

INNOVATION, GROWTH AND ASSET PRICES

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Abstract

Asset prices reflect anticipations of future growth. Likewise, long-term growth prospects mirror an economy's innovative potential. In this paper, we examine the asset pricing implications of a stochastic model of endogenous growth. In the model, asset prices and long-term growth prospects are endogenously determined by innovation and risky investments in R&D. In equilibrium, R&D endogenously drives a small, but persistent component in productivity growth. These productivity dynamics induce persistent uncertainty about the long-term growth prospects in the economy, which is reflected in long-term cycles and growth waves in quantities and asset market valuations. With recursive preferences, households are very averse to such persistent movements and command high risk premia in asset markets that help the model match key asset price data. In short, equilibrium growth is risky. Importantly, high equilibrium returns provide strong intertemporal incentives for innovation and thus provide a strong propagation mechanism in the model. Empirically, we find substantial evidence for innovation-driven low-frequency movements in aggregate growth rates and asset market valuations.

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1 Introduction

Asset prices reflect anticipations of an economy’s future growth prospects. Innovation and the development of new technologies are among the main drivers of long-term growth prospects. At the aggregate level, such innovation is reflected in the sustained growth of productivity. Empirical measures of innovation, such as R&D expenditures, are typically quite volatile, fairly persistent, and tend to be procyclical. Such movements in the driving forces of growth prospects should naturally be reflected in the dynamics of growth rates themselves. Indeed, in US post-war data productivity growth has undergone long and persistent swings.¹ Similarly, innovation-driven growth waves associated with the arrival of new technologies such as television, computers, the internet, to name a few, are well documented.² All these movements in growth prospects have the common feature that they do not arise at business cycle frequencies but appear to exhibit substantial variation at lower frequencies. Asset prices naturally reflect this low-frequency variation in growth prospects. In particular, if agents fear that a persistent slowdown in economic growth will lower asset prices, such movements will give rise to high risk premia in asset markets.

In this paper, we examine the determination of asset prices in a general equilibrium setting in which the economy’s long-term growth prospects are endogenously determined by innovation and R&D. Using a tractable model of innovation and R&D we link asset prices and aggregate risk premia to endogenous movements in long-term growth prospects. Importantly, in this environment, asset prices provide agents with incentives to engage in innovation, resulting in endogenous growth dynamics which will in turn be reflected in aggregate risk premia. More specifically, our setup has two distinguishing features. First, we embed a stochastic model of endogenous growth based on industrial innovation³ into a macroeconomic model, in which in contrast to the latter type of models, firms actively engage in innovation and R&D in order to generate sustained growth. Second, we assume that households have recursive preferences,⁴ so that they care about uncertainty about long-term growth prospects.

Understanding the links between growth, productivity and asset prices are also important from a policy perspective. Several authors predict a persistent slowdown in economic growth in the US for the next decades (e.g. Gordon (2010), Jorgenson (2010), Baker, DeLong, Krugman (2005)). For example, Gordon (2010) writes that the growth rate forecast until 2027 “represents the slowest growth of the measured American standard of living recorded during the past two centuries.” What are the implications of these forecasts for asset prices? What is a reasonable forecast for future asset returns in such scenarios? As pointed out by Baker, DeLong and Krugman (2005), such a reasonable forecast for asset returns is an important determi-

¹See e.g. Gordon (2010), and Jermann and Quadrini (2007)

²See e.g. Helpman (1998), and Jovanovic and Rousseau (2003)

³Following Romer (1990) and Grossman and Helpman (1991)

⁴Epstein and Zin (1989), Weil (1990)

nant of the choice between pay-as-you-go and prefunded social insurance systems. Based on historical data, this paper offers a tractable yet quantitatively successful framework linking growth, productivity and asset returns.

Our results suggest that accounting for the endogeneity of innovation, and therefore long-run growth, goes a long way towards establishing a macroeconomic framework that quantitatively captures the joint dynamics of quantities and asset prices in a general equilibrium setting. At the center of this framework is a strong propagation and amplification mechanism for shocks which is tightly linked to the joint dynamics of innovation and asset prices. High equilibrium returns provide strong incentives for agents to engage in innovation and investing in risky R&D. This pricing effect reinforces the impact of exogenous shocks through risk taking, thus providing an amplification mechanism. On the other hand, R&D leads to the development of new technologies which will persistently boost aggregate growth, so that aggregate growth appears in long waves, thus providing a propagation mechanism.

Such endogenous persistence feeds back into asset prices in our environment with recursive preferences. More specifically, in line with much of the recent asset pricing literature, we assume that agents have recursive Epstein-Zin utility with a preference for an early resolution of uncertainty. In this context, agents are very averse to such low-frequency dynamics because they fear that a persistent reduction in growth lowers asset prices so that slowdowns in economic growth are accompanied by low asset valuations. Hence they will command high risk premia in asset markets. These endogenous innovation-driven dynamics allow the model to be quantitatively consistent not only with basic macro and growth data, but also with a variety of stylized facts about asset markets, such as a high equity premium. Our results suggest that there is a tight connection between macroeconomic risk, growth and risk premia in asset markets. Endogenous feedback between risk-taking, growth and risk premia provides quantitatively significant amplification and propagation effects absent in standard macroeconomic models where growth is given exogenously. Put differently, we view stochastic models of endogenous growth as useful tools for both quantitative macroeconomic modeling and general equilibrium asset pricing.

Formally, we first show that in the model innovation and R&D endogenously drive a small, but persistent component in the growth rate of measured aggregate productivity. That is, productivity will exhibit an endogenous stochastic trend. More specifically, productivity will contain a high-frequency component driven by exogenous shocks, as well as an endogenous component that is linked to R&D activity in the economy. Crucially, this endogenous component operates at lower frequencies than the exogenous component; hence, productivity endogenously exhibits high- and low-frequency movements. Interestingly, these dynamics arise in a model that is driven by a single exogenous shock to the level of technology. While this shock induces fluctuations at business cycle frequency comparable to standard macroeconomic settings, the innovation

process in the model translates this disturbance into an additional, slow-moving component in productivity growth. Hence, a shock can trigger a persistent wave in productivity growth and generates a term structure of growth. Recently, in the finance literature, such dynamics have been referred to as long-run productivity risk⁵ and they arise endogenously in our model. Naturally, these productivity dynamics induce persistent uncertainty about the economy’s long-term growth prospects that will be reflected in the dynamics of aggregate quantities. In contrast to standard macro models, our setting endogenously generates fluctuations at various frequencies, including significant movements at a low frequencies. Such persistent movements are inherently linked to innovative activity. Indeed, as predicted by the model, we empirically show that a measure of innovation, namely what we will refer to as the R&D intensity has significant forecasting ability for aggregate growth rates including productivity, consumption, output and cash flows.

Such consumption and cash flow dynamics have important implications for risk premia in asset markets given our assumption of agents’ preference specification. Our assumption of recursive Epstein-Zin utility with a preference for early resolution of uncertainty implies that not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth. Consequently, agents are very averse to the persistent innovations to expected growth rates implied by the propagation mechanism of the model. Hence, the model’s propagation mechanism translates into endogenous consumption risks that are akin to long-run risks proposed in the asset pricing literature (Bansal and Yaron (2004)). In other words, equilibrium growth is risky. Quantitatively, this is reflected in a substantial equity premium and a low and stable risk-free rate. Moreover, persistence in cash flow dynamics will also be reflected in asset valuations. In this respect, the model predicts productivity-driven low-frequency cycles in stock market values and price-dividend ratios, and therefore gives an innovation-based explanation of the empirical evidence on long swings in asset market valuations.

Our paper is related to a number of different strands of literature in asset pricing, economic growth and macroeconomics. The economic mechanisms driving the asset pricing implications are closely related to Bansal and Yaron (2004). In a consumption-based model, Bansal and Yaron specify both consumption and dividend growth to contain a small, persistent component, which exogenously leads to long and persistent swings in the dynamics of these quantities. This specification along with the assumption of Epstein-Zin recursive utility with a preference for early resolution of uncertainty, allows them to generate high equity premia as compensation for these “long-run risks”. The ensuing literature on long-run risk quantitatively explains a wide range of patterns in asset markets, such as those in equity, government, corporate bond, foreign exchange and derivatives markets⁶. While somewhat hard to detect in the data, we show that the

⁵See e.g. Croce (2008), or Gomes, Kogan and Yogo (2008)

⁶Bansal and Yaron (2004) address the equity premium and time series predictability of stock returns, Bansal and Shaliastovich (2008) address bond and foreign exchange markets, Bhamra, Kuehn and Strebulaev (2009) and Chen (2009) look at corporate bond markets, while Drechsler and Yaron (2008) investigate derivatives markets.

growth rate dynamics that Bansal and Yaron specify directly follow naturally from agents' equilibrium R&D decisions in our endogenous growth model. In other words, the equilibrium growth rate dynamics that a broad class of stochastic endogenous growth models generate are precisely what Bansal and Yaron refer to as long-run risk. The consumption-based framework of Bansal and Yaron (2004) builds on earlier work by Barsky and DeLong (1993), who show that a predictable and persistent component in dividend growth can explain a large fraction of stock market fluctuations. The economic importance of such long-run risks is also reinforced by Alvarez and Jermann (2004, 2005), who empirically point out the importance of persistence in pricing kernels and show that low-frequency movements in consumption growth entail high welfare costs measured from asset market data, in contrast to business cycle fluctuations.

A number of papers examine AK models of endogenous growth with recursive preferences in analytically tractable settings. Obstfeld (1994) analyzes an analytically tractable setting in continuous time, but focuses on international diversification and risk-taking and its impact on growth in a two-country environment. Barro (2009) analyzes a model with Epstein-Zin preferences with an intertemporal elasticity larger than unity, but focuses on the effects of rare disasters on asset prices and welfare. Eberly and Wang (2009, 2010) study two-sector AK models with recursive preferences, but focus on the effects of capital reallocation on asset prices and growth.

Similarly, the paper is closely related to recent contributions examining the link between technological innovation and asset pricing (Garleanu, Panageas and Yu (2009), Garleanu, Kogan and Panageas (2009), Pastor and Veronesi (2009)). These contributions examine the implications of the adoption of new technologies for asset prices. Garleanu, Panageas and Yu (2009) and Garleanu, Kogan and Panageas (2009) examine the implications for the cross-section of stock returns, while Pastor and Veronesi (2009) consider the formation of bubbles. In these models of technology adoption, the arrival of new technologies, and therefore the long-run growth rate dynamics, follow exogenously specified processes, whereas we endogenize these long-run growth dynamics. Therefore, because we focus on the creation of new technologies, our approach is complementary. Additionally, another key difference is that these papers use preference specifications such that long-run fluctuations in long-run growth rates are not priced. Lin (2009) examines the link between endogenous technological change and the cross-section of returns in a partial equilibrium model.

Our paper is also closely related to the literature on general equilibrium asset pricing with production. Starting with Jermann (1998) and Boldrin, Christiano, Fisher (2001), many papers have pointed out the difficulties of rationalizing asset price data in production economies. Our paper is particularly closely related to Croce (2007, 2008), Backus, Routledge, Zin (2007) and Gomes, Kogan, Yogo (2008) who examine the implications of long-run productivity risk with recursive preferences for the equity premium, and the cross-section of stock returns, respectively. While they specify long-run productivity risk exogenously, our model

shows how such risk arises endogenously and can be linked to innovation. Tallarini (2000) considers the separate effects of risk and risk aversion on quantities with recursive preferences, while we investigate how risk and risk premia affect growth. Other contributions studying asset pricing in production economies with recursive preferences include Uhlig (2006, 2010), Backus, Routledge, Zin (2010), Campanale, Castro and Clementi (2008), Kaltenbrunner and Lochstoer (2008), Ai (2008), Kuehn (2008) and Gourio (2009, 2010). Methodologically, our paper is related to the large literature on endogenous growth. Here we build directly on the literature that stresses the importance of industrial innovation for growth, such as the seminal work by Romer (1990) and Grossman and Helpman (1991). Indeed, in our model we embed such a model of product innovation into a standard neoclassical production economy. In this sense, our paper is closely related to recent work identifying medium term cycles in macroeconomic fluctuations (Comin and Gertler (2007), Comin, Gertler and Santacreu (2009)) and exploring business cycle implications of endogenous product variety (Bilbiie, Ghironi and Melitz (2007, 2008)). Our notion of long-run fluctuations in growth is linked to Comin and Gertler's notion of a medium term cycle. However, while they consider low-frequency movements are around a trend, we focus on the low frequency movements of the trend growth rate, which is an important distinction from an asset pricing perspective. More generally, our paper also contributes to the literature linking the endogenous growth literature and the business cycle literatures (Jones, Manuelli, Siu, Stacchetti (2004)).

The paper is structured as follows. In section 2 we describe our benchmark model. In section 3 we qualitatively explore the growth and productivity processes arising in equilibrium and detail their links with the real business cycle model. We examine its quantitative implications for productivity, macroeconomic quantities and asset prices in section 4, along with a number of empirical predictions. Section 5 concludes.

2 Model

We start by describing our benchmark endogenous growth model. We embed a model of industrial innovation in the tradition of Romer (1990) and Grossman and Helpman (1991) into a fairly standard macroeconomic model with convex adjustment costs and recursive Epstein-Zin preferences. Additionally, to highlight the propagation mechanism, we compare the endogenous growth model with a standard neoclassical growth model with labor augmenting technology that has the same aggregate production function and capital adjustment costs as our benchmark model. In the following, we will refer to our benchmark model as the ENDO model and refer to the neoclassical growth model as the EXO model.

2.1 Benchmark Endogenous Growth Model (ENDO)

In our benchmark model, rather than assuming exogenous technological progress, growth instead arises through research and development (R&D) investment. R&D investment leads to the creation of intermediate goods or new patents used in the production of a consumption good. An increasing number of intermediate goods is the ultimate source of sustained growth, hence the model is a version of an expanding-variety model of endogenous growth.

The model features a representative final good firm which produces the single consumption good and behaves competitively. The production of the consumption good requires capital, labor and a composite intermediate good. Furthermore production of the final good is subject to stationary exogenous shocks. Intermediate goods are produced by a continuum of monopolistic producers. As in Romer (1990), introduction of new intermediate goods is the ultimate source of sustained productivity growth. Creation of new intermediate goods depends on research and development activity. We assume that the representative household has Epstein-Zin preferences, whose consumption and savings problem is fairly standard.

Household The representative household has Epstein-Zin preferences defined over consumption:

$$U_t = \left\{ (1 - \beta) C_t^{\frac{1-\gamma}{\theta}} + \beta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \quad (1)$$

where γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. When $\psi \neq \frac{1}{\gamma}$, the agent cares about news regarding long-run growth prospects. In the long-run risks literature, the parametrization $\psi > \frac{1}{\gamma}$ is assumed; that is, the agent has a preference for early resolution of uncertainty, so that the agent dislikes shocks to long-run expected growth rates.

The household maximizes utility by participating in financial markets and by supplying labor. Specifically, the household can take positions Z_t in the stock market, which pays an aggregate dividend \mathcal{D}_t , and in the bond market, B_t . Accordingly, the budget constraint of the household becomes

$$C_t + Q_t Z_{t+1} + B_{t+1} = W_t L_t + (Q_t + \mathcal{D}_t) Z_t + R_t B_t \quad (2)$$

where Q_t is the stock price, R_t is the gross risk free rate, W_t is the wage and L_t denotes hours worked.

As described above, the production side of the economy consists of several sectors, so that the aggregate dividend can be further decomposed into the individual payouts of these sectors, in a way to be described below.

As usual, the setup implies that the stochastic discount factor in the economy is given by

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \frac{[\mathbb{E}_t(U_{t+1}^{1-\gamma})]^{\frac{\gamma-1/\psi}{1-\gamma}}}{U_{t+1}^{\gamma-1/\psi}}. \quad (3)$$

where the second term, involving continuation utilities, captures preferences concerning uncertainty about long-run growth prospects. Furthermore, since the agent has no disutility for labor, she will supply her entire endowment, which we normalized to unity.

Final Goods Sector There is a representative firm that uses capital K_t , labor L_t and a composite of intermediate goods G_t to produce the final (consumption) good. Also, assume that the final goods firm owns the capital stock and has access to the production technology

$$Y_t = (K_t^\alpha (\Omega_t L_t)^{1-\alpha})^{1-\xi} G_t^\xi \quad (4)$$

where the composite G_t is defined according to the CES aggregator,

$$G_t \equiv \left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} di \right]^\nu \quad (5)$$

and $X_{i,t}$ is intermediate good $i \in [0, N_t]$, where N_t is the measure of intermediate goods in use at date t . Furthermore, α is the capital share, ξ is the intermediate goods share, and ν is the elasticity of substitution between the intermediate goods. Note that $\nu > 1$ is assumed so that increasing the variety of intermediate goods raises the level of productivity in the final goods sector. This property is crucial for sustained growth. The productivity shock Ω_t is assumed to follow a stationary Markov process. Because of the stationarity of the forcing process, sustained growth will arise endogenously from the development of new intermediate goods. We will describe the R&D policy below.

Define dividends of the final goods firm as

$$D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} di \quad (6)$$

where I_t is capital investment, W_t is the wage rate, and $P_{i,t}$ is the price per unit of intermediate good i , which the final goods firm takes as given. The last term captures the costs of buying intermediate goods at time t .

In line with the literature on production-based asset pricing, we assume that investment is subject to capital

adjustment costs. The capital stock then evolves as

$$K_{t+1} = (1 - \delta)K_t + \Lambda \left(\frac{I_t}{K_t} \right) K_t \quad (7)$$

where δ is the depreciation rate of capital and where the capital adjustment cost function $\lambda(\cdot)$ is specified as in Jermann (1998)

$$\Lambda \left(\frac{I_t}{K_t} \right) \equiv \frac{\alpha_1}{1 - \frac{1}{\zeta}} \left(\frac{I_t}{K_t} \right)^{1 - \frac{1}{\zeta}} + \alpha_2 \quad (8)$$

The parameter ζ represents the elasticity of the investment rate; in particular the limiting cases $\zeta \rightarrow 0$ and $\zeta \rightarrow \infty$ represent infinitely costly adjustment and frictionless adjustment, respectively. The parameters α_1 and α_2 are set so that there are no adjustment costs in the deterministic steady state.

Taking the stochastic discount factor M_t as given, the firm's problem is to choose investment, labor and intermediate goods input to maximize shareholder's wealth. This can be formally stated as

$$\max_{\{I_t, L_t, K_{t+1}, X_{i,t}\}_{t \geq 0, i \in [0, N_t]}} E_0 \left[\sum_{t=0}^{\infty} M_t D_t \right] \quad (9)$$

subject to

$$D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} di \quad (10)$$

$$K_{t+1} = (1 - \delta)K_t + \Lambda \left(\frac{I_t}{K_t} \right) K_t \quad (11)$$

Accordingly, denoting the multiplier on the capital accumulation constraint by q_t , the Lagrangian for the firm's problem is

$$\mathcal{L} = E_0 \left[\sum_{t=0}^{\infty} M_t \left\{ Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} di + q_t \left((1 - \delta)K_t + \Lambda \left(\frac{I_t}{K_t} \right) K_t - K_{t+1} \right) \right\} \right] \quad (12)$$

with corresponding first order conditions:

$$q_t = \frac{1}{\Lambda'_t} \quad (13)$$

$$W_t = (1 - \alpha)(1 - \xi) \frac{Y_t}{L_t} \quad (14)$$

$$1 = E_t \left[M_{t+1} \left\{ \frac{1}{q_t} \left(\alpha(1 - \xi) \frac{Y_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta) - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} \Lambda_{t+1} \right) \right\} \right] \quad (15)$$

$$P_{i,t} = (K_t^\alpha (\Omega_t L_t)^{1-\alpha})^{1-\xi} \nu^\xi \left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} di \right]^{\nu\xi-1} \frac{1}{\nu} X_{i,t}^{\frac{1}{\nu}-1} \quad (16)$$

where $\Lambda_t \equiv \Lambda\left(\frac{I_t}{K_t}\right)$ and $\Lambda'_t \equiv \Lambda'\left(\frac{I_t}{K_t}\right)$. The last equation determines the final good producer's demand for intermediate input. Importantly, that demand is procyclical, as it depends positively on Ω_t .

Intermediate Goods Sector Intermediate goods producers have monopoly power. Given the demand schedules set by the final good firm, monopolists producing the intermediate goods set the prices in order to maximize their profits. Intermediate goods producers transform one unit of the final good in one unit of their respective intermediate good. In this sense production is “roundabout” in that monopolists take final good as given as they are tiny themselves. This fixes the marginal cost of producing one intermediate good at unity.

The first-order condition with respect to $X_{i,t}$ implicitly gives the demand schedule for intermediate good i as a function of the price $P_{i,t}$. The monopolist producing $X_{i,t}$ takes the demand schedule $X_{i,t}(P_{i,t})$ as given and produces at unit cost using the final good input. Thus, the monopolist solves the following static profit maximization problem each period

$$\max_{P_{i,t}} \Pi_{i,t} \equiv P_{i,t} \cdot X_{i,t}(P_{i,t}) - X_{i,t}(P_{i,t}) \quad (17)$$

The monopolistically competitive characterization of the intermediate goods sector a la Dixit and Stiglitz (1977) results in the symmetric industry equilibrium conditions

$$X_{i,t} = X_t \quad (18)$$

$$P_{i,t} = P_t = \nu \quad (19)$$

That is, each intermediate goods producer produces the same amount and charges a markup $\nu > 1$ over marginal cost. Substituting these two equilibrium conditions into the definition for G_t and the F.O.C. w.r.t. $X_{i,t}$ yields:

$$G_t = N_t^\nu X_t \quad (20)$$

$$X_t = \left(\frac{\xi}{\nu} (K_t^\alpha (\Omega_t L_t)^{1-\alpha})^{1-\xi} N_t^{\nu\xi-1} \right)^{\frac{1}{1-\xi}} \quad (21)$$

and hence

$$\Pi_t = (\nu - 1)X_t \quad (22)$$

Consequently, the intermediate good input and hence monopoly profits are procyclical. The value of owning exclusive rights to produce intermediate good i is equal to the present discounted value of the current and

future monopoly profits

$$V_{i,t} = \Pi_{i,t} + \phi E_t[M_{t+1}V_{i,t+1}] \quad (23)$$

where ϕ is the survival rate of an intermediate good. Imposing the symmetric equilibrium conditions, we can drop the i subscript and write

$$V_t = \Pi_t + \phi E_t[M_{t+1}V_{t+1}] \quad (24)$$

Again, given the procyclicality of profits, this implies that the values of patents are procyclical. Since the value of patents are the payoff to innovation, as described below, this implies that the returns to innovation are procyclical and risky.

R&D Sector Innovators develop intermediate goods for the production of final output. They do so by conducting research and development, using the final good as input at unit cost. For simplicity, we assume that households can directly invest in research and development. They develop new intermediate goods, whose patents can then be sold in the market for intermediate goods patents. A new intermediate goods producer will buy the new patent. Assuming that this market is competitive, the price of a new patent will equal the value of the new patent to the new intermediate goods producer. As described above, this implies that investment in R&D is risky.

The R&D sector develops new intermediate goods and sells them to firms in the intermediate goods sector. It has access to linear technology and uses the final good as input to produce new varieties. Specifically, the law of motion for the measure of intermediate goods N_t is

$$N_{t+1} = \vartheta_t S_t + \phi N_t \quad (25)$$

where S_t denotes R&D expenditures (in terms of the final good) and ϑ_t represents the productivity of the R&D sector that is taken as exogenous by the R&D sector. In similar spirit as Comin and Gertler (2006), we assume that this technology coefficient involves a congestion externality effect capturing decreasing returns to scale in the innovation sector

$$\vartheta_t = \frac{\chi \cdot N_t}{S_t^{1-\eta} N_t^\eta} \quad (26)$$

where $\chi > 0$ is a scale parameter and $\eta \in [0, 1]$ is the elasticity of new intermediate goods with respect to R&D. Since there is free entry into the R&D sector, the following break-even condition must hold:

$$E_t[M_{t+1}V_{t+1}](N_{t+1} - \phi N_t) = S_t \quad (27)$$

This says that the expected sales revenues equals costs. This condition can be equivalently formulated, at the margin, as

$$\frac{1}{\vartheta_t} = E_t[M_{t+1}V_{t+1}] \quad (28)$$

which states that marginal cost equals expected marginal revenue.

Market Clearing Final output is used for consumption, investment in physical capital, factor input used in the production of intermediate goods, and R&D:

$$Y_t = C_t + I_t + N_t X_t + S_t \quad (29)$$

Alternatively, this can be written as

$$Y_t = C_t + I_t + N_t^{1-\nu} G_t + S_t \quad (30)$$

where the term $N_t^{1-\nu} G_t$ captures the costs of intermediate goods production. Given that $\nu > 1$ reflecting monopolistic competition, it follows that increasing product variety increases the efficiency of intermediate goods production, as the costs fall as N_t grows.

Stock Market We assume that the stock market value includes all the production sectors, namely the final good sector, the intermediate goods sector, as well as the research and development sector. The aggregate dividend then becomes

$$\mathcal{D}_t = D_t + \Pi_t N_t - S_t \quad (31)$$

Defining the stock market value to be the discounted sum of future aggregate dividends, exploiting the optimality conditions, this value can be rewritten as

$$\mathcal{Q}_t = q_t K_{t+1} + N_t (V_t - \Pi_t) + E_t \left[\sum_{i=0}^{\infty} M_{t+1+i} V_{t+1+i} (N_{t+1+i} - \phi N_{t+i}) \right] \quad (32)$$

as in Comin, Gertler, Santacreu (2009). The stock return is defined accordingly. Therefore, the stock market value comprises the current market value of the installed capital stock, reflected in the first term, the market value of currently used intermediate goods interpreted as patents or blueprints, reflected in the second term, as well as the market value of intermediate goods to be developed in the future, as reflected in the third term. Therefore, in addition to the tangible capital stock, the stock market values intangible capital as well as the option value of future intangibles.

Forcing Process We introduce uncertainty into the model by means of an exogenous shock Ω_t to the level of technology, that is assumed to evolve as

$$\Omega_t = e^{a_t} \quad (33)$$

$$a_t = \rho a_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2) \quad (34)$$

where $\rho < 1$. Note first that this process is strictly stationary, so that sustained growth in the model will not arise through exogenous trend growth in exogenous productivity, but endogenously. Second, while formally, Ω_t resembles labor augmenting technology, it does not represent measured TFP in our setting. Rather, measured TFP in the model can be decomposed in an exogenous component, driven by Ω_t , and an endogenous component which is driven by the accumulation of intermediate goods and hence innovation, which is also the source of sustained growth. We discuss the dynamics of productivity in detail in section 3.

2.2 Exogenous Growth Model (EXO)

To contrast with our benchmark endogenous growth model (ENDO), we consider the neoclassical growth model with exogenous labor augmenting technology. We will examine the connection between the two models in detail in the following section. Section 3 will present qualitative results, while section 4 examines the models quantitatively.

Specifically, rather than growth being endogenously determined through accumulation of intermediate goods N_t , we instead exogenously specify the evolution of the corresponding \tilde{N}_t as an exponential time trend. Furthermore, there is no more role of innovation and production will simply be a one-sector representative firm that uses only capital and labor as factor inputs. This specification of production and technology for the EXO model is standard in the real business cycle (RBC) literature. The capital accumulation equation and capital adjustment cost function are exactly the same as in the ENDO model. Furthermore, the household's problem is unchanged, we will simply describe the production sector.

Production A representative firm produces the final (consumption) good using capital K_t and labor effort L_t with constant returns to scale technology

$$Y_t = K_t^\alpha (\tilde{Z}_t L_t)^{1-\alpha} \quad (35)$$

where productivity \tilde{Z}_t is specified as a trend stationary process

$$\tilde{Z}_t = \bar{B} \Omega_t \tilde{N}_t \quad (36)$$

$$\tilde{N}_t = e^{\mu t} \quad (37)$$

for some constant $\bar{B} > 0$ to be determined and the productivity shock Ω_t is specified exactly as in the ENDO model:

$$\Omega_t = e^{a_t} \quad (38)$$

$$a_t = \rho a_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2) \quad (39)$$

Dividends are defined as

$$D_t \equiv Y_t - W_t L_t - I_t \quad (40)$$

where I_t is capital investment and W_t is the wage rate. The capital stock evolves as

$$K_{t+1} = (1 - \delta)K_t + \Lambda \left(\frac{I_t}{K_t} \right) K_t \quad (41)$$

Taking the pricing kernel M_t as given, the firm's problem is to maximize shareholder's wealth, which can be formally stated as

$$\max_{\{I_t, L_t, K_{t+1}\}} E_0 \left[\sum_{t=0}^{\infty} M_t D_t \right] \quad (42)$$

subject to

$$D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} di \quad (43)$$

$$K_{t+1} = (1 - \delta)K_t + \Lambda \left(\frac{I_t}{K_t} \right) K_t \quad (44)$$

with corresponding first-order conditions are:

$$q_t = \frac{1}{\Lambda'_t} \quad (45)$$

$$W_t = (1 - \alpha) \frac{Y_t}{L_t} \quad (46)$$

$$1 = E_t \left[M_{t+1} \left\{ \frac{1}{q_t} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta) - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\Lambda_{t+1} \right) \right\} \right] \quad (47)$$

$$(48)$$

where $\Lambda_t \equiv \Lambda \left(\frac{I_t}{K_t} \right)$ and $\Lambda'_t \equiv \Lambda' \left(\frac{I_t}{K_t} \right)$.

Output is used for consumption and investment:

$$Y_t = C_t + I_t \quad (49)$$

We now turn to a direct comparison of the two models, ENDO and EXO. Qualitatively, the respective dynamics are driven by different productivity processes, as we discuss in section 3. The remaining sections provide quantitative results.

3 Equilibrium Growth and Productivity

In our benchmark model, sustained growth is an equilibrium phenomenon resulting from agents' decisions. Moreover, these decisions generate growth rate and productivity dynamics contrasting with those implied by more standard macroeconomic frameworks. These dynamics will be reflected in asset prices. In this section we describe these patterns qualitatively, and contrast them with the exogenous growth specification (EXO) described above. We will provide empirical evidence supporting these patterns and a quantitative analysis in the next section.

First, it is convenient to represent the aggregate production function in the ENDO model in a form that permits straightforward comparison with the EXO specification. To that end, note that using the equilibrium conditions derived above, final output can be rewritten as follows:

$$Y_t = \left(\frac{\xi}{\nu} \right)^{\frac{\xi}{1-\xi}} K_t^\alpha (\Omega_t L_t)^{1-\alpha} N_t^{\frac{\nu\xi-\xi}{1-\xi}} \quad (50)$$

For sustained growth to obtain in this setting we need to impose a parametric restriction. Technically, to ensure balanced growth, we need the aggregate production function to be homogeneous of degree one in the

accumulating factors K_t and N_t . Thus, the following parameter restriction needs to be satisfied:

$$\alpha + \frac{\nu\xi - \xi}{1 - \xi} = 1 \quad (51)$$

Imposing this restriction, we have a production function that resembles the standard neoclassical one with labor augmenting technology:

$$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha} \quad (52)$$

where total factor productivity (TFP) is

$$Z_t \equiv \bar{A} \Omega_t N_t \quad (53)$$

and $\bar{A} \equiv \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}} > 0$ is a constant. From the specification of the EXO model, we recall that

$$\tilde{Z}_t = \bar{B} \Omega_t \tilde{N}_t \quad (54)$$

$$\tilde{N}_t = e^{\mu t} \quad (55)$$

where we now set $\bar{B} = \bar{A} \equiv \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}} > 0$. Hence, the fundamental difference between the ENDO model and the canonical real business cycle (RBC) framework is that the trend component of the TFP process, N_t , is endogenous and fluctuates in the ENDO model but exogenous and deterministic in the RBC model. Our benchmark model thus endogenously generates a stochastic trend, which is consistent with the evidence for OECD countries in Cogley (1990).

This stochastic trend is naturally reflected in the dynamics of productivity growth rates in the respective models. Clearly, for log productivity growth rates we have

$$\Delta z_t = \Delta n_t + \Delta a_t \quad (56)$$

in the ENDO model, and similarly

$$\Delta \tilde{z}_t = \mu + \Delta a_t \quad (57)$$

in the EXO model, where lowercase letters denote logs. Realistically, a_t will be a fairly persistent process, so that we can write

$$\Delta z_t \approx \Delta n_t + \epsilon_t \quad (58)$$

$$\Delta \tilde{z}_t \approx \mu + \epsilon_t \quad (59)$$

Accordingly, while in the EXO model productivity growth will be roughly i.i.d., it will inherit a second component in the benchmark model which depends on the accumulation of patents. Clearly, in the model, the accumulation of patents is determined by the dynamics of R&D. Therefore, qualitatively and quantitatively, the dynamics of productivity growth reflect the dynamics of innovation.

To see this, rewrite the growth rate of productivity, ΔZ_t , as $\Delta Z_t = \Delta N_t \cdot \Delta \Omega_t$. Given a realistically persistent calibration of $\{\Omega_t\}$ in logs, we have $\Delta \Omega_t \approx e^{\epsilon_t}$. On the other hand, given the accumulation of N_t as $N_t = \vartheta_t \cdot S_{t-1} + \phi N_{t-1}$, the growth rate of patents becomes $\Delta N_t = \vartheta_t \cdot \hat{S}_{t-1} + \phi$, where we set

$$\hat{S}_t \equiv \frac{S_t}{N_t}$$

We will refer to \hat{S}_t as the R&D *intensity*. Accordingly, we find $\Delta Z_t \approx (\vartheta_t \cdot \hat{S}_{t-1} + \phi)(e^{\epsilon_t})$. Thus,

$$E_t[\Delta Z_{t+1}] \approx E_t \left[(\vartheta_t \cdot \hat{S}_t + \phi)(e^{\epsilon_{t+1}}) \right] \quad (60)$$

$$= (\vartheta_t \cdot \hat{S}_t + \phi) E_t [e^{\epsilon_{t+1}}] \quad (61)$$

$$\approx \vartheta_t \cdot \hat{S}_t + \phi \quad (62)$$

Thus, while in the EXO model, expected productivity growth is approximately constant, the ENDO model exhibits variation in expected growth driven by the R&D intensity. Empirically, the R&D intensity is a fairly persistent and volatile process. We will therefore expect productivity growth in the ENDO model to exhibit substantial low-frequency variation. That is, productivity in the model exhibits a stochastic trend. In other words, this difference between the models leads to a very different propagation of the productivity shock a_t ; namely, the ENDO model generates low- and high-frequency cycles whereas the RBC model only generates high-frequency cycles. We provide empirical and quantitative evidence on this channel in the next section. The equilibrium productivity growth dynamics implied by the model resemble closely those specified by Croce (2008). Croce specifies productivity to contain an i.i.d. component as well as a small, but persistent component. He refers to that latter component as long-run productivity risk. While he exogenously specifies these dynamics, we show that such long-run productivity risk arises naturally in a setting with endogenous growth and that it is linked to innovation.

We can get further insights into the determinants of the stochastic trend by exploiting the specification of the innovation sector. From the accumulation equation for new intermediate goods and the optimality condition for R&D it follows that the growth rate of the measure of intermediate goods satisfies

$$\frac{N_{t+1}}{N_t} = \phi + E_t [\chi M_{t+1} V_{t+1}]^{\frac{\eta}{1-\eta}} \quad (63)$$

$$= \phi + E_t \left[\chi^{\frac{1}{\eta}} \sum_{j=1}^{\infty} M_{t+j|t} \phi^{j-1} \Pi_{t+j} \right]^{\frac{\eta}{1-\eta}} \quad (64)$$

where $M_{t+j|t} \equiv \prod_s^j M_{t+s|t}$ is the j -step ahead stochastic discount factor and $M_{t|t} \equiv 1$. This implies that growth is directly related to the discounted value of future profits in the intermediate goods sector. This observation has two important implications. First, growth rates will naturally inherit the procyclicality of profits. Second, the average growth rate is endogenously related to the discount rate. Quantity dynamics in this model therefore reflect aspects of risk, in particular, the amount of risk and the price of risk, and hence risk premia. With recursive preferences, equilibrium growth will also depend on the endogenous amount of persistent long-term uncertainty. This is quite in contrast to the real business cycle model, where, as shown by Tallarini (2000), risk premia do not affect quantity dynamics.

4 Findings

In this section we explore the quantitative implications of the model using simulations. We use perturbation methods to solve the model. To account for risk premia and potential time variation in them, we use a higher order approximation around the stochastic steady state.

4.1 Calibration

In this part, we present the benchmark quarterly calibration used to assess the quantitative implications of the endogenous growth model (ENDO). Worth emphasizing, the core set of results are robust to reasonable deviations around the benchmark calibration and alternative parameterizations are explored in the sensitivity analysis section below. Recursive preferences have been used extensively in recent work in asset pricing.⁷ We follow this literature and set preference parameters to standard values that are also supported empirically.⁸ The parameters related to the final goods sector are set to standard values in the business cycle literature. We follow Comin and Gertler (2006) in calibrating the parameters related to the intermediate goods and R&D sectors. These choices are also consistent with empirical evidence in the growth literature. Critically,

⁷See Bansal and Yaron (2004).

⁸See Bansal, Yaron, and Kiku (2007) uses Euler conditions and a GMM estimator to provide empirical support for the parameter values.

satisfying balanced growth helps provide further restrictions on parameter values.

We begin with a description of the calibration of the preference parameters. The elasticity of intertemporal substitution ψ is set to value of 1.85⁹ and the coefficient of relative risk aversion γ is set to a value of 10, which are standard values in the long-run risks literature. An intertemporal elasticity of substitution larger than one is consistent with the notion that an increase in uncertainty lowers the price-dividend ratio. Note that in this parametrization, $\psi > \frac{1}{\gamma}$, which implies that the agent dislikes shocks to expected growth rates and is particularly important for generating a sizeable risk premium in this setting. The subjective discount factor β is set to an annualized value of 0.984 so as to be consistent with the level of the riskfree rate. In the endogenous growth setting β also has important effect on the level of the growth rate. In particular, increasing β (the agent is more patient) increases the steady-state growth rate. Holding all else constant (including β), the direct effect of an increase in growth is an increase in the level of the riskfree rate. On the other hand, the direct effect of increasing β and holding the level of the growth rate fixed is a decrease in the level of the riskfree rate.

We now describe the calibration of the parameters pertaining to the final goods sector. The capital share α is set to .35, the intermediate goods (materials) share ξ is set to 0.5, and the depreciation rate of capital δ is set to 0.02. These three standard business cycle parameters are calibrated to match steady-state evidence. The capital adjustment cost function is standard in the production-based asset pricing literature.¹⁰ The adjustment cost parameter ζ is set at 0.70 to match the relative volatility of consumption growth to output growth.

We now turn to the calibration of the parameters relating to the stationary productivity shock $a_t \equiv \log(\Omega_t)$. Note that this shock is different than the Solow residual since the final goods production technology includes a composite input consisting of an expanding variety of intermediate goods. A decomposition of total factor productivity in our benchmark model is provided below, which provides a mapping between the neoclassical model and our growth model. The persistence parameter ρ is set to an annualized value of 0.95 and is calibrated to match the first autocorrelation of R&D intensity, which is the key driver of expected growth rates and in turn, a critical determinant of asset prices. Furthermore, this value for ρ allows us to be consistent with the first autocorrelations of the key quantity growth rates and productivity growth.¹¹ The volatility parameter σ is set at 1.75% to match output growth volatility.

⁹This choice is also consistent with the estimation evidence in Fernandez-Villaverde, Koijen, Rubio-Ramirez and van Binsbergen (2011).

¹⁰See, for example, Jermann (1998), Croce (2008), Kaltenbrunner and Lochstoer (2008) or Fernandez-Villaverde, Koijen, Rubio-Ramirez and van Binsbergen (2011) for estimation evidence.

¹¹To provide further discipline on the calibration of ρ , note that since the ENDO model implies the TFP decomposition, $\Delta z_t = \Delta a_t + \Delta n_t$, we can project log TFP growth on log growth of the R&D stock to back out the residual Δa_t . The autocorrelations of the extracted residual $\Delta \hat{a}_t$ show that we cannot reject that it is white noise. Hence, in levels, it must be the case that a_t is a persistent process to be consistent with this empirical evidence. In our benchmark calibration, the annualized value of ρ is .95.

Now we describe the remaining parameters. The markup in the intermediate goods sector ν is set to a value of 1.65 and the elasticity of new intermediate goods with respect to R&D η is set to a value of 0.85. The markup of intermediate inputs is difficult to measure, so the rationale for the calibration is that the specialized nature would suggest that the markup is at the high end of the estimates across various goods. Nevertheless, varying the parameter around a reasonable range does not change our key quantitative results. The parameter η is within the range of panel and cross-sectional estimates from Griliches (1990). Since the variety of intermediate goods can be interpreted as the stock of R&D (a directly observable quantity), we can then interpret one minus the survival rate ϕ as the depreciation rate of the R&D stock. Hence, we set ϕ to 0.9625 which corresponds to an annualized depreciation rate $1 - \phi$ of 15% which is a standard value and that assumed by the BLS in the the R&D stock calculations. The scale parameter χ is used to help match balanced growth evidence and set at a value of 0.332.

We provide two slightly different calibrations of the exogenous growth model (EXO) to facilitate direct comparison with our benchmark model. For both calibrations, we set a trend growth parameter μ equal to 1.90% to match average output growth. We label the first calibration EXO I. In this calibration, we adjust the volatility of the forcing process to exactly match the volatility of consumption growth of the benchmark model. Without that adjustment, consumption volatility in a calibration of the EXO model with exactly the same parameters as in the benchmark model is higher than in the benchmark, because of the different resource constraints. We label such a calibration of the exogenous growth model EXO II. As our results indicate, our results are robust to such differences¹².

4.2 Productivity Dynamics

Many of the key implications of the benchmark model can be understood by looking at the endogenous dynamics of total factor productivity (TFP) growth, ΔZ_t , which we outlined in section 3:

$$E_t[\Delta Z_{t+1}] \approx \vartheta_t \cdot \widehat{S}_t + \phi \quad (65)$$

where $\widehat{S}_t \equiv \frac{S_t}{N_t}$ is the R&D intensity. Therefore, qualitatively, the dynamics of TFP are driven by the endogenous movements in R&D.

Quantitatively, the implications of the model will thus depend on the ability of our calibration to match basic stylized facts about R&D activity and innovation. As table 3 documents, the model is broadly consistent with volatilities and autocorrelations of R&D investment, the stock of R&D and R&D intensities in the data. Crucially, as in the data, the R&D intensity is a fairly volatile and persistent process. Specifically, we match

¹²Extensive further robustness checks with other model specifications are available in a separate appendix on request.

its annual autocorrelation of 0.93.

The above decomposition of the expected growth rate of TFP therefore suggests a highly persistent component in TFP growth. Table 4 confirms this prediction, both in the data as well as in the model. While uncovering the expected growth rate of productivity as a latent variable in the data (as in Croce (2008)) suggests an annual persistence coefficient of 0.93, our model closely matches this number with a persistence coefficient of 0.95. Moreover, the volatilities of expected TFP growth rates in the data and in the model roughly match. Naturally, matching the persistence properties of TFP in the data is important in a production economy, as this will affect the dynamics of macroeconomic variables. Note that in contrast to our benchmark model, the EXO specifications imply that TFP growth is roughly i.i.d. and not consistent with empirical evidence.

Qualitatively, the above decomposition and the persistence of R&D intensity suggests that R&D intensity should track productivity growth and expected productivity growth rather well. Figure 1 visualizes this pattern for expected productivity growth, using a simulated sample path. For realized growth rates, this is confirmed in figures 2 and 3, both in the model as well as in the data. The plots visualize the small, but persistent component in TFP growth induced by equilibrium R&D activity.

Accounting for an endogenous persistent component in productivity growth helps our benchmark model match observed productivity dynamics relative to the exogenous specifications, as table 5 documents. While the benchmark model roughly matches the annual autocorrelation of productivity growth, the EXO models are an order of magnitude off. The persistent component ENDO specification also accounts for a fairly volatile conditional mean of productivity growth as well as higher long-term volatilities than in the i.i.d. case.

On the other hand, from an empirical point of view, these results suggest, that R&D activity, and especially the R&D intensity should forecast productivity growth rather effectively. We confirm this prediction in table 6, which documents results from projecting productivity growth on R&D intensity, over several horizons. In the data, R&D intensity forecasts productivity growth over several years, significantly, and with R^2 's increasing with the horizon. Qualitatively, the model replicates this pattern rather well.

The intuition for these results comes from the endogenous R&D dynamics that the model generates. This can be readily gleaned from the impulse responses to an exogenous shock displayed in figure 4. It exhibits responses of quantities in the patents sector. Crucially, after a shock to profits rise persistently. Intuitively, a positive shock in the final goods sector raises the demand X_t for intermediate goods, and with $\Pi_t = (\nu - 1)X_t$, this translates directly into higher profits. Naturally, given persistently higher profits, the value of a patent goes up, as shown in the third panel. Then, in turn, as the payoff to innovation is the value of patents, this triggers a persistent increase in the R&D intensity. This yields the persistent endogenous component

in productivity growth displayed above. Crucially, the exogenous shock has two effects. It immediately raises productivity of the final output firm temporarily (due to the mean-reverting nature of the shock), leading to standard fluctuations at business cycle frequency. In addition, it also induces more R&D which will be reflected in more patents which has a permanent effect on the level of productivity. Moreover, the increases in R&D are persistent, leading to fluctuations at lower frequencies. Intuitively, in this setting, an exogenous shock to the level of productivity endogenously generates a persistent shock to the growth rate of the economy, or in other words, it generates growth waves.

4.3 Consumption Dynamics and Endogenous Long-Run Risk

In the previous section, we documented that the benchmark model has rich implications for the dynamics of measured TFP, which will naturally be reflected in the quantity dynamics of our production economy. With a view towards asset pricing, we focus on the implications for consumption dynamics in this section. In particular, we examine the dynamics of expected consumption growth that the model generates. While Bansal and Yaron (2004) have shown in an endowment economy that persistent variation in expected consumption growth coupled with recursive preferences can generate substantial risk premia in asset markets, the empirical evidence regarding this channel is still controversial. In this light, providing theoretical evidence in production economies supporting the mechanism would be reassuring. While several papers have considered how such long run risks can arise endogenously in production economies (Croce (2008), Kaltenbrunner and Lochstoer (2008), Campanale, Castro and Clementi (2008)), these studies operate in versions of the real business cycle model (proxied by the EXO specifications here) and typically do not generate sufficient endogenous risks to match asset market statistics.

Table 7 documents basic properties of consumption growth in the model. While the model is calibrated to match the volatility of consumption growth (as reported in table 2), it also roughly replicates its annual autocorrelation. This is in sharp contrast to the EXO specifications, where consumption growth is barely autocorrelated. This is somewhat symptomatic for the standard real business cycle model, which typically fails to match the observed autocorrelation of growth rates, or, in other words, lacks propagation (Cogley and Nason, 1995). More importantly, the table also documents that the benchmark model produces substantial variation in expected consumption growth, considerably more than the EXO specifications. On the other hand, the impulse response in figure 8 shows that this variation is also very persistent, and again in sharp contrast to the EXO models. Similarly, this is also reflected in the substantial long-term volatilities that consumption growth exhibits in the model, as reported in the table. In line with the asset pricing literature, we will refer to the volatility of consumption growth as business cycle or short-run risk and persistent variation in expected consumption growth as long-run risk. This suggests that the benchmark model generates

quantitatively significant long-run risks in consumption growth. This will be reflected in the model’s asset pricing implications which are discussed below. Note that while Bansal and Yaron (2004) (in an endowment economy setting) and Croce (2008) (in a production economy setting) generate long-run risks by introducing independent, persistent shocks to consumption and productivity growth respectively, in our model fluctuations in realized consumption growth and persistent variation in expected consumption growth are due to one source of exogenous uncertainty only. Hence, the model translates this disturbance in substantial low-frequency movements in consumption growth, or, in other words, provides a strong mechanism to propagate this shock. Accordingly, shocks to realized consumption growth also act as shocks to expected consumption growth.

Table 8 reports long-horizon autocorrelations of consumption growth, in the data and in the model. We restrict the empirical sample to 1953 to 2008, to ensure consistence with the availability with R&D data. While our model matches the first autocorrelation of consumption growth almost exactly, the second and third autocorrelation are negative in the data and positive in the model, and more importantly, outside the 95% confidence interval. On the other hand, all longer horizon autocorrelations are within that confidence interval.

Naturally, persistence in expected consumption growth is just a reflection of persistent dynamics in productivity growth. Hence, our model suggests that the dynamics of innovation is ultimately the source of long-run risk in consumption growth. Innovation generates persistent productivity gains, which are reflected in long growth waves in consumption. Figure 5 visualizes how closely expected consumption growth tracks the R&D intensity, which is the key determinant of expected productivity growth, in a sample path simulated from the model. Empirically, this suggests that measures related to innovation, and the R&D intensity in particular, should have forecasting for consumption growth. We verify this in table 9, which reports results from projecting future consumption growth over various horizons on the R&D intensity. Empirically, R&D intensity predicts future consumption growth over horizons up to 5 years, with significant point estimates, and R^2 ’s are increasing with the horizon. Qualitatively, the model reflects this pattern reasonably well. This gives empirical support to the notion of innovation-driven low-frequency variation in consumption growth. Using purely statistical techniques, Bansal, Kiku and Yaron (2007), extract series for expected consumption growth in the data. Notably, the correlation between the R&D intensity and their model-free series is about 70%, which points to a macroeconomic foundation for predictability in consumption growth.

Another way of capturing low-frequency variation in consumption growth is by using appropriate filters. We use bandpass filters (Christiano and Fitzgerald (2003)) to identify high and low frequency cycles in growth rates. More specifically, we identify high frequency or business cycle movements with a bandwidth of 2 to 32 quarters, and low frequency components with a bandwidth of 32 to 200 quarters. In a similar spirit, Comin

and Gertler (2006) identify medium term cycles in quantities using bandpass filters. However, they identify cycles as low-frequency deviations from a linear trend, while we focus on cycles in growth rates. This is an important distinction from an asset pricing perspective. In particular, there is a close correspondence between our macroeconomic notion of a low-frequency component in consumption growth and the finance notion of long-run risks in consumption growth. This is visualized in figure 6, which plots consumption growth and its low frequency component in a simulated sample path as well as in the data.

4.4 Fluctuations and Propagation

While consumption dynamics are important for asset pricing, endogenous persistent variation in expected productivity growth suggests a propagation mechanism for quantities more generally, which, as alluded to above, standard macro models typically lack. We therefore now turn to a more systematic discussion of the macroeconomic implications of the model. Table 2 reports basic statistics implied by the model. Concentrating on the quantities for the moment, the table shows that the model is reasonably consistent with basic quantity statistics. In particular, our benchmark model does just as well as the EXO models, which are versions of a standard real business cycle model. One way of interpreting this finding is by saying that the ENDO model generates high-frequency dynamics or business cycle statistics in line with the canonical real business cycle model. On the other hand, all specifications predict investment to be too smooth. This is a general difficulty with production models with recursive preferences.

While table 2 suggests that our benchmark endogenous growth model performs equally well as the real business cycle model when it comes to basic quantity statistics, the endogenous productivity dynamics of the model lead to a strong propagation mechanism absent in the latter model, as we explore now.

Table 10 reports autocorrelations of basic growth rates, in the data, as well as in the ENDO and the EXO models. Note first that while all growth rates exhibit considerable positive autocorrelation at annual frequencies, the corresponding persistence implied by the EXO models is virtually zero, and sometimes even negative. This is one of the main weaknesses of the real business cycle model (as pointed out e.g. in Cogley and Nason (1995)). In stark contrast, our ENDO model generates substantial positive autocorrelation in all quantities, and in general are quantitatively close to their data counterparts. Note that the exogenous component of productivity is the same in both model. Accordingly, the ENDO model possesses a strong propagation mechanism induced by the endogenous component of productivity, e.g. by R&D.

The intuition for this endogenous propagation is of course simple, and tightly linked to the dynamics of TFP documented in the previous section. To the extent that innovation induces a persistent component in productivity, this will be reflected in quantity dynamics. Recall however, that the TFP dynamics implied by the model are consistent with the empirical evidence. As for consumption growth, this suggests that the

driver of expected productivity growth, namely the R&D intensity, should forecast quantity growth. This is verified in table 13 for output growth.

The propagation mechanism implies that macroeconomic quantities display markedly different behavior at different frequencies, in other words, it implies a rich intertemporal distribution of growth rates. This can be seen from tables 11 and 12, which capture the intertemporal distribution of growth rates. While the implied volatilities of growth rates of the EXO and ENDO models are basically undistinguishable at short horizons, in the ENDO model they grow fairly quickly over longer horizons. Another way of interpreting this finding is that the ENDO model generates significant dynamics at lower frequencies, while the EXO models do not. Empirically, this can be captured by looking at low frequency components of growth rates. The correspondence of high and low frequency movements in output growth in both the data and the model is visualized in figure 7.

Another implication of the model is that it generates cash flow dynamics in line with the empirical evidence. First of all, it generates strongly procyclical profits. This can be seen from figure 4. This is in line with recent work on expanding variety models in Bilbiie, Ghironi, Melitz (2007), but typically presents a challenge to macro models. In our setting, this is driven by the procyclical demand for intermediate goods. Second, the model generates a persistent component in dividend growth. This can be seen in table 11, which documents considerable volatility in conditional expected dividend growth, which implies substantial variation in the conditional mean of cash flow growth. This is visualized in figure 8. Again, this is in stark contrast to the exogenous growth specifications. This will be important from an asset pricing perspective, as only the benchmark model generates sufficient long-term uncertainty about dividend growth.

One way of summarizing the results of this section is by saying that while matching business cycle statistics well, the benchmark endogenous growth model generates substantial movements in quantities at lower frequencies, in contrast to the real business cycle model. In other words, the endogenous growth model exhibits a strong propagation mechanism absent in the real business cycle model.

4.5 Asset Pricing Implications

In order to use our model to shed some light on the link between macroeconomic risk and growth via asset price data, we must first make sure that the model is quantitatively consistent with basic asset market statistics. As we will show shortly, the quantity dynamics discussed in the previous section will be key in generating high risk premia roughly in line with historical data. Since we assume that the agent has Epstein-Zin utility with a preference for an early resolution of uncertainty, this implies that not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth.

Table 2 reports asset market statistics along with the quantity statistics referred to earlier, both for the benchmark model and the exogenous growth specifications. Quantitatively, the benchmark model is broadly consistent with basic asset price data: It generates a low and smooth risk-free rate and a sizeable equity premium. The equity premium is in excess of 4%. The volatility of the equity premium is a close to 7% annually, which is close to half the historical volatility of the market excess return. Given our productivity-based model, this number can be thought of as the productivity-driven fraction of historical excess return volatility. On the other hand, it is well known that dividend-specific shock explain a good part of stock return volatility, which we do not account for in the model. In particular, Ai, Croce and Li (2010) report that empirically the productivity-driven fraction of return volatility is just around 6%, which is roughly consistent with our quantitative finding.

In order to understand these results, it is instructive to compare the asset pricing implications of the benchmark model with those of the exogenous growth specifications. While, as discussed previously, the quantity implications of the models are similar at high-frequencies, the pricing implications are radically different. As can be seen from the table, the risk free rate is counterfactually high in the exogenous growth specifications, and the equity premium is close to zero and only a tiny fraction of what obtains in the benchmark model. These differences are intimately connected to differences in low-frequency dynamics that the two models generate, as reported in tables 11 and 12. Intuitively, in the settings with exogenous growth, expected growth rates are roughly constant (as in the real business cycle model), therefore diminishing households' precautionary savings motive. In such a setting, households want to borrow against their future income, which in equilibrium can only be prevented by a prohibitively high interest rate. In the endogenous growth setting, however, taking advantage of profit opportunities in the intermediate goods sector leads to long and persistent swings in aggregate growth rates, and higher volatility over longer horizons. In this context, households optimally save for low growth episodes, leading to a lower interest rate in equilibrium.

Moreover, the model also generates a substantial equity premium. The reason is twofold. First, as discussed above, the model generates endogenous long-run risks in consumption growth reflected in the stochastic discount factor. Second, the level of the equity premium also implies that in equilibrium, dividends are risky. The reason is that dividends naturally inherit a persistent component from the endogenous component of productivity. These cash flow dynamics not only affect risk premia, but naturally, also asset market valuations. In particular, the model roughly matches the dynamics of stock market values, in the data, as measured by Tobin's Q. This can be seen from tables 2 and 10. Not only does the model generate considerably higher stock market volatility than its EXO counterparts, it also matches its autocorrelation. This suggests that innovation-driven cycles rationalize long-term movements in stock market valuations.

These effects can also be seen in figure 8. Figure 8 shows conditional expected growth rates for macro

variables. Specifically, the figure documents that following a productivity shock expected growth rates respond strongly in a persistent fashion in the ENDO model whereas in the EXO models expected growth rates are virtually unresponsive to the shock. In particular, expected dividend growth rates endogenously exhibit substantial persistent variation. This endogenous variation is consistent with the setups in Barsky and DeLong (1993), show that such a process can explain long swings in stock markets, and in Bansal and Yaron (2004).

The impulse responses also show that innovations to realized consumption and dividend growth are tightly linked to innovations to expected growth. Both of these innovations are priced when agents have Epstein-Zin utility with a preference for early resolution of uncertainty. In this case, agents fear that persistent slowdowns in growth coincide with a fall in asset prices. Therefore bad shocks are simultaneously bad shocks for the long run, which renders equity claims very risky.

This can be seen in figure 9. The figure reports impulse responses of the stochastic discount factor, returns and marginal q in the benchmark model, and in the exogenous growth specifications. Notice first that in the benchmark model the response of the stochastic discount factor is substantially larger on impact than in the exogenous growth counterparts. This is due to the rich intertemporal distribution of consumption growth that the model generates. In the benchmark model, as discussed above, a shock to realized consumption growth is simultaneously a shock to expected consumption growth, through the model's propagation mechanism. This revision in consumption growth expectations is picked up in the stochastic discount factor as a revision to expected continuation utility. On the other hand, in the exogenous growth counterparts, consumption growth is essentially iid, so that a shock to realized consumption growth does not lead to a revision in growth expectations, and continuation utilities are unaffected. A shock's impact on continuation utilities therefore leads to significantly different responses in the stochastic discount factor, making it substantially more volatile in the benchmark model.

Similarly, while qualitatively in both benchmark and exogenous growth models the responses of returns and prices to a shock go in the same directions, quantitatively the effects are much more pronounced in the former, again owing to the increased persistence and long-term volatility that it displays. Another way of understanding the asset pricing implications of the models is to recall that the equity premium is $E[r_d - r_f] \approx -cov(m, r_d)$. This implies that equity must offer the higher a premium, the more equity returns and the discount factor move in opposite directions. We can see from figure 10 that the benchmark model displays stronger co-movements of equity returns and discount factor, leading to a higher equity premium.

Taken together, these results suggest that an endogenous growth model with recursive preferences is a natural environment to understand asset pricing in a general equilibrium setting. More specifically, the mechanisms that allow the model to endogenously generate high risk premia, namely long persistent swings in growth

rates coupled with recursive preferences, are exactly those that Bansal and Yaron (2004) specify exogenously and refer to as long-run risks. While the dynamics they specify are somewhat hard to detect in the data, they are a natural implication of agents' optimal innovation and R&D decisions in our model. The model suggests that these effects are quantitatively significant. This also suggests that the growth rate dynamics that Bansal and Yaron specify naturally arise endogenously in a wide class of stochastic endogenous growth models, thus making them a very natural way to link asset prices to long-term growth prospects.

4.6 Risk and Growth

While the endogenous growth rate dynamics in the model in conjunction with recursive preferences help explain large equity risk premia in historical data, asset prices also have important feedback effects on the macroeconomy. In particular, realistic risk premia in the model provide realistic compensation for risk-taking. This mechanism therefore channels resources towards investments in risky assets, and hence, in our model, provides incentives for innovation and investment in R&D. Realistic compensation for risk therefore fosters growth by encouraging risk-taking. More specifically, in our model, the average growth rate is increasing in the Sharpe ratio, so long as $\psi > \frac{1-\gamma}{\gamma}$. This feedback effect is negligible in standard models of endogenous growth because they do not generate significant risk premia, and is assumed away in standard macro models as they specify growth exogenously. However, our model suggests that this effect can be quantitatively significant.

We provide quantitative evidence on this feedback effect in table 14, where we report sensitivity of model implications with respect to the key preference parameters, risk aversion and intertemporal elasticity of substitution. Varying risk aversion and intertemporal elasticity of substitution separately allows us to trace out the effects of changes in risk prices and changes in the amount of risk independently.

Consider first varying risk aversion, in the first 2 columns. Consistent with the results in Tallarini (2000), varying risk aversion barely affects standard business cycle statistics, that is, second moments. In other words, varying risk aversion does not affect the amount of risk in the economy. Importantly, this holds for both business cycle risks as measured by consumption growth volatility as for long-run risks as measured by the persistence of consumption growth and the volatility of its conditional expectation. However, what varies with risk aversion is the price of risk, which is reflected in substantial differences in the equity premium, and hence risk compensation, across risk aversion specifications. Therefore, relative to Tallarini, the benchmark endogenous growth model exhibits a new effect, namely sensitivity of the average growth rate relative to the risk aversion. Specifically, raising risk aversion fosters growth. This has a simple intuition: Higher compensation for the same risks, or similarly, higher price for the same magnitude of risks reflected by in a higher Sharpe ratio, makes investment in risky assets more attractive and therefore channels resources towards

innovation and R&D. This is reflected in higher R&D investment, as measured by the R&D intensity, and hence higher growth. Formally, this effect can be traced to equation (64), which relates equilibrium growth to the present value of future profits in the intermediate goods sector, thereby making the link between growth and the stochastic discount factor and hence risk, explicit.

In the last two columns, we keep risk aversion fixed at the benchmark level, but vary the intertemporal elasticity of substitution. Note that for all specifications we have $\psi > \frac{1}{\gamma}$, so that irrespective of the specification, agents have a preference for early resolution of uncertainty. Varying the IES changes the amount of risk in the economy, and its intertemporal distribution. Raising the IES is akin to increasing the propensity to substitute over time, which increases the response of investment to productivity and expected productivity growth and accordingly its volatility. In turn this smoothes consumption growth and increases its persistence. This raises the volatility of the conditional mean of consumption growth. Raising the IES therefore reduces short-run risk and increases long-run risk, while lowering the IES increases short-run risk and reduces long-run risk. With a high price of long-run risk, the net effect is an increase of the equity premium in the first case, and a fall in the latter case. As above, the average growth rate of the economy is increasing in the Sharpe ratio. However, there are two effects at work. As usual, increasing the propensity to substitute over time increases the growth rate. On top of that, the same risk-taking channel as above is at work. The intertemporal distribution of risks has shifted, and was reallocated towards long-run risk with higher price of risk and compensation, which channels resources towards risky investments. This further increases R&D and hence growth.

These results suggest that there is a tight connection between macroeconomic risk, growth and risk premia in asset markets. Importantly, in our model, the dynamics of risk-taking affect the dynamics of aggregate growth rates, which, in conjunction with recursive preferences, give rise to large risk premia. In turn, of course, these premia compensate for risk-taking. Our results suggest that these endogenous feedback effects are quantitatively significant.

4.7 Long-Term Comovement

So far, we have discussed how the benchmark model generates fluctuations in quantities and prices at various frequencies. However, the model also has interesting and realistic implications for comovement between prices and quantities at lower frequencies. This is displayed in figures 10, 11 and 12.

Figure 10 reveals that the model replicates the low-frequency comovements between productivity and quantities in the data. This is noteworthy because it reveals the significant variation macro data exhibit at lower frequencies and the significant comovement between productivity and quantities, which is mirrored by the ENDO model.

Figure 11 shows the close match between the price-dividend ratio and productivity growth in the data and the benchmark model at low-frequencies. This strongly suggests productivity-driven slow movements in asset market valuations in the data. In the model, these movements are driven by variation in expected cash flows, induced by time variation in R&D intensity. The long swings in price-dividend ratios are consistent with the evidence in Barsky and Delong (1993).

At lower frequencies we also find strong cross-correlations between stock returns and consumption growth. This is displayed in figure 12, indicating the lag-lead structure between returns and consumption growth. In the data and at low frequencies, returns lead consumption growth by several quarters and the lead correlations die away more slowly (relative to the lag correlations). In other words, lower-frequency movements in returns contain important information regarding long-run movements in future growth. The ENDO model replicates this feature whereas the EXO models do not. This important divergence between the two models is due to the fact that in the ENDO model, growth rates contain a predictable component, which is absent in the EXO models, that is a key determinant of asset prices. In sum, the benchmark model is able reconcile the long-term relationship between returns and growth that the neoclassical growth model fails to produce.

5 Conclusion

Starting from the notion that asset prices reflect expectations about future growth, we provide a quantitative analysis of a stochastic model of endogenous growth in which long-term growth prospects are endogenously determined. In the model, innovation and investment in research and development are the ultimate sources of sustained growth. In the data, R&D is quite volatile, persistent and procyclical. Our model then predicts that these movements in the sources of growth will be reflected in the dynamics of the aggregate economy, leading to long-term cycles and persistent growth waves. More precisely, the model predicts a small, but persistent endogenous component in productivity growth, determined by innovation, leads to long and persistent swings in macroeconomic quantities. Therefore, in spite of only a single exogenous shock, the model generates significant cycles at high- and low-frequencies generated by the endogenous response of innovation to the shock. In other words, the innovation process generates a strong propagation mechanism absent in standard macroeconomic models. Empirically, we find strong support for innovation driven low frequency fluctuations in aggregate growth rates in the data.

Under the assumption of recursive preferences, such that households care about uncertainty about long-term growth prospects, these quantity dynamics have strong implications for asset prices and risk premia. With such a preference specification agents are very averse to the low-frequency variation in expected growth rates that the model generates, which yields high risk premia in asset markets and a low risk and stable free rate. As such, the model provides a macroeconomic foundation for long run risks in asset markets, as pioneered by

Bansal and Yaron (2004), and suggests that a strong propagation mechanism in macro models and high risk premia are inherently linked. In particular, realistically high premia on risk-taking provide strong intertemporal incentives to engage in innovation and investing in R&D. Moreover, the model predicts innovation and productivity driven low frequency movements in stock market values and price-dividend ratios, in line with the empirical evidence.

Our model has further implications for the welfare costs of fluctuations. In particular, the model suggests that the welfare costs of fluctuations are large, substantially larger than suggested by Lucas' (1987) calculations. This reflects that, in our model, shocks to current productivity induce persistent uncertainty about future growth. Such long-run uncertainty carries a high price of risk, and hence leads to substantial welfare losses, consistent with the evidence in Alvarez and Jermann (2004). Importantly, short-run macroeconomic uncertainty and uncertainty about future growth prospects are inherently linked. We examine the implications of the model for the cost of fluctuations along with some policy implications in a companion paper (Kung and Schmid (2010))¹³.

In short, our model implies that there are tight links between macroeconomic risk, growth and risk premia in asset markets and hence suggests that stochastic models of endogenous growth are a useful framework for quantitative macroeconomic modeling and asset pricing. In other words, as emphasized by Baker, DeLong and Krugman (2005), "In our view, the links between asset returns and economic growth are strong". This paper offers a tractable and empirically successful framework to analyze and quantify them.

¹³See Barlevy (2004) for a related analysis for the cost of fluctuations in a model with endogenous growth

6 References

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Appendix A. Data

Annual and quarterly data for consumption, capital investment, and GDP are from the Bureau of Economic Analysis (BEA). Annual data on private business R&D investment is from the survey conducted by National Science Foundation (NSF). Annual data on the stock of private business R&D is from the Bureau of Labor Statistics (BLS). Annual productivity data is obtained from the BLS and is measured as multifactor productivity in the private nonfarm business sector. The sample period is for 1953-2008, since R&D data is only available during that time period. Consumption is measured as expenditures on nondurable goods and services. Capital investment is measured as private fixed investment. Output is measured as GDP. The variables are converted to real using the Consumer Price Index (CPI), which is obtained from the Center for Research in Security Prices (CRSP). Monthly nominal return and yield data are from CRSP. The real market return is constructed by taking the nominal value-weighted return on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) and deflating it using the CPI. The real risk-free rate is constructed by using the nominal average one-month yields on treasury bills and taking out expected inflation.¹⁴ Aggregate market and book values of assets are from the Flow of Funds account.

¹⁴We model the monthly time series process for inflation using an AR(4).

Table 1: Calibration

Parameter	Description	ENDO	EXO I	EXO II
β^4	Subjective Discount Factor	0.984	0.984	0.984
ψ	Elasticity of Intertemporal Substitution	1.85	1.85	1.85
γ	Risk Aversion	10	10	10
ξ	Intermediate Goods Share	0.5	-	-
ν	Elasticity of Substitution Between Intermediate Goods	1.65	-	-
α	Capital Share	0.35	0.35	0.35
ρ^4	Autocorrelation of Ω	0.95	0.95	0.95
χ	Scale Parameter	0.332	-	-
ϕ	Survival Rate of Intermediate Good	0.9625	-	-
η	Elasticity of New Intermediate Goods wrt R&D	0.83	-	-
δ	Depreciation Rate of Capital Stock	0.02	0.02	0.02
σ	Volatility of Productivity Shock ϵ	1.75%	0.97%	1.75%
ζ	Elasticity of Capital Investment Rate	0.70	0.70	0.70
$\mu * 4$	Trend Growth Rate	-	1.90%	1.90%

This table reports the benchmark quarterly calibration used for the endogenous growth (ENDO) and exogenous growth (EXO) models.

Table 2: Summary Statistics

	Data	ENDO	EXO I	EXO II
First Moments				
$E[\Delta y]$	1.90%	1.90%	1.90%	1.90%
$E[r_f]$	1.62%	1.21%	2.61%	2.58%
$E[r_d^* - r_f]$	5.57%	4.10%	0.08%	0.19%
Second Moments				
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.61	0.61	1.13	1.13
$\sigma_{\Delta i}/\sigma_{\Delta c}$	4.38	2.23	0.79	0.79
$\sigma_{\Delta s}/\sigma_{\Delta y}$	2.10	1.64	-	-
$\sigma_{\Delta z}/\sigma_{\Delta y}$	1.22	1.52	1.54	1.54
$\sigma_{\Delta c}$	1.42%	1.42%	1.42%	2.58%
σ_{r_f}	0.67%	0.30%	0.05%	0.09%
$\sigma_{r_d - r_f}$	14.98%	7.06%	2.27%	4.10%
σ_Q	12.85%	16.46%	2.89%	5.19%

This table presents annual first and second moments from the endogenous growth (ENDO) model, the exogenous growth (EXO) model, and the data. The models are calibrated at a quarterly frequency and the moments are annualized. Since the equity risk premium from the models is unlevered, we follow Boldrin, Christiano, and Fisher (2001) and compute the levered risk premium from the model as: $r_{d,t+1}^* - r_{f,t} = (1 + \kappa)(r_{d,t+1} - r_{f,t})$, where r_d is the unlevered return and κ is the average aggregate debt-to-equity ratio, which is set to $\frac{2}{3}$. Annual macro data are obtained from the BEA, BLS, and NSF. Q is ratio of the market value to book value of assets (Tobin's Q). Monthly return data are from CRSP and the corresponding sample moments are annualized. Annual market and book values of assets are from the Flow of Funds account. The data sample is 1953-2008.

Table 3: Innovation Dynamics

	Data	ENDO
$\sigma_{\Delta s}$	4.89%	3.82%
$AC1(\Delta s)$	0.21	0.06
$AC1(\Delta n)$	0.90	0.94
$AC1(S/N)$	0.93	0.93

This table reports summary statistics for innovation-related variables: log R&D growth, log stock of R&D growth and R&D intensity. The first column presents the statistics from the data and the second column is from the endogenous growth model (ENDO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. R&D stock data are from the BLS. R&D flow data are from the NSF.

Table 4: Expected Productivity Growth Dynamics

	Estimate	ENDO
$\rho_{\tilde{x}}$	0.93	0.95
$\sigma(\tilde{x})$	1.10%	1.20%

This table reports the annual persistence and standard deviation of the expected growth rate component of productivity growth from the data and from the endogenous growth (ENDO) model. The estimates are taken from Croce (2010), where the expected growth rate component of productivity \tilde{x}_{t-1} is a latent variable that is assumed to follow an AR(1). In contrast, in the ENDO model the expected growth rate component is the growth rate of the variety of intermediate goods Δn_t , a endogenous structural variable of the model. In particular, since the shock Ω_t is persistent, log productivity growth can be written approximately as $\Delta z_t = \tilde{x}_{t-1} + \epsilon_t$, where $\tilde{x}_{t-1} \equiv \Delta n_t$ and ϵ_t is an iid disturbance. The ENDO model endogenously generates a productivity process that is the same as the exogenous specification of Croce (2010), which is supported empirically.

Table 5: Productivity Growth Dynamics

	Data	ENDO	EXO I	EXO II
$AC1(\Delta z)$	0.09	0.11	-0.02	-0.02
$\sigma(E_t[\Delta z_{t+1}])$		0.38%	0.15%	0.27%
$\sigma_{\Delta z(5)}$		9.29%	4.15%	7.45%
$\sigma_{\Delta z(10)}$		15.79%	5.55%	9.92%
$\sigma_{\Delta z(20)}$		25.24%	6.86%	12.67%

This table reports summary statistics for productivity growth: Annual autocorrelation, volatility of the conditional mean, and 5, 10 and 20 year volatilities. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual multifactor productivity data are from the BLS.

Table 6: Productivity Growth Forecasts

Horizon (k)	Data			ENDO	
	$\hat{\beta}$	S.E.	\hat{R}^2	β	R^2
1	0.014	0.009	0.031	0.075	0.039
2	0.031	0.015	0.080	0.142	0.062
3	0.049	0.024	0.120	0.204	0.077
4	0.069	0.032	0.174	0.261	0.088
5	0.091	0.041	0.232	0.314	0.095

This table presents productivity growth forecasting regressions from the data and from the endogenous growth (ENDO) model for horizons (k) of one year to five years. Specifically, we project real per capita output growth on log R&D intensity, $\Delta z_{t,t+1} + \dots + \Delta z_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}$. In the data, the regression is estimated via OLS with Newey-West standard errors with $k - 1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Multifactor productivity data and R&D stock data are from the BLS. R&D flow data are from the NSF.

Table 7: Consumption Dynamics

	Data	ENDO	EXO I	EXO II
$AC1(\Delta c)$	0.40	0.39	-0.002	-0.002
$\sigma(E_t[\Delta c_{t+1}])$		0.51%	0.09%	0.17%
$\sigma_{\Delta c(5)}$		6.63%	3.14%	5.63%
$\sigma_{\Delta c(10)}$		11.97%	4.30%	7.70%
$\sigma_{\Delta c(20)}$		21.18%	5.58%	10.22%

This table reports summary statistics for consumption growth: Annual autocorrelation, volatility of the conditional mean, and 5, 10 and 20 year volatilities. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual consumption data are from the BEA.

Table 8: Consumption Autocorrelations

	Data	ENDO	95% confidence interval	
			lower	upper
$AC1(\Delta c)$	0.40	0.39	0.05	0.53
$AC2(\Delta c)$	-0.09	0.26	-0.01	0.49
$AC3(\Delta c)$	-0.17	0.21	-0.06	0.46
$AC4(\Delta c)$	-0.11	0.17	-0.11	0.42
$AC5(\Delta c)$	0.06	0.13	-0.15	0.39
$AC6(\Delta c)$	0.10	0.11	-0.17	0.38
$AC7(\Delta c)$	-0.02	0.09	-0.20	0.35
$AC8(\Delta c)$	-0.16	0.05	-0.24	0.33
$AC9(\Delta c)$	-0.17	0.03	-0.25	0.32
$AC10(\Delta c)$	-0.01	0.02	-0.27	0.31

This table reports long-horizon autocorrelations of consumption growth. The first column presents the statistics from the data for the sample 1953-2008, the second column is from the endogenous growth model (ENDO), with lower and upper boundaries of the 95% confidence interval. Model estimates are obtained from 200 simulations of 56 years of data at quarterly frequency, time-aggregated to annual frequency. Annual consumption data are from the BEA.

Table 9: Consumption Growth Forecasts

Horizon (k)	Data			ENDO	
	$\hat{\beta}$	S.E.	\hat{R}^2	β	R^2
1	0.017	0.006	0.070	0.068	0.141
2	0.034	0.012	0.105	0.118	0.161
3	0.048	0.017	0.131	0.168	0.179
4	0.062	0.023	0.163	0.200	0.175
5	0.077	0.030	0.202	0.224	0.168

This table presents consumption growth forecasting regressions from the data and from the endogenous growth (ENDO) model for horizons (k) of one year to five years. Specifically, we project real per capita output growth on log R&D intensity, $\Delta c_{t,t+1} + \dots + \Delta c_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}$. In the data the regression is estimated via OLS with Newey-West standard errors with $k - 1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Consumption data is from the BEA, R&D flow data is from the NSF, and R&D stock data is from the BLS.

Table 10: First Autocorrelations

	Data	ENDO	EXO I	EXO II
$AC1(\Delta z)$	0.09	0.11	-0.020	-0.020
$AC1(\Delta c)$	0.40	0.46	-0.002	-0.002
$AC1(\Delta y)$	0.37	0.21	0.001	0.001
$AC1(\Delta i)$	0.25	0.14	0.012	0.012
$AC1(Q)$	0.95	0.96	0.89	0.89

This table reports first autocorrelations of annual variables. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual macro data are from the BEA, BLS, and NSF. Annual market and book values of assets are from the Flow of Funds account.

Table 11: Volatility of Expected Growth Rates

	ENDO	EXO I	EXO II
$\sigma(E_t[\Delta z_{t+1}])$	0.38%	0.15%	0.27%
$\sigma(E_t[\Delta c_{t+1}])$	0.51%	0.09%	0.17%
$\sigma(E_t[\Delta y_{t+1}])$	0.42%	0.08%	0.14%
$\sigma(E_t[\Delta i_{t+1}])$	0.37%	0.05%	0.10%
$\sigma(E_t[\Delta d_{t+1}])$	0.92%	0.18%	0.31%

This table reports annualized volatilities of expected growth rates from the endogenous growth (ENDO) and exogenous growth (EXO) models.

Table 12: Volatility of High- and Low-Frequency Components

	High				Low			
	Data	ENDO	EXO I	EXO II	Data	ENDO	EXO I	EXO II
$\sigma_{\Delta y}$	1.95%	2.17%	1.12%	2.18%	0.69%	0.62%	0.29%	0.54%

This table reports the annualized volatilities of high- and low-frequency components of output growth from the data and from the ENDO and EXO models. The bandpass filter from Christiano and Fitzgerald (2003) is used to isolate the components of the various frequencies. The high-frequency component is defined as a bandwidth of 2 to 32 quarters. The low-frequency component is defined as a bandwidth of 32 to 200 quarters. Quarterly output data is from the BEA.

Table 13: Output Growth Forecasts

Horizon (k)	Data			ENDO	
	$\hat{\beta}$	S.E.	\hat{R}^2	β	R^2
1	0.020	0.013	0.040	0.085	0.105
2	0.046	0.022	0.084	0.163	0.161
3	0.068	0.029	0.119	0.236	0.195
4	0.089	0.041	0.158	0.306	0.217
5	0.114	0.051	0.210	0.372	0.231

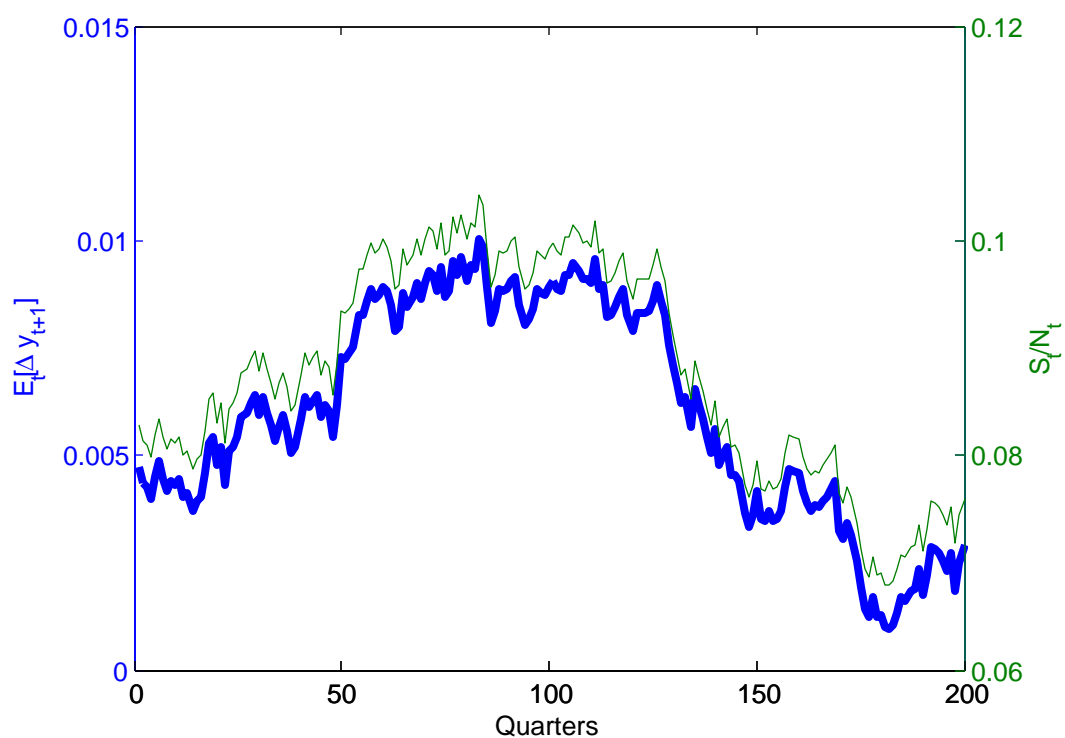
This table presents output growth forecasting regressions from the data and from the endogenous growth (ENDO) model for horizons (k) of one year to five years. Specifically, we project real per capita output growth on log R&D intensity, $\Delta c_{t,t+1} + \dots + \Delta c_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}$. In the data, the regression is estimated via OLS with Newey-West standard errors with $k - 1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Output data is from the BEA, R&D flow data is from the NSF, and R&D stock data is from the BLS.

Table 14: Sensitivity Analysis: Preference Parameters

	$\gamma = 2$	$\gamma = 15$	$\psi = 0.5$	$\psi = 2.2$
First Moments				
$E[\Delta y]$	1.87%	2.02%	0.86%	2.38%
$E[r_f]$	2.40%	0.50%	2.23%	0.87%
$E[r_d^* - r_f]$	0.68%	6.27%	1.28%	5.06%
$E[S/N]$	0.081	0.084	0.077	0.086
Other Moments				
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.61	0.61	1.09	0.52
$\sigma_{\Delta i}/\sigma_{\Delta c}$	2.23	2.23	0.57	2.37
$\sigma_{\Delta s}/\sigma_{\Delta y}$	1.64	1.64	1.11	1.73
$\sigma_{\Delta c}$	1.42%	1.42%	2.61%	1.21%
σ_{r_f}	0.30%	0.30%	0.38%	0.27%
$\sigma_{r_d - r_f}$	7.06%	7.06%	3.41%	7.35%
$AC1(\Delta c)$	0.46	0.46	0.07	0.62
$\sigma(E_t[\Delta c_{t+1}])$	0.51%	0.51%	0.19%	0.59%
Sharpe Ratio	0.10	0.88	0.38	0.69

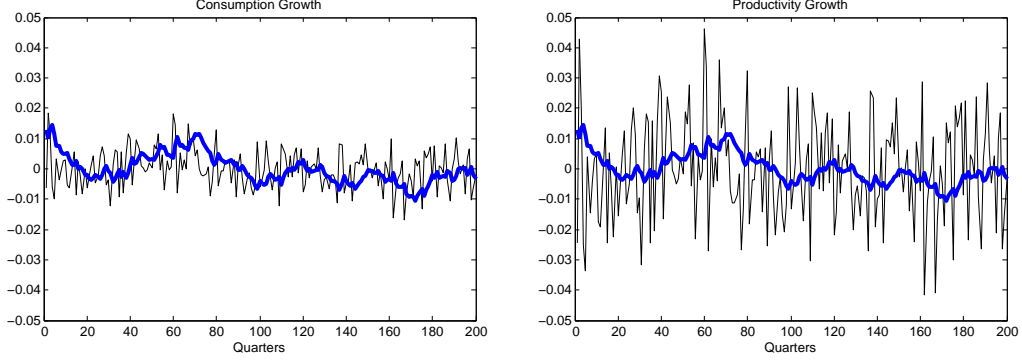
This table compares key summary statistics from alternate calibration of the endogenous growth model (ENDO) that vary the preference parameters, risk aversion γ and the elasticity of intertemporal substitution ψ , one at a time while holding all other parameters fixed at the benchmark calibration. Note that at the benchmark calibration, $\gamma = 10$ and $\psi = 1.85$. The models are calibrated at a quarterly frequency and the summary statistics are annualized. The risk premium is levered following Boldrin, Christiano, and Fisher (2001).

Figure 1: Expected Productivity Growth and R&D Intensity



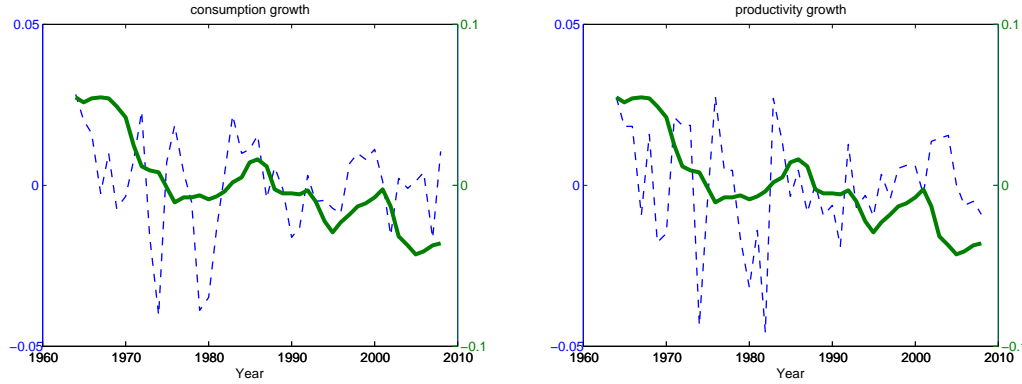
This figure plots expected log productivity growth $E_t[\Delta z_{t+1}]$ (thick line) and R&D intensity $\frac{s_t}{N_t}$ (thin line) from the ENDO model for a sample simulation of 200 quarters. In the model, R&D intensity is the key determinant of expected growth rates.

Figure 2: Growth Rates and R&D Intensity



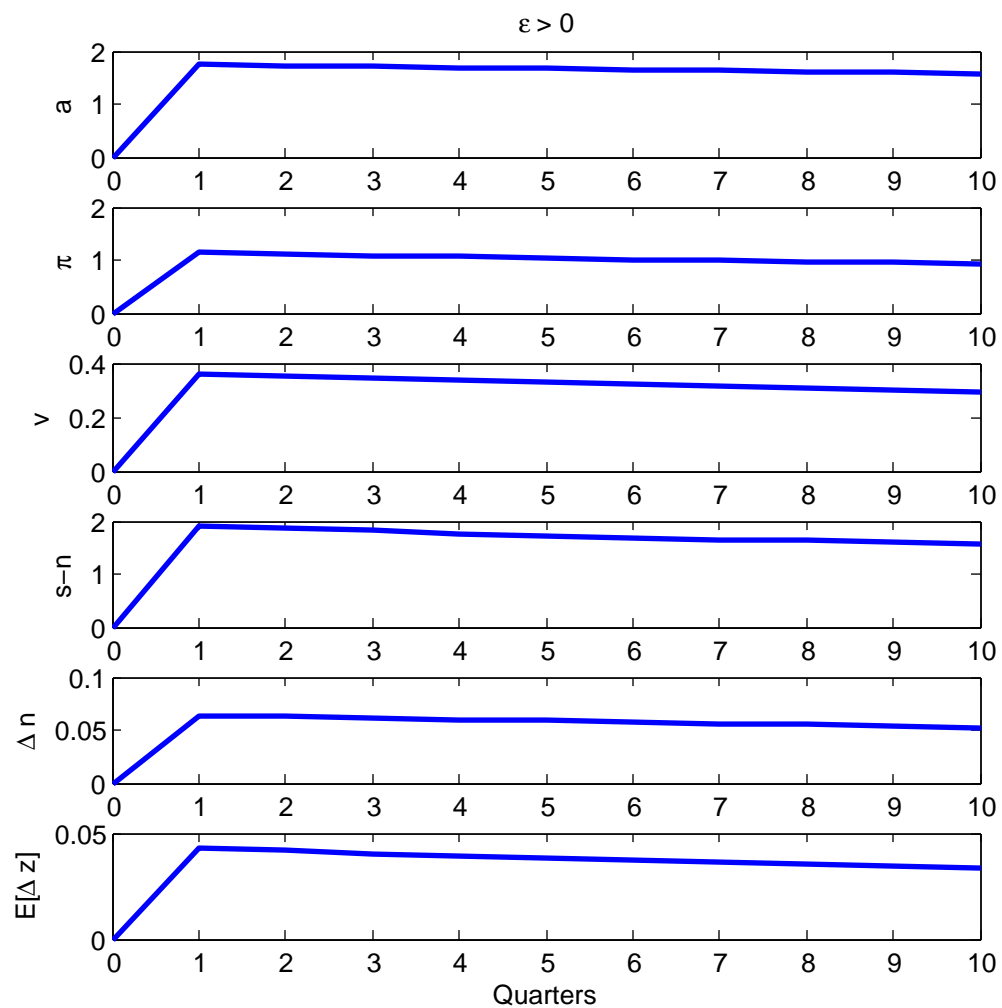
The left panel plots demeaned log consumption growth Δc_t (thin line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (thick bold line) from the ENDO model for a sample simulation of 200 quarters. The right panel plots demeaned log output growth Δy_t (thin line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (thick bold line) from the ENDO model for a sample simulation of 200 quarters. In the model, R&D intensity is the key determinant of expected growth rates.

Figure 3: Growth Rates and R&D Intensity from Data



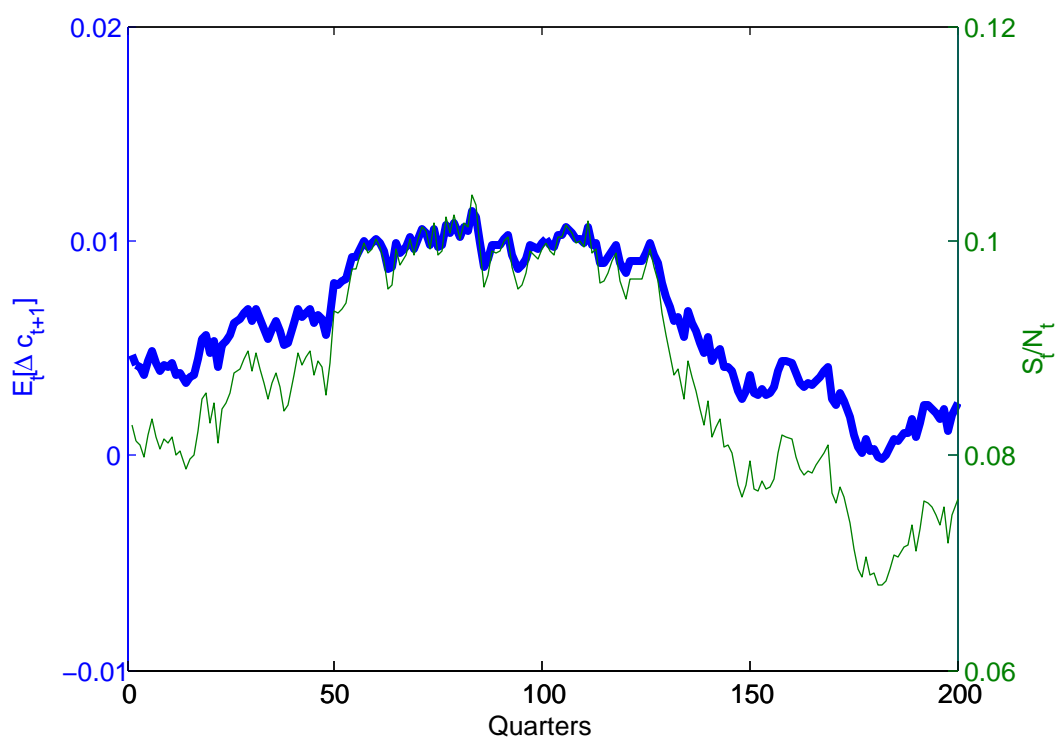
The left panel plots demeaned log consumption growth Δc_t (dashed line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (bold line) from the data. The right panel plots demeaned log output growth Δy_t (dashed line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (bold line) from the data. Annual data on aggregate output and consumption is from the BEA. Annual data on R&D expenditures are from the NSF and data on R&D stocks are from the BLS. In the model, R&D intensity is the key determinant of expected growth rates.

Figure 4: Endogenous Growth Mechanism



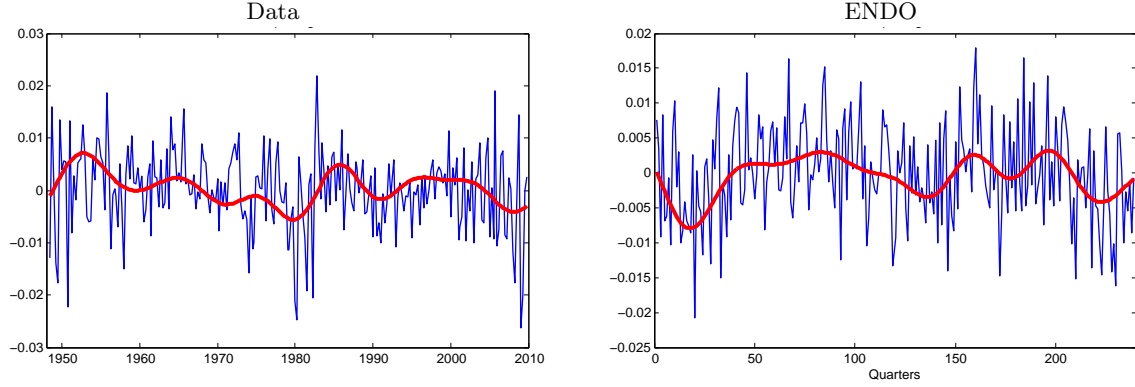
This figure shows quarterly log-deviations from the steady state for the ENDO model. All deviations are multiplied by 100.

Figure 5: Expected Consumption Growth and R&D Intensity



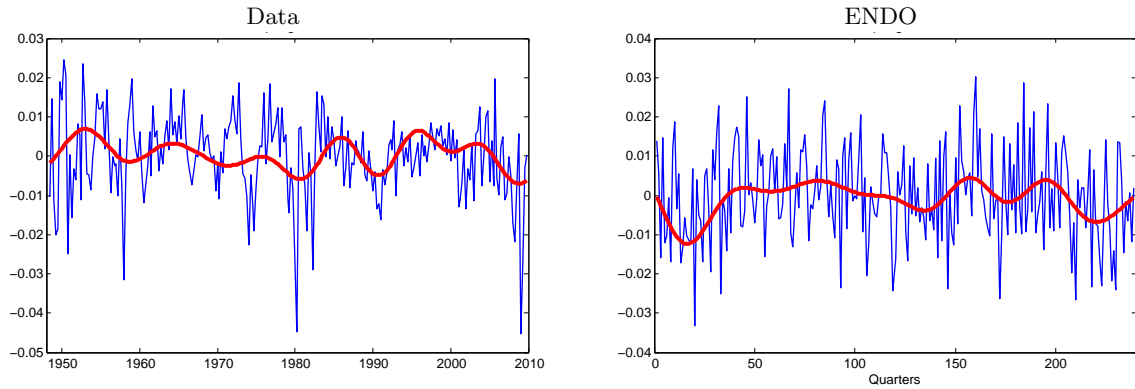
This figure plots expected log consumption growth $E_t[\Delta c_{t+1}]$ (thick line) and R&D intensity $\frac{s_t}{N_t}$ (thin line) from the ENDO model for a sample simulation of 200 quarters. In the model, R&D intensity is the key determinant of expected growth rates.

Figure 6: Consumption Growth with Low-Frequency Component



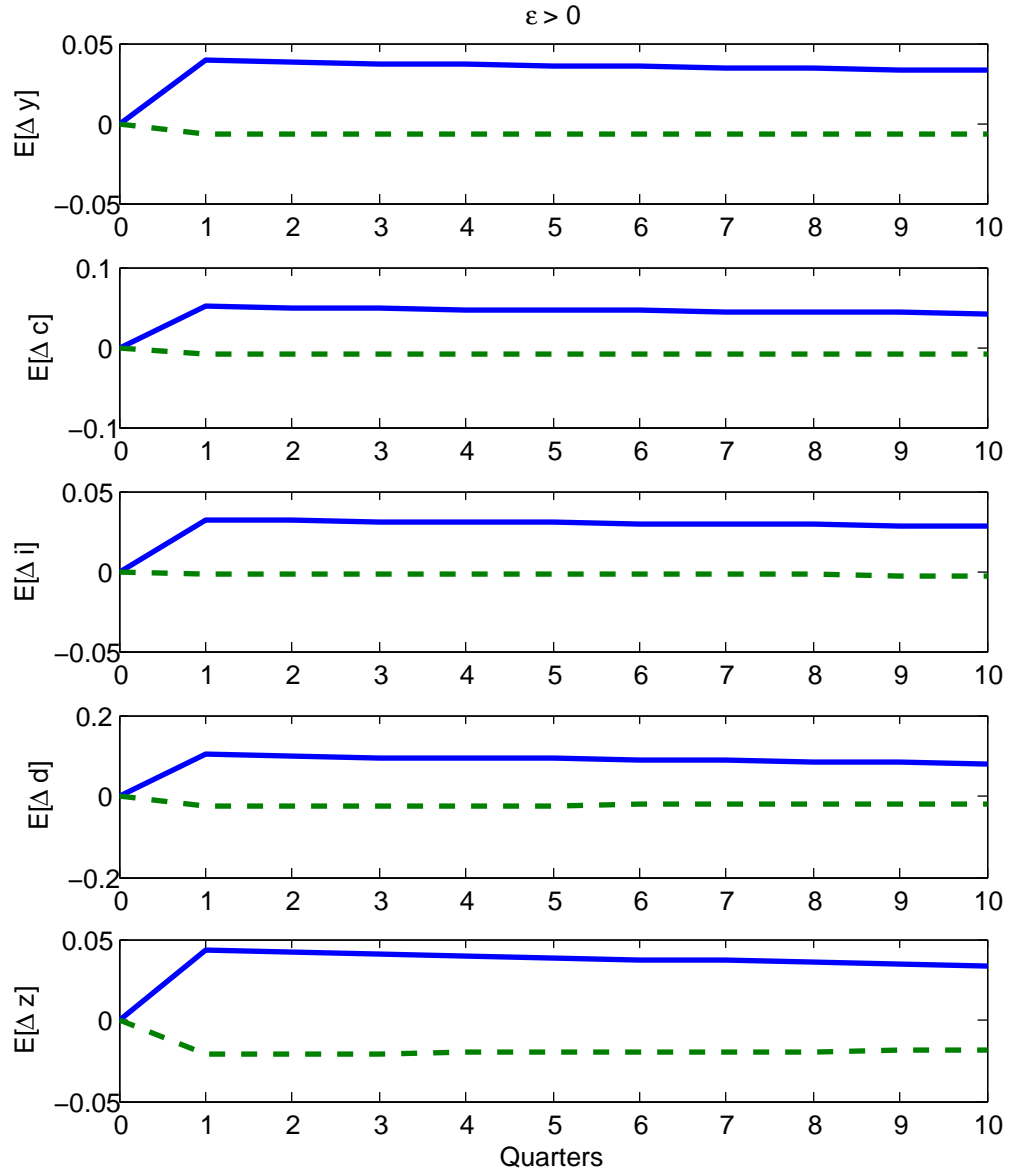
The left panel plots the demeaned log consumption growth rate (thin line) with the low-frequency component (thick line) from the data. The right panel plots the demeaned log consumption growth rate (thin line) with the low-frequency component (thick line) from the ENDO model. The low-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (2003) and selecting a bandwidth of 32 to 200 quarters. Quarterly consumption data are obtained from the BEA.

Figure 7: Output Growth with Low-Frequency Component



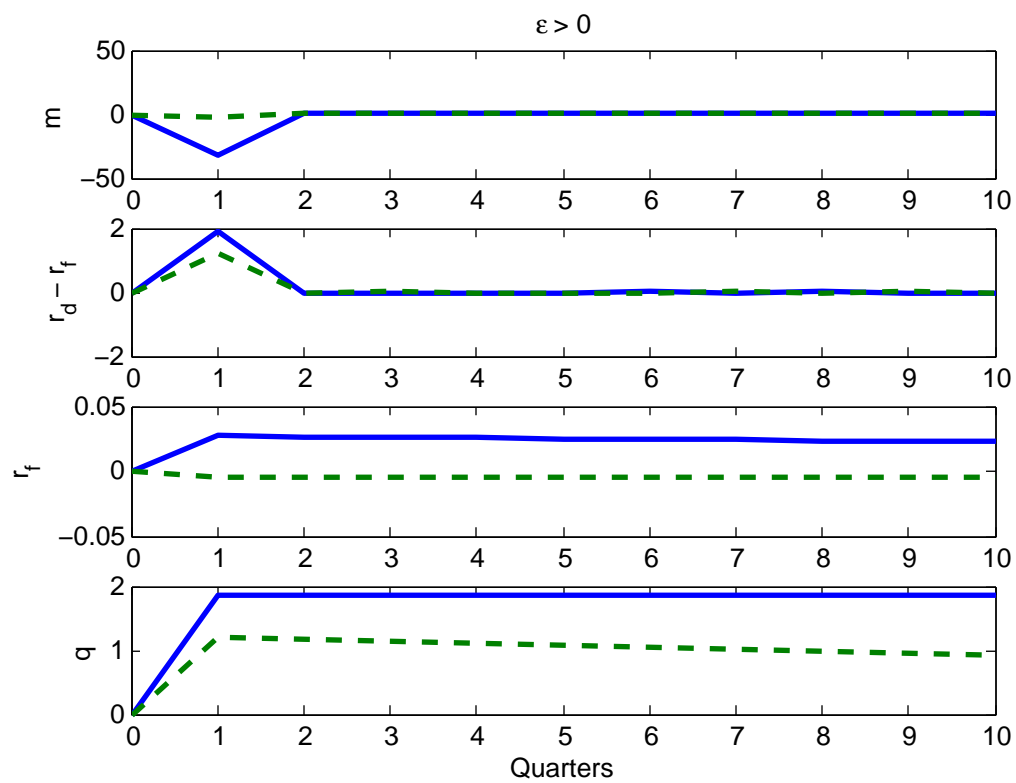
The left panel plots the demeaned log output growth rate (thin line) with the low-frequency component (thick line) from the data. The right panel plots the demeaned log output growth rate (thin line) with the low-frequency component (thick line) from the ENDO model. The low-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (2003) and selecting a bandwidth of 32 to 200 quarters. Quarterly output data are obtained from the BEA.

Figure 8: Expected Growth Rates



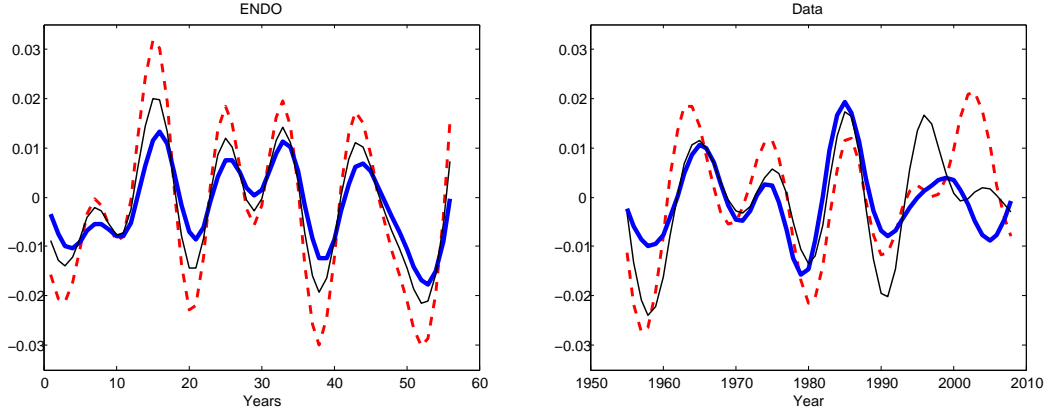
This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.

Figure 9: Asset Prices



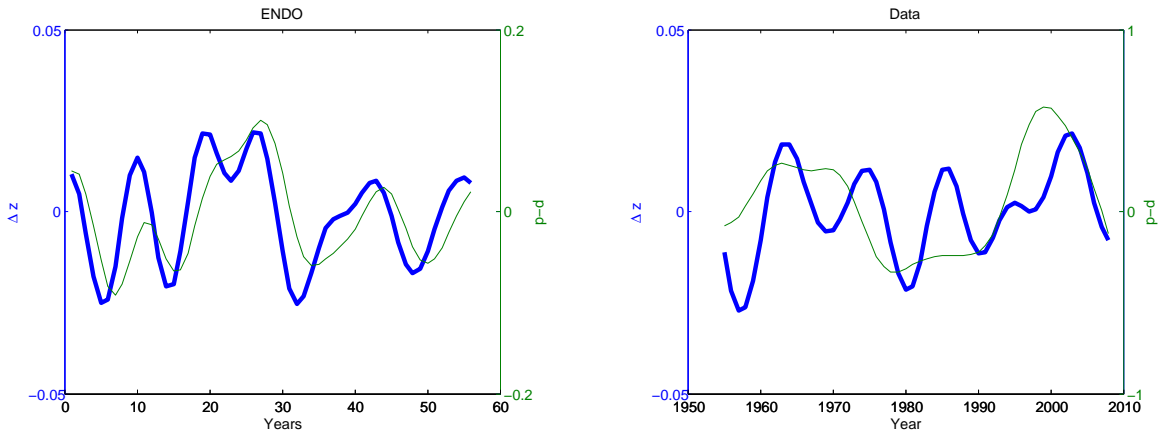
This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.

Figure 10: Low-Frequency Growth Components



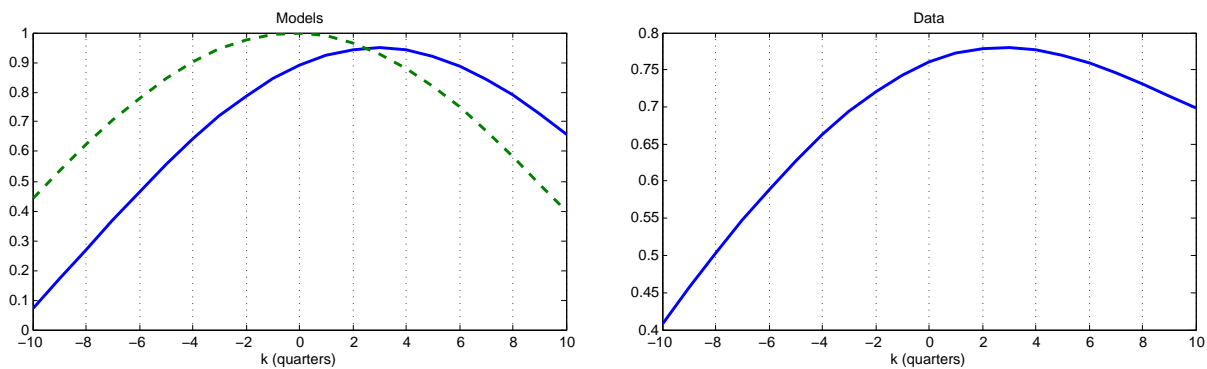
This figure plots the low-frequency growth components for productivity (dashed line), output (thin line), and consumption (bold line). The left panel corresponds to a sample simulation from the ENDO model and the right panel corresponds to the data. The low-frequency component is obtained by applying the bandpass filter from Christiano and Fitzgerald (2003) to annual data and selecting a bandwidth of 8 to 50 years. Annual data on GDP and consumption are from the BEA and annual productivity data are from the BLS.

Figure 11: Low-Frequency Component of Productivity Growth and Price-Dividend Ratio



This figure plots the low-frequency components for productivity growth (bold line) and for the price-dividend ratio (thin line). The left panel corresponds to a sample simulation from the ENDO model and the right panel corresponds to the data. The low-frequency component is obtained by applying the bandpass filter from Christiano and Fitzgerald (2003) to annual data and selecting a bandwidth of 8 to 50 years. The correlation between the two series is 0.46 in the data and 0.67 in the model. Annual data on productivity are from the BLS and price-dividend data are from CRSP.

Figure 12: Low-Frequency Cross-Correlation of Returns and Consumption Growth



The left panel plots cross-correlations of the medium-frequency component of the equity return and the low-frequency component of consumption growth for the ENDO (bold line) and EXO (dashed line) models: $\text{corr}(r_{d,t}, \Delta c_{t+k})$. The right panel plots the same cross-correlations from the data. The low-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (2003) and selecting a bandwidth of 32 to 200 quarters. Quarterly consumption data is obtained from the BEA. Monthly return data is obtained from CRSP and then compounded to a quarterly frequency.