchmark case, in which neither buyer is subject to a capital constraint, the equilibrium price is $\min\{2\Delta_1, 2\Delta_2\}$. This is the standard outcome for settings with public buyer valuations. The buyer with the highest valuation acquires the asset at a price determined by the buyer with the second-highest valuation.[21] This result generalizes easily to the case in which both buyers have low leverage, so that their capital constraints are not binding in equilibrium.

Lemma 6 *If* $\max\{r_1, r_2\} \leq \min\{2\Delta_1, 2\Delta_2\}$*, the seller sells everything to the buyer with the higher valuation for a price* $\min\{2\Delta_1, 2\Delta_2\}$*.*

6.2 Equilibrium Outcomes When Buyers Are Leveraged

We start with the case in which at least one of the two buyers has a low-enough leverage such that he would acquire the asset if he were the only buyer. Without loss of generality, let this buyer be buyer 1; formally, $r_1 \leq 2\Delta_1$. Our main result is:

Proposition 6 *Assume* $r_1 \leq 2\Delta_1$*. Then:*

[21] See, for example, Ho and Stoll (1983).

(A) If buyer 2 has relatively low leverage, $r_2' \leq r_1$, the seller sells everything for price $\max\{r_1, \min\{2\Delta_1, 2\Delta_2\}\}$*. Whenever the price exceeds the no-leverage benchmark* $\min\{2\Delta_1, 2\Delta_2\}$*, only buyer* 1 *acquires the asset.*

(B) If buyer 2 has relatively high leverage, $r_2' > r_1$, the seller sells everything for price $\max\{r_2, \min\{2\Delta_1, 2\Delta_2\}\}$*. In particular, if $r_2 \in (2\Delta_2, 2\Delta_1)$, the seller sells everything to buyer* 1 *at price r_2; if $r_2 \geq$* $\max\{2\Delta_1, 2\Delta_2\}$*, the seller sells everything to buyer* 2*, who makes negative profits.*

Recall that in the benchmark case without capital constraints, the equilibrium price is $\min\{2\Delta_1, 2\Delta_2\}$. In part (A), capital constraints interact with competition in a straightforward way: the important capital constraint is buyer 1's, and sometimes forces buyer 1 to increase his offer to r_1.

In part (B), in contrast, the interaction between capital constraints and competition is less straightforward, and can lead to a form of "contagion" of capital constraints. That is, if buyer 2's leverage falls in the interval $r_2 \in (2\Delta_2, 2\Delta_1)$, buyer 2's capital constraint leads him to compete more aggressively with buyer 1, and consequently buyer 1 ends up paying an amount r_2 that is determined by buyer 2's capital constraint. If $r_2 \geq \max(2\Delta_1, 2\Delta_2)$, buyer 1 can no longer compete, and so buyer 2 acquires everything at a price r_2. In the latter case, buyer 2 makes negative profits, even though he would not bid at all if he were the only buyer. Buyer 2 is forced to make this bid, since otherwise the seller will trade with buyer 1 and the capital constraint of buyer 2 will be violated.

Finally, we consider the case in which both buyers are so leveraged, that, if bidding individually, trade collapses. Clearly, no trade is an equilibrium, since given that one buyer is not willing to acquire the asset, the unique best response for the other buyer is also not to acquire. For some parameter values we may obtain additional equilibria in which trade occurs.[22] In these equilibria one buyer makes an offer, knowing that it will not be accepted in equilibrium, and the second buyer makes a "preemptive" offer to rule out a situation in which the seller trades with the first buyer and the capital constraint of the second buyer is violated. These equilibria, however, do not survive iterated elimination of weakly dominated strategies.

[22] For example, if $r_1' > r_2$, there is an equilibrium in which $(p_1, q_1) = (r_1, x)$ and $p_2 = (r_2, x)$ and there is an equilibrium in which $(p_1, q_1) = (r_1, x)$ and $p_2 = (r_2, \underline{q})$. If $r_2' > r_1$, there is an equilibrium in which $(p_2, q_2) = (r_2, x)$ and $p_1 = (r_1, x)$ and there is an equilibrium in which $(p_2, q_2) = (r_2, x)$ and $p_1 = (r_1, \underline{q})$.

Proposition 7 *If $r_i > 2\Delta_i$ for $i \in \{1, 2\}$, the only equilibrium that survives iterated elimination of weakly dominated strategies is no trade.*

7 Extensions and Robustness

7.1 What Can the Market Observe?

So far, we have assumed that market participants observe the terms of the buyer's offer, regardless of whether the offer is accepted or rejected. What if, instead, market participants observe the offer terms only if the offer is accepted and trade actually occurs? Perhaps surprisingly, the equilibrium outcomes are exactly the same under this alternative information assumption.

To see this, consider first the one-period case. Let (p, q) be an equilibrium offer in the partial information case. If the offer is rejected, the market's beliefs about the asset value are based on the equilibrium offer. Consequently, (p, q) is an equilibrium offer if and only if it maximizes expected profits $q\pi(p)$ subject to the constraint that the capital constraint is satisfied after the offer is accepted. But this is exactly the same as the equilibrium condition for the case in which the terms of rejected offers are observed, since in this case, the capital constraint is satisfied after any rejected offer. Consequently, the equilibrium outcomes are the same in the two cases, as claimed. An identical argument applies in the dynamic case.[23]

7.2 Limited Borrowing Capacity

What if the buyer and seller can borrow against only a fraction of the market value of their assets, as opposed to the full value as we have thus far assumed? The analysis of the one-period case is easily extended to capture this. Replace $h_i(\psi)$ in the capital constraint with $\alpha_i h_i(\psi)$, where α_i is a constant, such that $\alpha_i \in (0, 1]$. The parameter α_i represents "haircuts" set by a regulator or by potential lenders to account for the asset's risk. Alternatively, $\alpha_i < 1$ represents limitations on the ability of potential lenders to seize borrowers' assets.

[23] Recall that in the dynamic case, Lemma 3 tells us that the capital constraint is always slack enough after rejection to allow the buyer to make the benchmark offer $(p, q) = (\Delta, x)$. So even when the terms of rejected offers are observed, what happens after an offer is rejected plays no role in the maximization problem.

One can show (both in the seller's case and the buyer's case) that increasing α_i increases the region in which trade can happen.[24] Thus, a regulator might be able to defrost the market by increasing α_i, for example, by reducing capital requirements or providing loan guarantees. However, once α_i is large enough so that trade can occur, but not too large so that the benchmark solution is not achieved, a further increase in α_i reduces the bid price and the probability of trade. Intuitively, a higher α_i increases the market value of existing assets and therefore has a similar effect to that of reducing initial leverage.

7.3 Marking to Market

In our main analysis above, we assume that the market value of assets is derived using all available information; that is, using Bayes' rule. However, one obtains qualitatively similar results if instead assets are "marked to market"; that is, valued at the most recent transaction price.

Denote by $p_0 \in (0,1]$ the price of the last offer accepted. Under marking to market with a borrowing capacity $\widehat{\alpha} \in (0,1]$,

$$\widehat{\alpha} h(\psi) = \begin{cases} \widehat{\alpha} p & \text{if offer accepted} \\ \widehat{\alpha} p_0 & \text{if offer rejected.} \end{cases} \tag{13}$$

As before, assume that the capital constraint is initially satisfied, but instead of an initial valuation $\alpha_i(\frac{1}{2} + \gamma\Delta)$, we now use the price p_0, so the assumption is that the capital constraint is satisfied if $\widehat{\alpha} h(\psi) = \widehat{\alpha} p_0$ and $q_i = 0$.

Consider first the one-period case. Since the capital constraint after an offer is rejected plays no role, the case $\widehat{\alpha} < 1$ is a special case of our general analysis with $\alpha_i = 2\widehat{\alpha}$ and $\gamma = 0$. The special case $\widehat{\alpha} = 1$ is not covered by our prior analysis (from footnote 24 it requires $\alpha_i \in (0,2)$) but is nonetheless especially tractable. Under marking to market and $\widehat{\alpha} = 1$, the capital constraint becomes $pM \geq L - Z$, both in the buyer's case and the seller's case. The number of units sold/purchased plays no role in the capital constraint because the borrowing capacity of these units is exactly the amount paid; on net, they neither add nor subtract value in the capital constraint. Denoting initial leverage (of the buyer's or seller's, depending on the case)

[24] For the results to hold, it is enough to assume $\alpha_i \in (0,2)$. In the seller's case, equation (6) becomes $p \geq \frac{\delta_s}{\alpha_s + (2-\alpha_s)\frac{q}{M_s}}$. Trade can occur if and only if $\delta \leq 2\Delta[\alpha_s + (2-\alpha_s)\frac{x}{M_s}]$, and if trade occurs, the buyer offers to buy x units at a price per unit $\max(\Delta, \frac{\delta_s}{\alpha_s + (2-\alpha_s)\frac{x}{M_s}})$. The analysis for the buyer case is in the proof of Propositions 3 and 4.

by $\delta_0 \equiv \frac{L-Z}{p_0 M}$, the capital constraint becomes $p \geq \delta_0 p_0$. If $2\Delta \geq \delta_0 p_0$, the buyer offers to buy x units at a price per unit $\max(\Delta, \delta_0 p_0)$. Otherwise, the buyer does not make any offer.

Next, consider the two-period case. For conciseness, we focus on the special case $\widehat{\alpha} = 1$. In this case, the capital constraint becomes

$$\sum_{i=1}^{2} h(\psi_i) M \geq L_b - Z_b, \tag{14}$$

where $h(\psi_i) = \phi_i p_i + (1 - \phi_i) p_0$. That is, if an offer is accepted ($\phi_i = 1$), existing inventories are valued based on the sale price p_i, and if an offer is rejected ($\phi_i = 0$), existing inventories are valued based on the price p_0.

Suppose first that market conditions are worsening, so p_0 is high compared with current valuations; for example, $p_0 \geq 1$. Then the buyer cannot relax the capital constraint by bidding a high price. Instead, any accepted offer tightens the constraint. The analysis is very similar to the main model. The buyer must decide whether to make just one bid or two. When he makes two bids, the first bid is higher than the benchmark and the second bid is even higher. In addition, the buyer always bids for the maximum amount and makes nonnegative profits in each period. Note, however, that since rejected offers do not increase the market value of inventories, Lemma 3 no longer holds; instead, the capital constraint implies that $p_r = \max(\Delta, \frac{L_b - Z_b}{M} - p_0)$.[25]

In contrast, if $p_0 < 1$, the buyer may try to support the market by making a small trade at a high price on which he expects to lose money. As in the main model (see part (iii) of Proposition 5), losing money in the first period allows the buyer to make a profitable trade in the second period so that on net, the buyer's expected profits are nonnegative.

8 Summary

We analyze how existing stocks of assets — inventories — affect trade, information dissemination, and price formation. When market participants are close to their maximal leverage, concerns about the revelation of bad news prevent socially beneficial

[25] If the first offer is rejected and the second offer is accepted, the capital constraint (14) becomes $(p_0 + p_r)M \geq L_b - z_b$, which reduces to $p_r \geq \frac{L_b - Z_b}{M} - p_0$. As in the one-period case, the optimal choice of p_r is as close as possible to the benchmark Δ, subject to the capital constraint being satisfied.

trade and information dissemination. However, when market participants are further from their maximal leverage, inventories lead to overbidding (in the sense that the buyer pays more than he would like), which stimulates socially beneficial trade. Because trade increases buyer inventories and often increases buyer leverage, these predictions imply that prices and trade volumes may first increase before collapsing. We use our model to comment on several prominent policy questions.

9 Appendix

Proof of Propositions 3 and 4. The proof is for the general case $\alpha_b \in (0,2)$, discussed in Section 7, when Assumption 1 is replaced with $x < \frac{\alpha_b}{2-\alpha_b} M_b$.

Define $\theta \equiv \frac{L_b - z_b}{\alpha_b M_b} - \gamma\Delta$, $\beta \equiv \frac{2-\alpha_b}{\alpha_b M_b}$, and $c \equiv \frac{2\gamma\Delta}{M_b}$. For use below, note that $\theta = \delta_b \frac{1+2\gamma\Delta}{2\alpha_b} - \gamma\Delta$, where $\delta_b = \frac{L_b - Z_b}{(\frac{1}{2}+\gamma\Delta)M_b}$ as defined in the main text. Observe that $1 - \beta q > 0$, since $x < \frac{\alpha_b}{2-\alpha_b} M_b$. Hence, defining $p(q) \equiv \frac{2\theta - cq}{1-\beta q}$, the capital constraint can be written as $p \geq p(q)$. (The definition of $p(q)$ in the text is a special case.) Observe that $p'(q) = \frac{2\beta\left(\theta - \frac{c}{2\beta}\right)}{(1-\beta q)^2} = \frac{2\beta\left(\theta - \frac{\alpha_b\gamma\Delta}{2-\alpha_b}\right)}{(1-\beta q)^2}$. Thus,

$$\min_{q\in[0,x]} p(q) = \begin{cases} p(0) & \text{if } \theta \geq \frac{\alpha_b\gamma\Delta}{2-\alpha_b} \\ p(x) & \text{if } \theta \leq \frac{\alpha_b\gamma\Delta}{2-\alpha_b}. \end{cases} \tag{15}$$

Trade is possible if and only if there exists a quantity $q \in (0,x]$, such that $p(q) \leq 2\Delta$, so that the buyer makes nonnegative profits; that is, if either $p(x) \leq 2\Delta$ or $p(0) = 2\theta < 2\Delta$. Hence, trade is possible if and only if θ falls in some lower interval.

Case 1: $\frac{\alpha_b\gamma}{2-\alpha_b} < 1$ (contains Proposition 4 as a special case)

At $\theta = \Delta$, $p(x) > p(0) = 2\Delta$. Thus, trade is impossible for $\theta \geq \Delta$ but is possible for all $\theta < \Delta$, or equivalently, $\delta_b < \frac{2\alpha_b(1+\gamma)\Delta}{1+2\gamma\Delta}$. Next, we characterize the buyer's offer when $\theta < \Delta$. If $p(x) \leq \Delta$, which is equivalent to $\theta \leq \frac{1}{2}(\Delta - \beta\Delta x + cx)$, the capital constraint does not bind and the buyer makes the benchmark offer (x, Δ). If $\theta \leq \frac{\alpha_b\gamma}{2-\alpha_b}\Delta$, increasing q relaxes the capital constraint, and since $\theta < \Delta$ we know the buyer has a strictly profitable trade. Consequently, the buyer bids for the entire amount x available and chooses a price $\max\{\Delta, p(x)\}$. This price is weakly increasing in θ, and hence in initial leverage δ_b.

The remainder of the proof of this case deals with the open interval of θ values above $\max\left\{\frac{\alpha_b\gamma}{2-\alpha_b}\Delta, \frac{1}{2}(\Delta - \beta\Delta x + cx)\right\}$ but below Δ. Since $\theta < \Delta$, we know that the buyer has a strictly profitable trade. Moreover, any strictly profitable trade in which the capital constraint is slack is strictly dominated by one in which it binds: either $q < x$, in which case q can be increased, or $q = x$ and $p > p(x) > \Delta$, in which case p can be decreased. Consequently, the buyer's best offer is the solution to the more constrained maximization problem in which he must keep the capital constraint binding; that is,

$$\max_{q \in [0,x]} q\pi(p(q)). \tag{16}$$

Observe that $\frac{\partial}{\partial q} q\pi(p(q)) = \pi(p(q)) + qp'(q)\pi'(p(q))$, and recall $p'(q) > 0$ in the interval under consideration. Hence, (16) has a unique solution, as follows: If $q\pi(p(q)) > 0$ and $\frac{\partial}{\partial q} q\pi(p(q)) \leq 0$ for some q, then $\pi'(p(q)) < 0$, and so by the strict concavity of π and the strict convexity of p, it follows that $\frac{\partial}{\partial q} q\pi(p(q)) < 0$ for all higher q. Moreover, the maximizer of (16) must be such that $\pi'(p(q)) < 0$ (if the maximizer is the corner $q = x$, this follows from $p(x) > \Delta$). Hence, in the interval from which the maximizer of (16) is drawn, $\frac{\partial}{\partial q} q\pi(p(q))$ strictly decreases in θ. Hence, the buyer's choice of q weakly decreases as θ (and hence initial leverage δ_b) increases. Note also that if q is strictly decreasing at any θ, the same is true for all higher θ.

For the effect of θ (and hence leverage) on the price offered, note first that if the buyer offers to buy everything ($q = x$) for price $p(x)$, it follows immediately that the price increases. If instead the buyer offers to buy $q < x$, the price satisfies $p = p(q)$, and the optimal p solves $\max_{p \in [0,1]} q(p)\pi(p)$, where $q(p) = \frac{p-2\theta}{\beta p - c}$ is the inverse function of $p(q)$. Observe that $\frac{\partial}{\partial p} q(p)\pi(p) = q'(p)\pi(p) + q(p)\pi'(p)$; $q'(p) = \frac{2\theta\beta - c}{(\beta p - c)^2} > 0$; and recall that the optimal p satisfies $\pi'(p) < 0$. Hence, in the interval in which the (unique) optimal p is drawn, $\frac{\partial}{\partial p} q(p)\pi(p)$ strictly increases in θ and strictly decreases in p (the last part follows from the concavity of π and q). Hence, the optimal p increases in θ.

From the analysis above, the buyer's offer is continuous as a function of θ. Finally, as θ approaches Δ, only offers with q close to 0 can satisfy the capital constraint (with a price below 2Δ). It follows easily that as θ approaches Δ, the buyer's offer converges to $(q, p) = (0, 2\Delta)$. The expected volume (pq) converges to 0.

To show that expected volume first increases in leverage, it is enough to show that there exists some interval to the right of $\frac{1}{2}(\Delta - \beta\Delta x + cx)$ such that when θ falls in this interval the buyer offers to buy everything, $q = x$. If $\frac{\alpha_b \gamma}{2 - \alpha_b}\Delta > \frac{1}{2}(\Delta - \beta\Delta x + cx)$, this is immediate from the analysis above. Otherwise, note that at $\theta = \frac{1}{2}(\Delta - \beta\Delta x + cx)$, we know $p(x) = \Delta$; thus, $\frac{\partial}{\partial q} q\pi(p(q))\Big|_{q=x} = \pi(\Delta) + xp'(x)\pi'(\Delta) = \pi(\Delta) > 0$.

By continuity, it follows that $\frac{\partial}{\partial q} q\pi\left(p\left(q\right)\right)\Big|_{q=x} > 0$ over some interval to the right of $\frac{1}{2}(\Delta - \beta\Delta x + cx)$, implying that the buyer offers to buy $q = x$ in this interval.

From the analysis above, q must eventually be strictly decreasing in θ, (and if it is strictly decreasing at some θ, the same is true for all higher θ up to Δ, when trade becomes impossible). In this case, expected volume changes by $\frac{\partial(qp(q))}{\partial\theta} = \frac{\partial q}{\partial\theta}p(q) + qp'(q)\frac{\partial q}{\partial\theta} = \frac{\partial q}{\partial\theta}[p(q) + qp'(q)]$, which is strictly negative in the interval under consideration.

Case 2: $\frac{\alpha_b\gamma}{2-\alpha_b} \geq 1$ (contains Proposition 3 as a special case)

In this case, at $\theta = \Delta$, $p\left(x\right) \leq p\left(0\right) = 2\theta$, and so $p\left(x\right) \leq 2\Delta$. Hence, trade is certainly possible up to $\theta = \Delta$. Hence, trade is possible for all θ weakly below the cutoff value of θ such that $p\left(x\right) = 2\Delta$. The characterization of the buyer's offer for θ below this cutoff is the same as for the first part of the case $\frac{\alpha_b\gamma}{2-\alpha_b} < 1$.

Changes in α_b: From the capital constraint $\alpha_b(\frac{1}{2}p + \gamma\Delta)(M_b + q) - pq \geq L_b - Z_b$, it follows that if trade is possible when $\alpha_b = \alpha$, it is also possible when $\alpha_b \geq \alpha$. Also observe that β, $p\left(q\right)$, and $p'(q)$ strictly decrease in α. Hence, following similar steps as above, one can show that increasing α has a similar effect on the price and quantity as reducing θ. Q.E.D.

Proof of Lemma 1. Since the capital constraint is satisfied at the start of the second period (as F is sufficiently large), it follows from Proposition 3 that $q_a, q_r \in \{0, x\}$. For the first period, note that the quantity q_1 enters the capital constraint with a coefficient $(\frac{1}{2}p_1 + \Delta) - p_1$, which is the expected value of the asset acquired minus the price paid. This expression has the same sign as the per-unit profit $\pi\left(p_1\right)$. Consequently, if $\pi\left(p_1\right) \geq 0$, the buyer offers to buy the maximum amount x, since doing so relaxes the capital constraint and increases profits; while if $\pi\left(p_1\right) < 0$, the buyer offers to buy the minimum amount $\underline{q}$ or nothing. Offering $\underline{q}$ might be optimal because if the offer is rejected, the capital constraint is loosened and the buyer starts the second period with a lower leverage. Q.E.D.

Proof of Lemma 2. Suppose the buyer offers (p_i, q_i). The acceptance of this offer has two effects: First, the value of existing assets falls from $(\frac{1}{2} + \Delta)M$ to $(\frac{1}{2}p_i + \Delta)M$, with a net effect $\frac{1}{2}(1 - p_i)M > \frac{1}{2}(1 - p_i)x$. Second, the buyer adds q_i units, each with a borrowing capacity of $\frac{1}{2}p_i + \Delta$, but he also pays p_i per unit. If profits $\frac{1}{2}p_i + \Delta - p_i$ are negative, this second effect is also negative, and the proof is complete. Otherwise, the added borrowing capacity from this is $(\Delta - \frac{1}{2}p_i)q_i$, which is at most $\frac{1}{2}(1-p_i)x$, since $q_i \leq x$ and $\Delta < \frac{1}{2}$. Combining the two effects, it follows that the overall effect is negative and the capital constraint is tightened. Q.E.D.

Proof of Lemma 3. Since choosing $(p_r, q_r) = (\Delta, x)$ maximizes second-period profits, it is enough to show that the capital constraint is not violated after choosing this pair. That is, we need to show that if the first offer (p_1, q_1) is rejected and the second offer $(p_r, q_r) = (\Delta, x)$ is accepted, equation (9) holds. In this case, $h(\psi_1) = \frac{1}{2} + \frac{1}{2}p_1 + \Delta$ and $h(\psi_2) = \frac{1}{2}p_r + \Delta = \frac{3}{2}\Delta$. Thus, we need to show that

$$[(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)M] + [(\frac{3}{2}\Delta)(M+x) - \Delta x] \geq L_b - Z_b. \tag{17}$$

Since the coefficient on x in (17) is positive and $q_1 \leq x$, it is enough to show that

$$[(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)M] + [(\frac{3}{2}\Delta)(M+q_1) - \Delta q_1] \geq L_b - Z_b. \tag{18}$$

To do so, we use the fact that the offer (p_1, q_1) satisfies the first-period capital constraint (equation (10)) if accepted. That is,

$$[(\frac{1}{2}p_1 + \Delta)(M+q_1) - p_1 q_1] + [(\frac{1}{2} + \Delta)M] \geq L_b - Z_b. \tag{19}$$

Equation (18) can be rewritten as

$$[(\frac{1}{2}p_1 + \Delta)(M+q_1) - p_1 q_1] + (\frac{1}{2} + \Delta)M + \frac{1}{2}\Delta(M - q_1) + \frac{1}{2}p_1 q_1 \geq L_b - Z_b. \tag{20}$$

Since $M > x \geq q_1$, equation (19) implies equation (20). Q.E.D.

Proof of Proposition 5. Define $H \equiv \frac{L_b - Z_b - 2\Delta(M+x)}{\frac{1}{2}(M-x)}$ and $\sigma \equiv \frac{\frac{1}{2}M - \Delta x}{\frac{1}{2}(M-x)}$. Observe that $\sigma > 1$, since $x < M$ and $\Delta < \frac{1}{2}$. In addition, H is a monotone transformation of the buyer's initial leverage, defined as $\delta = \frac{L_b - Z_b}{(\frac{1}{2}+\Delta)2M}$. For use below, note that when $q_1 = x$, the first-period capital constraint, (10), is equivalent to $p_1 \geq H - \sigma$.

Case 1: $H \leq 2\Delta + \sigma$. We first show that in this case $q_1 = x$. Define $\overline{p} = \max(\Delta, H - \sigma)$. Since the offer $(p_1, q_1) = (\overline{p}, x)$ satisfies the first-period capital constraint and provides nonnegative profits (since $H - \sigma \leq 2\Delta$), trade can always happen. Thus, by Lemma 1, it is enough to show that it is suboptimal to choose $q_1 = \underline{q}$. The proof is by contradiction. Suppose to the contrary that the optimal bidding strategy is $(p_1, q_1; p_a, q_a; p_r, q_r)$ with $q_1 = \underline{q}$. From Lemmas 1 and 3, $p_1 >$

2Δ; $(p_r, q_r) = (\Delta, x)$; and we can assume, without loss of generality, that either (i) $p_a = q_a = 0$, or (ii) $p_a \in (0, 2\Delta]$ and $q_a = x$. We obtain a contradiction as follows: If $p_a = 0$, we can increase the buyer's expected utility by choosing the strategy $(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) = (\overline{p}, x; p_a, q_a; p_r, q_r)$, as follows:

$$\begin{aligned} U(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) &= x\pi(\overline{p}) + (1 - \overline{p})x\pi(\Delta) \\ &> q_1\pi(p_1) + (1 - p_1)x\pi(\Delta) = U(p_1, q_1; p_a, q_a; p_r, q_r). \end{aligned} \tag{21}$$

The inequality follows since $H - \sigma \leq 2\Delta < p_1$. If instead $p_a > 0$, we can increase the buyer's expected utility by choosing the strategy $(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) = (p_a, q_a; p_1, q_1; p_r, q_r)$, as follows:

$$\begin{aligned} U(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) &= q_a\pi(p_a) + p_a q_1\pi(p_1) + (1 - p_a)x\pi(\Delta) \\ &> p_1 q_a\pi(p_a) + q_1\pi(p_1) + (1 - p_1)x\pi(\Delta) = U(p_1, q_1; p_a, q_a; p_r, q_r). \end{aligned} \tag{22}$$

The first equality follows since the strategy $(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r)$ satisfies capital constraints (9) and (10); in particular, since the capital constraint is satisfied after the offers (p_1, q_1) and (p_a, q_a) are accepted, it follows from Lemma 2 that it is also satisfied if the buyer makes only one of these offers. The inequality follows since $p_a < p_1 \leq 1$, $\pi(p_1) < 0$, and $\pi(p_a) \geq 0$.

So far, we have established $q_1 = x$. For strategies in which $p_a > 0$, the second-period capital constraint, (9), reduces to $p_1 + p_a \geq H$. By Lemma 2, the second-period constraint implies the first-period constraint. Thus, if $H \leq 2\Delta$, the benchmark solution, $p_1 = p_a = \Delta$, is achieved.

The remainder of Case 1 deals with $H \in (2\Delta, 2\Delta + \sigma]$. Define $V(p_1, p_a) \equiv \pi(p_1) + p_1\pi(p_a) + (1 - p_1)\pi(\Delta)$, which is profits from (p_1, p_a) divided by the bid size x. Observe that if a solution has $p_a > 0$, then $p_1 + p_a = H$: Otherwise, $p_1 + p_a > H > 2\Delta$, and so either $p_1 > \Delta$ and $\frac{\partial V}{\partial p_1} = \pi'(p_1) + \pi(p_a) - \pi(\Delta) < 0$, which is a contradiction; or else $p_a > \Delta$, which is clearly suboptimal. Thus, the problem reduces to choosing p_1 to maximize $\max\{V(p_1, 0), V(p_1, H - p_1)\}$ subject to the first-period constraint $p_1 \geq H - \sigma$. Define $R_1(H) \equiv \max_{p_1 \geq H - \sigma} V(p_1, 0)$ and $R_2(H) \equiv \max_{p_1 \geq H - \sigma} V(p_1, H - p_1)$. Both R_1 and R_2 are continuous. Observe that $R_2(2\Delta) > R_1(2\Delta)$, and $4\Delta - \sigma < 2\Delta$ (since $\Delta < 1/2$). For $H \in (4\Delta, 2\Delta + \sigma)$, if both $p_1, p_a > 0$, then at least one exceeds 2Δ, which we know is suboptimal (Lemma 1); and since the pair $p_1 < 2\Delta$, $p_a = 0$ gives strictly positive profits, $R_2(H) < R_1(H)$ in this range. Hence, there exist $H_1, H_2 \in (2\Delta, 4\Delta)$, such that $R_2(H) > R_1(H)$ whenever $H < H_1$; and $R_1(H) > R_2(H)$ whenever $H > H_2$. When $H \in (2\Delta, H_1)$, the buyer chooses his first-period bid to make sure he can bid in the second period even if his first-period bid is accepted. When $H \in (H_2, 2\Delta + \sigma)$, the buyer withdraws from the market if his first offer is accepted.

Next, we show that if $H \in (2\Delta, H_1)$, then $p_a > p_1 > \Delta$, that is, $p_1 \in (\Delta, \frac{H}{2})$. In this case, $V(p, H-p)$ is a cubic in p, and the coefficient on the cubic term is negative. Thus, the result follows if $\frac{d}{dp_1}V(p_1, H-p_1)\Big|_{p_1=\Delta} > 0 > \frac{d}{dp_1}V(p_1, H-p_1)\Big|_{p_1=H/2}$. Evaluating, $\frac{d}{dp_1}V(p_1, H-p_1) = \pi'(p_1) + \pi(H-p_1) - p_1\pi'(H-p_1) - \pi(\Delta)$. Since π is a quadratic with its maximum at Δ, for any p, $\pi(p) = \pi(\Delta) + \frac{1}{2}(p-\Delta)\pi'(p)$. Given this,

$$\begin{aligned}
\frac{d}{dp_1}V(p_1, H-p_1)\Big|_{p_1=\Delta} &= \pi(H-\Delta) - \Delta\pi'(H-\Delta) - \pi(\Delta) \qquad (23)\\
&= \left(\frac{1}{2}(H-\Delta-\Delta) - \Delta\right)\pi'(H-\Delta),
\end{aligned}$$

which is positive, since $H \in (2\Delta, H_1)$ and $H_1 < 4\Delta$. Similarly,

$$\begin{aligned}
\frac{d}{dp_1}V(p_1, H-p_1)\Big|_{p_1=H/2} &= (1-\frac{H}{2})\pi'(\frac{H}{2}) + \pi(\frac{H}{2}) - \pi(\Delta) \qquad (24)\\
&= (1-\frac{H}{2}+\frac{1}{2}(\frac{H}{2}-\Delta))\pi'(\frac{H}{2})\\
&= (1-H/4-\Delta/2)\pi'(H/2),
\end{aligned}$$

which is negative, since $H \in (2\Delta, H_1)$, $H_1 < 4\Delta$, and $\Delta < 1/2$.

Finally, we characterize the optimal offer when $H \in (H_2, 2\Delta+\sigma)$. The optimal p_1 maximizes $V(p_1, 0)$, subject to $p_1 \geq H - \sigma$. Observe that $V(p_1, 0)$ is quadratic and that the unconstrained solution is $\widetilde{p}_1 = \Delta - \frac{1}{2}\Delta^2$. Thus, the optimal solution is $p_1 = \max(\widetilde{p}_1, H-\sigma)$, which is increasing in H (and hence, in leverage). In addition, $p_1 > \Delta$ when $H > \Delta + \sigma$.

Case 2: $H > 2\Delta + \sigma$. We first claim that if there is trade, $p_1 > 2\Delta$. To establish this, suppose to the contrary that $p_1 \leq 2\Delta$, so from Lemma 1, $q_1 = x$ is optimal. But then (p_1, q_1) violates the first-period capital constraint, giving a contradiction. It then follows from Lemma 1 that if trade occurs, $q_1 = \underline{q}$; and from Lemma 3, we know that $(p_r, q_r) = (x, \Delta)$.

Next, we show that if the first offer is accepted, it is optimal not to make any offer in the second period. Recall that accepted offers reduce value (Lemma 2), so if the buyer makes an offer in the first period and the offer is accepted, he starts the second period with a capital constraint that is tighter than the one he had at the start of the first period. Hence, the fact that the buyer had to choose $p_1 > 2\Delta$ implies that either $p_a = 0$ or $p_a > 2\Delta$. Since choosing $p_a > 2\Delta$ is suboptimal, we must have $p_a = 0$.

The buyer's expected utility reduces to $\underline{q}\pi(p_1) + (1 - p_1)x\pi(\Delta)$. This expression is strictly decreasing in p_1 when $p_1 > 2\Delta$. Thus, if trade occurs, the optimal p_1 satisfies the first-period capital constraint with equality, and it follows that $p_1 = \underline{H}$, where $\underline{H} \equiv \frac{L_b - Z_b - 2\Delta(M+\underline{q})}{\frac{1}{2}(M-\underline{q})} - \frac{\frac{1}{2}M - \Delta\underline{q}}{\frac{1}{2}(M-\underline{q})}$. Observe that $\underline{H}$ is a monotone transformation of the buyer's initial leverage. Thus, if there is trade, the initial bid is increasing and continuous in leverage. To establish that trade can happen if and only if leverage is sufficiently low, observe that at $\underline{H} = 2\Delta$, the buyer's expected utility is strictly positive, whereas at $\underline{H} = 1$, the buyer's expected utility is strictly negative. Q.E.D.

Proof of Lemma 4. The capital constraint (7) can be written as $(\frac{1}{2}p + \Delta_i)M_i + q_i(\Delta_i - \frac{1}{2}p) \geq 0$. By definition, $p_i(0)$ and $p_i(x)$ are the minimum solutions in p for $q_i = 0$ and $q_i = x$, respectively. (Note that by Assumption 1, the left-hand side is increasing in p.) Hence, if (i) $p_i(0) = 2\Delta_i$, then also $p_i(x) = 2\Delta_i$; (ii) if $p_i(0) > 2\Delta_i$, we must have $p_i(x) > p_i(0)$; and (iii) if $p_i(x) < 2\Delta_i$, then $p_i(0) > p_i(x)$, which combined with (ii) gives $p_i(x) < p_i(0) < 2\Delta_i$. The result then follows. Q.E.D.

Proof of Lemma 5: Part (A) and (B): If $p_i > 2\Delta_i$, reducing the offer to $p_i = 2\Delta_i$ always weakly improves profits, and does so strictly whenever $-i$'s offer is lower. If $p_i < r_i$, increasing the offer to $p_i = r_i$ always weakly increases utility and does so strictly whenever $-i$'s offer is lower. If $r_i < 2\Delta_i$, $p_i \in [r_i, 2\Delta_i)$ and $q_i < x$, the buyer can weakly increase his utility by raising the quantity to x, and does so strictly if $-i$'s offer is lower. The same is true if $r_i = 2\Delta_i = p_i$ and $q_i < x$; in the latter case, the buyer does so ensure that the offer of the other agent is not accepted. Finally, if $r_i < 2\Delta_i$ and $p_i = 2\Delta_i$, the buyer can weakly increase his utility by reducing the offer to $p_i = 2\Delta_i - \varepsilon$, and he can do so strictly if $-i$'s offer is lower than $2\Delta_i$.

Part (C): (i) Offering $p_i \in (0, r_i)$ is weakly dominated by offering $p_i = 0$: If $p_{-i} > p_i$ and $q_{-i} = x$, buyer i is indifferent; otherwise, he strictly prefers $p_i = 0$. (ii) Offering $p_i > r_i$ is weakly dominated by offering $p_i = r_i$ with the same q_i: If $p_{-i} > p_i$ and $q_{-i} = x$, buyer i is indifferent; otherwise, he strictly prefers $p_i = r_i$. (iii) Offering $p_i = r_i$ and $q_i \in (\underline{q}, x)$ is weakly dominated by offering $(p_i, q_i) = (r_i, \underline{q})$: If $p_{-i} > p_i$ and $q_{-i} = x$, buyer i is indifferent; otherwise, buyer i is strictly better off. Q.E.D.

Proof of Lemma 6. First, consider the case $\Delta_1 \neq \Delta_2$, and assume, without loss of generality, that $\Delta_1 > \Delta_2$, so in equilibrium the highest bid is posted by buyer 1. From Lemma 5 and standard competition arguments, if $r_2 < 2\Delta_2$, the highest bid is $2\Delta_2$ (in particular, $p_2 = 2\Delta_2$ is weakly dominated, and so $p_2 = 2\Delta_2 - \varepsilon$ and $p_1 = 2\Delta_2$), and if $r_2 = 2\Delta_2$, the highest bid is $2\Delta_2 + \varepsilon$. Since $r_1 \leq \min(2\Delta_1, 2\Delta_2)$, the capital constraint of buyer 1 is satisfied. The capital constraint of buyer 2 is also satisfied, since by Lemma 4, r_2' is less than the equilibrium price. Next, consider the case $\Delta_1 = \Delta_2 = \Delta$. If $\max(r_1, r_2) = 2\Delta$, then by Lemma 5, the equilibrium price is 2Δ and is posted by the buyer with the highest r_i. From Lemma 4, both capital constraints are satisfied. If $\max(r_1, r_2) < 2\Delta$, Lemma 4 implies that $\max(r_1, r_2) < \max(r_1', r_2') \leq 2\Delta - \varepsilon$. Lemma 5 and competition then imply that both buyers post the price $2\Delta - \varepsilon$ for the full amount. Clearly, both capital constraints are satisfied given the equilibrium price. Q.E.D.

Proof of Proposition 6: Part (A), $r_2' \leq r_1$**:** First, consider the case $2\Delta_2 < r_1$, in which case $\max\{r_1, \min\{2\Delta_1, 2\Delta_2\}\} = r_1$. From Lemma 5, buyer 1 bids at least a price r_1. If $2\Delta_2 < r_2$, then by Lemma 5 and $r_1 > r_2'$, iterated elimination of weakly dominated strategies implies that buyer 2 bids nothing, and hence buyer 1 bids $(p_1, q_1) = (r_1, x)$.[26] If instead $2\Delta_2 \geq r_2$, then by Lemma 5, buyer 2 bids at most $2\Delta_2$, and hence buyer 1 bids $(p_1, q_1) = (r_1, x)$. Second, consider the case $2\Delta_2 \geq r_1$. Since $2\Delta_2 \geq r_2'$, Lemma 4 implies that $2\Delta_2 \geq r_2' \geq r_2$. Moreover, $2\Delta_1 \geq r_1 \geq r_2' \geq r_2$. Hence, we can apply Lemma 6, to complete the proof of this case.

Part (B), $r_2' > r_1$**:** First, consider the case $r_2 > 2\Delta_2$, in which case $\max\{r_2, \min\{2\Delta_1, 2\Delta_2\}\} = r_2$. There is no equilibrium in which buyer 2 does not bid: in such an equilibrium buyer 1 would bid r_1, but then buyer 2 would be better off bidding r_2. So by Lemma 5, in equilibrium buyer 2 must bid r_2. If $r_2 < 2\Delta_1$, then the unique equilibrium outcome is that buyer 1 bids $(p_1, q_1) = (r_2 + \varepsilon, x)$. (Note that from Lemma 4, $r_2 + \varepsilon > r_2' > r_1$.) If instead $r_2 \geq 2\Delta_1$, the unique equilibrium outcome is that buyer 2 bids $(p_2, q_2) = (r_2, x)$. (To see this, suppose to the contrary that there is an equilibrium in which buyer 2 bids $(r_2, \underline{q})$. In such an equilibrium buyer 1 must bid (r_1, x), so if the seller's valuation is below r_1, the seller acquires assets from both

[26] If $r_2 \in (r_1, 2\Delta_1)$, there also exists an equilibrium in which buyer 2 bids r_2 and buyer 1 bids $r_2 + \varepsilon$ for the full amount. However, this equilibrium does not survive iterated elimination of weakly dominated strategies. Note that this is the only part of Proposition 6 that relies on iterated elimination.

buyers. But then the capital constraint of buyer 2 is violated. Since we assume that the penalty for violating the constraint is high, buyer 2 is better off bidding for the full amount.

Second, consider the case $r_2 \leq 2\Delta_2$, so from Lemma 4, $r_2' \leq 2\Delta_2$. If $r_2 > 2\Delta_1$, Lemma 5 implies that the seller sells everything to buyer 2 at price r_2. If instead $r_2 \leq 2\Delta_1$, then $r_1 \leq 2\Delta_2$, and we can apply Lemma 6 to complete the the proof. Q.E.D.

Proof of Proposition 7. We can apply Lemma 5 to do the first round of iterated elimination of weakly dominated strategies. If $r_1' > r_2$, then $r_1 > r_2'$ by Lemma 4, and the second round of elimination implies that buyer 2 bids nothing, and hence buyer 1 also bids nothing. If $r_1' \leq r_2$, the second round of elimination implies that buyer 1 bids nothing, and hence buyer 2 bids nothing. Q.E.D.

References

[1] Acharya, Viral V., Douglas Gale, and Tanju Yorulmazer, forthcoming, Rollover risk and market freezes, *Journal of Finance.*

[2] Acharya, Viral V., and S. Viswanathan, forthcoming, Leverage, moral Hazard and liquidity, *Journal of Finance.*

[3] Adrian, Tobias, and Hyun-Song Shin, 2010, Liquidity and leverage, *Journal of Financial Intermediation*, 19, 418-437.

[4] Allen, Franklin, and Elena Carletti, 2008, Mark-to-market accounting and liquidity pricing, *Journal of Accounting and Economics*, 45, 358-378.

[5] Allen, Franklin, and Gerald R. Faulhaber, 1989, Signaling by underpricing in the IPO market, *Journal of Financial Economics*, 23, 303-323.

[6] Allen, Franklin, and Douglas Gale, 1994, Liquidity preference, market participation and asset price volatility, *American Economic Review*, 84, 933-955.

[7] Amihud, Yakov, and Haim Mendelson, 1980, Dealership market: Market making with inventory, *Journal of Financial Economics* 8, 31-53.

[8] Ashcraft, Adam, B., and Til Schuermann, 2008, Understanding the securitization of subprime mortgage credit, *Federal Reserve Bank of New York Staff Reports*, no. 318.

[9] Brunnermeier, Markus K., and Lasse H. Pedersen, 2009, Market liquidity and funding liquidity, *Review of Financial Studies*, 22, 2201-2238.

[10] Diamond, Douglas, W., and Raghuram G. Rajan, forthcoming, Fear of fire sales, illiquidity seeking, and credit freezes, *Quarterly Journal of Economics.*

[11] Downing, Chris, Dwight Jaffee, and Nancy Wallace, 2009, Is the market for mortgage-backed securities a market for lemons?, *Review of Financial Studies*, 22, 2457-2494.

[12] Grinblatt, Mark, and Chuan Yang Hwang, 1989, Signalling and the pricing of new issues, *Journal of Finance,* 44, 393-420.

[13] Glosten, Lawrence R., and Paul R. Milgrom, 1985, Bid, ask and transaction prices in a specialist market with heterogeneously informed traders, *Journal of Financial Economics*, 14, 71-100.

[14] Heaton, John, Deborah Lucas, and Robert McDonald, 2010, Is mark-to-market accounting destabilizing? Analysis and implications for policy, *Journal of Monetary Economics*, 57, 64-75..

[15] Ho, Thomas., and Hans R. Stoll, 1981, Optimal dealer pricing under transactions and return uncertainty, *Journal of Financial Economics*, 9, 47-73.

[16] Ho, Thomas, and Hans R. Stoll, 1983, The dynamics of dealer markets under competition, *Journal of Finance*, 38, 1053-1074.

[17] Kiyotaki, Nobuhiro, and John Moore, 1997, Credit cycles, *Journal of Political Economy*, 105, 211-248.

[18] Kyle, Albert S. and Wei Xiong, 2001, Contagion as a wealth effect, *Journal of Finance,* 56, 1401-1440.

[19] Lewis, Michael, 2010, *The Big Short: Inside the Doomsday Machine* (W. W. Norton & Company, New York, London).

[20] Manaster, Steven, and Steven C. Mann, 1996, Life in the pits: Competitive market making and inventory control, *Review of Financial Studies*, 9, 953-975.

[21] Milbradt, Konstantin, 2010, Level 3 assets: Booking profits, concealing losses, working paper.

[22] Plantin, Guillaume, Haresh Sapra, and Hyun Song Shin, 2008, Marking-to-market: Panacea or Pandora's box?, *Journal of Accounting Research*, 46, 435-460.

[23] Samuelson, William, 1984, Bargaining under asymmetric information, *Econometrica*, 52, 995-1005.

[24] Shleifer, Andrei, and Robert W. Vishny, 1992, Liquidation values and debt capacity: A market equilibrium approach, *Journal of Finance*, 47, 1343-1366.

[25] Welch, Ivo, 1989, Seasoned offerings, imitation costs, and the underpricing of initial public offerings, *Journal of Finance*, 44, 421-449.