Stochastic Idiosyncratic Operating Risk and Real Options: Implications for Stock Returns

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Abstract

We combine real options and stochastic idiosyncratic operating risk in a simple equity valuation model of firms to capture the cross-sectional variation of stock returns associated with idiosyncratic return volatility. Our model is able to simultaneously explain two main disparate empirical anomalies: the positive contemporaneous relation between risk-adjusted returns and changes in idiosyncratic return volatility, and the poor risk-adjusted performance of stocks with high idiosyncratic risks, among some others. The model further predicts that (i) risk-adjusted returns increase (decrease) following large rises (drops) in idiosyncratic return volatility – the switch effect – and that (ii) the anomalies and the switch effect are stronger for firms that are more abundant in real options and undergo larger changes in idiosyncratic return volatility. Simulations and empirical analysis strongly support these predictions.

Keywords: Idiosyncratic return volatility, cross section of stock returns, asset pricing, real options, growth options, stochastic volatility, regime switching, mixed jump-diffusion processes.

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1 Introduction

Modern portfolio theory and the capital asset pricing model (CAPM) suggest that investors diversify idiosyncratic risks and only systematic risk is priced in equilibrium. The empirical evidence on idiosyncratic stock return volatility ($IVol$) and stock returns is not readily explained by this simple intuition. One strand of the literature (e.g., Duffee (1995)) establishes that changes in monthly realized $IVol$ is contemporaneously positively related with risk-adjusted stock returns (positive $IVol$ anomaly hereafter).[1] A different strand (e.g., Ang, Hodrick, Xing, and Zhang (2006)), on the other hand, establishes that portfolios of stocks with high end-of-month realized $IVol$ significantly under-perform their low $IVol$ counterparts on a risk-adjusted basis (negative $IVol$ anomaly hereafter), casting doubts on the notion of a positive risk premium for idiosyncratic return risk.[2,3] Yet, a third strand of the literature establishes that the negative $IVol$ anomaly is due to strong return reversals among a subset of small stocks (e.g., Huang, Liu, Rhee, and Zhang (2009) and Fu (2009)). Given the lack of consensus, it is not surprising that progress in delivering a unified explanation for the above empirical findings has been difficult.

In this paper, we reconcile these seemingly disparate empirical regularities in a simple equity valuation model of firms with real options and stochastic idiosyncratic operating risk. The model is stylized to highlight key predictions, which are then validated by numerical simulations and supported empirically. We demonstrate that, contrary to conventional wisdom, the presence of stochastic idiosyncratic operating risk imparts a close relationship between stocks' risk-adjusted returns and $IVol$ if firm valuations incorporate convexities in the firms' output price, a feature that we attribute to the firms' real options. The firm's currently producing assets – the assets-in-place – have linear valuations in the profit flow, and therefore, are invariant with respect to the firm's idiosyncratic operating risk. As a

[1]Spiegel and Wang (2006), Fu (2009) and Huang, Liu, Rhee, and Zhang (2009) also report a positive correspondence between idiosyncratic return volatility and stock returns at the firm or portfolio level.

[2]Ang, Hodrick, Xing, and Zhang (2009) also report similar findings using international return data.

[3]Earlier empirical papers investigating idiosyncratic volatility and returns in the cross section are Lintner (1965), Tinic and West (1986) and Lehmann (1990).

consequence, we argue that the relation between risk-adjusted stock returns and $IVol$ is entirely attributed to the firms' reliance on real options and their exposure to stochastic firm-specific operating risk. We demonstrate that the proposed mechanism reconciles the conflicting anomalies related to $IVol$ in the cross-section of stock returns.

The intuition for our results is as follows. Assuming a Markov regime-switching process for the firm-specific operating risk allows the switches in idiosyncratic operating risk to be accompanied by changes in option valuations, hence stock returns exhibit discrete jumps in the direction of the switches.[4,5] As a consequence, expected returns are composed of two regime dependent components: a continuous drift term that prevails between switches, and a probability weighted jump term that is triggered when a switch occurs. Since the continuous drift component compensates for the jump term in normal times absent of jumps, realized returns are on average larger than expected when volatility is high, and lower than expected when volatility is low. When a switch arrives, the jump term dominates and realized returns are on average inversely related with idiosyncratic return risk. In the language of asset pricing, realized returns larger than expected translates to positive risk-adjusted or abnormal returns. The model produces continuation in risk-adjusted returns followed by large reversals in tandem with movements in idiosyncratic return volatility.

This regime dependency and the time-series pattern of the firm's operating risk help establish predictability in risk-adjusted returns that extends to capture the cross-sectional variation of stock returns associated with $IVol$. The positive $IVol$ anomaly is explained by the cross-sectional dispersion in abnormal returns driven by the firms that experience changes in idiosyncratic operating risk. The negative $IVol$ anomaly, on the other hand, is explained by the regime dependency of the betas[6], coupled by the predictability of the di-

[4] A 2-regime Markov switching process is assumed for tractability, but it is not with loss of generality. Qualitatively, our results should persist in a more general structure insofar as idiosyncratic risk exhibits mean reversion.

[5] Guo, Miao, and Morellec (2005) and Hackbarth, Miao, and Morellec (2006) also develop a 2-regime Markov switching process in state dynamics to investigate investment and capital structure decisions, respectively.

[6] The firm's equity beta is inversely related with the firm's idiosyncratic operating risk due to a low systematic component when the option value is high.

rection of the subsequent switch conditioned on the regime, which generates predictability of the direction of future reversals in risk-adjusted returns. In portfolio-based asset pricing tests, sorting and grouping stocks on month-end realized $IVol$ is akin to grouping stocks on the firms' most recent operating risk and portfolios of $IVol$-ranked stocks exhibit differences in risk-adjusted returns. Taken together, the model generates risk-adjusted returns that correlate positively with contemporaneous changes in $IVol$, but negatively with past realized $IVol$, reconciling two disparate and conflicting anomalies not previously considered jointly in a single framework.

We validate our intuition with numerical simulations. Using a panel of simulated data of risk-adjusted returns, we recreate results qualitatively similar to Duffee (1995) and Ang, Hodrick, Xing, and Zhang (2006), with more pronounced effects when we use a larger spread in idiosyncratic operating risk between regimes. When we specify a single volatility regime – the standard specification in most real option models – we find that the model generates no link between $IVol$ and risk-adjusted returns. Therefore, the simulations validate that our explanation is the driving mechanism behind the results, not other potentially opaque features of the model.

Our model also helps understand the findings that the negative intertemporal relation between $IVol$ and future stock returns is largely explained by the return reversal of stocks with high $IVol$ among a subset of small stocks (e.g., Huang, Liu, Rhee, and Zhang (2009)) and Fu (2009)). More specifically, small stocks with high $IVol$ exhibit stronger positive contemporaneous correlation with returns, subsequently leading to stronger reversals and lower abnormal returns. Pontiff (2006) offers an explanation based on high transaction costs and limits to arbitrage to point to the persistence of low returns among small and high $IVol$ stocks. An alternative possible explanation can be based on Daniel, Hirshleifer, and Subrahmanyam (1998), who offer an explanation of cognitive bias and persistent mispricings in financial markets. Our model is able to generate strong return reversals through the dynamics of the volatility structure embedded in the operations of small firms that posses growth opportunities, or real options. Our explanation is based on a rational theory of firms that face uncertain operating environments and observable firm characteristics, rather than on market imperfections or investors' cognitive biases.

The novel contribution of this paper is to highlight that real options, in conjunction with stochastic firm-specific operating risk, can explain the conflicting empirical relation between risk-adjusted returns and $IVol$. The bulk of our empirical analysis is focused on reporting this link and verifying the predictions of our theory. To investigate our conjecture on the positive contemporaneous relation between $IVol$ and returns, we revisit Grullon, Lyandres, and Zhdanov (2010) by recreating many of their empirical proxies for firms' reliance on growth options, and additionally, creating some of our own proxies. Our regression specifications are similar to those of Grullon, Lyandres, and Zhdanov as well, with additional specifications in which we include the difference between the stocks' 70th and 30th percentile in-sample breakpoint values of $IVol$ as an additional explanatory variable to proxy for the spread in idiosyncratic operating risk across volatility regimes. Using a battery of real option intensity proxies, we find that the positive contemporaneous relation between returns and changes in $IVol$ is stronger among firms that are more likely to incorporate real options and experience more extreme changes in $IVol$, results that lend strong support to our model.

In order to investigate our conjecture on the poor performance of high $IVol$ stocks, we revisit Ang, Hodrick, Xing, and Zhang (2006) by sorting and grouping stocks into portfolios based on the level of their month-end realized $IVol$, and independently, on the firms' real option proxy, and on the difference between the stocks' 70th and 30th in-sample percentile breakpoint values of $IVol$. As in most asset pricing tests, we assess the portfolio performances by investigating value-weighted risk-adjusted returns relative to the Fama and French 3-factor model. Again, using a battery of real option intensity proxies, we find that the poor future performance of high $IVol$ stocks is more pronounced among firms that are more likely to incorporate real options and experience more extreme changes in $IVol$, results that are in strong agreement with our model.

The model we propose captures additional empirical testable features. To the extent that stock returns incorporate firms' real options and stochastic firm-specific operating risks, stocks that experience an up (down) $IVol$-switch episode should be associated with higher (lower) post-switch risk-adjusted returns to reflect the regime dependency of the risk-adjusted returns. In order to investigate this novel conjecture, we compute the difference in 5-month risk-adjusted average returns around the month in which a stock's $IVol$ undergoes a sudden change larger than the difference between its 70th and 30th in-sample percentile breakpoint values and employ event studies. Using a battery of real option intensity proxies, we find that in the up-switch sample the difference between post and pre-switch returns is positive, while in the down-switch sample the difference in returns is negative. We also find that this 'switch effect' is amplified for more real option intensive firms and firms that experience more extreme changes in $IVol$. Here again, the results are in strong support of our theory.

Corporate investment decisions are commonly modeled in the context of growth options,[7] and growth options are shown to affect firm risks. Berk, Green, and Naik (1999) were among the first to establish a link between corporate investment-based characteristics and firm betas to explain anomalous regularities in the cross section of stocks.[8,9] We contribute to this literature by integrating a new dimension – stochastic operating risk – with real options to explain the empirical regularities related to $IVol$. A common theme in the extant literature investigates how much real options contribute to the firm's market beta relative to the firm's assets-in-place, a feature present in our model as well. Our contribution hinges on how firm-specific risks in operations affect the firms' market risk through

[7]This approach was first pioneered by MacDonald and Siegel (1985), MacDonald and Siegel (1986) and Brennan and Schwartz (1985), and later adopted by many others. Dixit and Pindyck (1994) summarizes the body of literature.

[8]Fama and French (1992) provide evidence on the ability of size and book-to-market to explain returns. Fama and French (1996) provides a cross-sectional landscape view of how average returns vary across stocks.

[9]Further work in this area have also focused on real options to build a bridge between firms' characteristics and market betas (e.g., Carlson, Fisher, and Giammarino (2004), Zhang (2005), Sagi and Seashole (2007) and Garlappi and Yan (2011), among many others).

the firms' real options. In our model, the idiosyncratic volatility of a firm' output price serves as an additional state variable that affects the market beta of the firm's real options, but not the market beta of the firm's assets-in-place, a channel previously not considered as a link between firm observable characteristics and expected stock returns. Furthermore, the literature has relied on imperfect measures of firm risk to reconcile predictions with the empirical evidence. Although we don't refute that betas are likely to be mismeasured in practice, in our model the $IVol$ anomalies are not anomalous relative to the correctly specified asset pricing model. We additionally contribute to the literature by investigating truly residuals effects since our predictions and empirical tests are on the cross-sectional variation of *risk-adjusted* returns associated with idiosyncratic return volatility.

We also make a contribution to the broader literature on the cross-section of stock returns. By explicitly considering time-varying firm-specific operating risk allows us to propose a novel channel between the operating environment that firms face and the firms' stock returns, providing fertile grounds for additional research. The novel features of our model yield additional testable predictions on the correspondence between $IVol$ and risk-adjusted stock returns, some of which we test empirically in this paper.[10]

To our knowledge little inroads have been made to link idiosyncratic operating risk to asset pricing, a void we hope to fill with this paper. The economics literature has recognized that firm-level idiosyncratic technology shocks aggregate to create macroeconomic effects (e.g. Caballero and Pindyck (1996)) and Bloom (2009)). Also, it has been shown that the presence of idiosyncratic shocks in a competitive industry of firms with growth options translates to the firms' potential to retain monopolistic rents (e.g. Ch. 8 of Dixit and Pindyck (1994)).[11] Incorporating time varying idiosyncratic operating risks ensures that firms have time-varying potential to retain monopolistic rents, motivating the importance of idiosyncratic operating risk as a determinant of firm value and stock returns.[12]

[10]Schwert (2003) highlights the merits of structural models in deriving new testable hypotheses.

[11]Idiosyncratic shocks contrasts from aggregate shocks since the latter are shared among competing firms, but the former provides a unique advantage that is not shared with the competitors.

[12]This paper does not explicitly consider a competitive industry equilibrium with multiple firms as in

The rest of the paper is organized as follows: Section II presents the model environment. Section III derives closed-form solutions for the valuations and returns. Section IV discusses model simulation results. Section V reports the empirical methodology along with the results. Section VI concludes. The Appendix contains all the proofs and other technical details omitted in the main body of the paper.

2 Model

We construct a growth option model similar in spirit to the models in Garlappi and Yan (2008) and Carlson, Fisher, and Giammarino (2004).[13] This section describes the firms' economic environment.

2.1 The Environment

We consider two types of firms. Mature firms are producing units in the economy and produce at full capacity. In contrast, young firms produce at a lower operating scale, but have the option to make an irreversible investment and increase production and become mature. Firms are all equity financed. The output price of each firm follows a geometric Brownian motion

$$\frac{dP}{P} = \mu dt + \sigma_{P,i} dB_1 + \sigma_A dB_2 \tag{2.1}$$

where μ is the growth rate, σ_A is the market volatility, $\sigma_{P,i}$ is the idiosyncratic volatility, and dB_1 and dB_2 are the increments of two independent Brownian motions. Time and firm subscripts throughout are omitted for convenience.

Caballero and Pindyck (1996) or Dixit and Pindyck (1994). In an earlier draft, we considered an industry equilibrium model of firms with entry and exit and idiosyncratic operating risk similar to the one consider in this paper. We found the qualitative implications for $IVol$ and stock returns to be the same.

[13] With no loss of generality, we rely specifically on growth options to incorporate convexity of firm valuations in the firms' output price. Other forms of real options that incorporate convexities would accommodate similar results.

We allow firms to have random and time-varying potential to realize monopolistic rents by allowing idiosyncratic operating risks to be time varying.[14] $\sigma_{P,i}$ follows a 2-state Markov switching process [15]

$$\triangle\sigma_{P,i} = \begin{cases} \sigma_{P,H} - \sigma_{P,L} & , \quad \text{with prob. } \lambda_H dt, \qquad \text{if } i = L \\ 0 & , \quad \text{with prob. } 1 - \lambda_H dt, \quad \text{if } i = L \\ \sigma_{P,L} - \sigma_{P,H} & , \quad \text{with prob. } \lambda_L dt, \qquad \text{if } i = H \\ 0 & , \quad \text{with prob. } 1 - \lambda_L dt, \quad \text{if } i = H \end{cases} \tag{2.2}$$

where $\sigma_{P,H} - \sigma_{P,L} > 0$, and λ_L and λ_H are known transition parameters between high and low volatility regimes H and L. The switches between the two regimes $\triangle\sigma_{P,i}$ are independent Poisson processes and independent across firms. Both P and the volatility regime i are observable for any given firm.[16] We subscript quantities with $i \in \{H, L\}$ throughout to denote their dependence on the idiosyncratic volatility regime at any given time.

All sources of operating uncertainty are driven by the uncertainty in the price of the firm' output. Investors in the stock market can hedge market risk in the firms' operations by trading on two securities. Let M_t denote the price of the risk free asset with dynamics

$$\frac{dM}{M} = rdt \tag{2.3}$$

[14] Dixit and Pindyck (1994) and Caballero and Pindyck (1996) show that idiosyncratic shocks translates to a firm's ability to retaining monopolistic rents – a firm that experiences a positive idiosyncratic technology shock experiences an advantage that cannot be stolen by its competitors, while a positive aggregate shock is shared with the firm's competitors. Some plausible micro-economic examples for a change in idiosyncratic operating risk are: shifts in consumer needs and wants, persistent changes in production technology, or changes in the general operating environment of the firm or the firm's industry, among others.

[15] Assuming a 2-state Markov- switching process is not without generality. A model with a more general volatility structure is possible, but at a cost of analytical tractability.

[16] Conditioned on being in the high volatility state, the probability that $\triangle\sigma_{P,i}$ will switch to the low volatility regime in the next short interval dt is $\lambda_L dt$. $\lambda_H dt$ is defined similarly. Based on standard properties of Poisson processes, the expected duration that the process dP will stay in the high volatility regime H and the low volatility regime L are λ_L^{-1} and λ_H^{-1}, respectively. The proportion of time spent in the high and low volatility regimes are $\frac{\lambda_H}{\lambda_H+\lambda_L}$ and $\left(1 - \frac{\lambda_H}{\lambda_H+\lambda_L}\right)$ respectively.

and let S be the price of a risky security with dynamics

$$\frac{dS}{S} = \mu_S dt + \sigma_S dB_2 \tag{2.4}$$

S has a beta equal to one and $\lambda = \frac{\mu_S - r}{\sigma_S}$ is the market price of risk. The proportion of S held in a replicating portfolio determines the beta of the portfolio. This greatly simplifies firm valuations and the determination of firm betas.

2.2 The Value of a Mature Firm

The equity value of a mature firm is composed of the profit stream from selling the output. The cost of producing a unit of output is c per unit of time. ξ_M denotes the scale of production, therefore the profit per unit of time is $\pi_M(P) = \xi_M(P - c)$. The equity value of a mature firm is given as follows

$$V_M(P) = \xi_M \left(\frac{P}{r - \mu^*} - \frac{c}{r} \right) \tag{2.5}$$

where $\mu^* = \mu - \sigma_A \lambda < r$. The firm value is the present value of a growing risky perpetuity, less the present value of a riskless perpetuity.

2.3 The Value of a Young Firm

Young firms produce at a lower capacity than mature firms, i.e. $\xi_Y < \xi_M$, therefore the profit per unit of time is $\pi_Y(P) = \xi_Y(P - c)$, but possess a perpetual option to increase production scale by $\xi = \xi_M - \xi_Y$ upon making a one time irreversible investment of I. For simplicity, we assume that financing is done by equity. Young firms derive value from assets currently in production, or assets-in-place, and the growth option. The value of the assets-in-place have the same functional form as equation (2.5) with ξ_M replaced by ξ_Y. The equity value of a young firm is given as follows

$$V_{Y,i}(P) = \xi_Y \left(\frac{P}{r - \mu^*} - \frac{c}{r} \right) + GO_i(P) \tag{2.6}$$

where the value of the growth option $GO_i(P)$ obeys the following Bellman equation

$$GO_i(P) = e^{-rdt} E^{\mathbb{Q}} \left[GO_i(P + dP, \sigma_{P,i} + \triangle\sigma) \right] \tag{2.7}$$

with a value realization upon exercise net of cost of $\xi \left(\frac{P}{r-\mu^*} - \frac{c}{r} \right) - I$, and $E^{\mathbb{Q}}[.]$ denotes the expectation operator under the $\mathbb{Q}$ measure. The convexity of the option value with respect to the firm's operating risk ensures that the firm value and the decision to expand are dependent on both P and the volatility regime i. Optimal exercise requires to choose when to invest, which occurs at time τ_i. Define P_i^* the price level at which a young firm exercises its growth option. The choice of P_i^* describes the strategy for a young firm, and the strategy chosen that satisfies the optimality conditions maximizes the value of the firm.

2.4 Equity betas and Expected Returns

Equity expected returns differ in the cross-section based on the firms' maturity, price of output, and the idiosyncratic volatility regime in effect. The expected return of a young firm can be expressed according to the CAPM as

$$E \left[\frac{\pi_Y(P)dt + dV_{Y,i}(P)}{V_{Y,i}(P)dt} \right] = r + \beta_{V_Y,i}(P)\sigma_S \lambda \tag{2.8}$$

where $\beta_{V_Y,i}(P)$ denotes the firm's CAPM beta conditional on i and P. Because the option value and the decision to invest is dependent on the realization of the volatility regime, the equity beta of young firms are also dependent on the volatility regime.

3 Model Solution

This section describes the model solution. We discuss the properties of the solution and their empirical predictions.

3.1 Valuation

The following proposition states the valuation and the exercise threshold of the investment option.

Proposition 1 *If the price process is given by (2.1) and (2.2), then the value of a growth option in the region of low values of P, $P \in (0, P_1)$, is*

$$F_H(P) = \frac{B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1})}{\lambda_H} + \frac{B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})}{\lambda_H} \tag{3.1}$$

if P is in the high volatility regime, and

$$F_L(P) = B_{L,1}P^{\beta_{2,1}} + B_{L,2}P^{\beta_{2,2}} \tag{3.2}$$

if P is in the low volatility regime.

In the region of intermediate values of P, $P \in (P_1, P_2)$, the option value is

$$G_H(P) = \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I\right) + C_{H,1}P^{\beta_{1,1}} + C_{H,2}P^{\beta_{1,2}} \tag{3.3}$$

if P is in the high volatility regime, and

$$G_L(P) = \xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I \tag{3.4}$$

if P is in the low volatility regime. Moreover, the optimal exercise boundaries P_1 and P_2

are the solution to the following system of equations

$$C_{H,1}P_1^{\beta_{1,1}} + C_{H,2}P_1^{\beta_{1,2}} - \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P_1}{r-\mu^*} - \frac{c}{r}\right) - I\right) = \frac{B_{L,1}P_1^{\beta_{2,1}}q_L(\beta_{2,1})}{\lambda_H} + \frac{B_{L,2}P_1^{\beta_{2,2}}q_L(\beta_{2,2})}{\lambda_H} \tag{3.5}$$

$$\beta_{1,1}C_{H,1}P_1^{\beta_{1,1}} + \beta_{1,2}C_{H,2}P_1^{\beta_{1,2}} + \frac{\xi P_1 \lambda_L}{(r-\mu^*)(\lambda_L + r)} = \frac{\beta_{2,1}B_{L,1}P_1^{\beta_{2,1}}q_L(\beta_{2,1})}{\lambda_H} + \frac{\beta_{2,2}B_{L,2}P_1^{\beta_{2,2}}q_L(\beta_{2,2})}{\lambda_H} \tag{3.6}$$

where the expressions for $B_{L,1}$,$B_{L,2}$,$C_{H,1}$, $C_{L,1}$, $q_L(\beta)$, $\beta_{1,1}$, $\beta_{1,2}$, $\beta_{2,1}$ and $\beta_{2,2}$ are given in the Appendix.

Proof: *See Appendix.* ∎

The proposition states that young firms have separate investment policies in each volatility regime. There are three distinct regions in the range of possible values of P to consider (Guo, Miao, and Morellec (2005)). In the region where $P \in (0, P_1)$, the value of the investment is below the cost in both volatility regimes, and the option value is given by (3.1) and (3.2). In the region where $P \in (P_1, P_2)$, the value of the investment exceeds the cost only in the low volatility regime, and the option is kept alive in the high volatility regime only.[17] The option value is given by (3.4) and (3.3) in this region. Lastly, in the region where $P > P_2$, the investment value exceeds the value to keep the option alive in both volatility regimes. The option value $\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I$ reflects immediate investment and the value of the incremental increase in production scale.

More importantly, the proposition reveals that growth options have distinct valuations across volatility regimes, a feature that contrasts starkly from the firm's assets-in-place or a mature firm whose values are invariant with respect to the volatility regime. The reliance of the firm's equity value on the regime of the idiosyncratic operating risk is attributed

[17]This property hinges on standard option pricing results that the value of an option is increasing in the volatility of the underlying asset.

entirely to the firm's growth options.

Insert Figure 1 here

Figure 1 provides a graphical illustration of Proposition 1 for different set of parameter values of $\sigma_{P,H}$ and $\sigma_{P,L}$. Comparing the graphs across panels reveals that the opportunity to expand has a larger valuation in regime $i = H$ than in regime $i = L$, and the difference is increasing in the spread between $\sigma_{P,H}$ and $\sigma_{P,L}$. The last panel reveals that the model results in a single valuation profile if $\sigma_{P,H} = \sigma_{P,L}$, which is the usual specification in standard growth option models.

3.2 Returns

This section reveals that the growth option's dependence on the firms' idiosyncratic operating risk has important implications for the time-series and the cross-sectional variation in stock returns.

Proposition 2 *If the price process is given by (2.1) and (2.2), and $F_i(P)$, $i \in \{H, L\}$, is given by (3.1) and (3.2), then $F_i(P)$ has the following dynamics*

$$\frac{dF_i(P)}{F_i(P)} = a_i(P)dt + b_i(P)dB_i + \nu_i(P)dz_i \tag{3.7}$$

where $dB_i = \frac{\sigma_{P,i}dB_1 + \sigma_A dB_2}{\sigma_i}$, $\sigma_i = \sqrt{\sigma_{P,i}^2 + \sigma_A^2}$, dz_i is a Poisson process that is perfectly functionally dependent on $\Delta\sigma_{P,i}$ as given in (2.2), and

$$a_H(P) > a_L(P) \tag{3.8}$$

$$b_H(P) > b_L(P) \tag{3.9}$$

$$\nu_H(P) < 0 < \nu_L(P) \tag{3.10}$$

Furthermore, $F_i(P)$ has risk-adjusted returns relative to the CAPM with the following *dynamics*

$$\frac{dF_i(P)}{F_i(P)} - [r + \beta_{F_i}(P)\sigma_S\lambda] = -\lambda_{i'}\nu_i(P)dt + b_i(P)dB_i + \nu_i(P)dz_i \tag{3.11}$$

where $i' = L$ if $i = H$ and vice-versa, and

$$\beta_H(P) < \beta_L(P) \tag{3.12}$$

where the inequalities (3.8), (3.9), (3.10), and (3.12) are increasing in the spread between $\sigma_{P,H}$ and $\sigma_{P,L}$, and the expressions for $a_H(P)$, $b_H(P)$, $\nu_H(P)$, $\beta_H(P)$, $a_L(P)$, $b_L(P)$, $\nu_L(P)$ and $\beta_L(P)$ are given in the Appendix.

Proof: *See Appendix.* $\blacksquare$

Proposition 2 conveys the central idea of our paper. The dynamics for the value of a growth option differs between volatility regimes, a feature inherited from the dependance of the value of the option on the volatility regime. Since firm values incorporate growth options, the value of a young firm obeys the laws of motion pertaining to the volatility regime in effect. In 'normal times' absent of a switch, the firm value dynamics is determined by the first two terms of (3.7), the drift and the diffusion terms $a_i(P)$ and $b_i(P)$ respectively. However, on average every $1/\lambda_i$ units of time, a sudden and relatively large change in firm value contributed by $\nu_i(P)$ in (3.7) is activated by the arrival of a switch, after which the dynamics obeys the law of motion pertaining to the new volatility regime. The dynamics stays the same until the next switch arrives. This property of the value process is attributed to the firm's growth option and amplified by the spread between $\sigma_{P,H}$ and $\sigma_{P,L}$. In contrast to growth options, the value of the assets-in-place and the value of mature firms are invariant with respect to the volatility regime. Consequently, they have continuous returns and do not exhibit jumps.[18]

[18]The dynamics for the value of the assets-in-place and mature firms is given in the technical appendix.

The expression for the option's risk-adjusted returns relative to the CAPM is given in equation (3.11) of the proposition. The equation imparts a close cross-sectional relationship between idiosyncratic stock return volatility and risk-adjusted returns that is testable empirically. In line with our intuition, the proposition shows that $b_H(P) > b_L(P)$, establishing a positive correlation between the firms' idiosyncratic stock return volatility and idiosyncratic operating risk. The jump term $\nu_i(P)$, if triggered, is positively associated with the sign of the switch in volatility but correlates inversely with the previous volatility regime, i.e. $\nu_H(P) < 0 < \nu_L(P)$. This suggests that the stock returns of growth option firms should exhibit positive contemporaneous correlation with changes in idiosyncratic return volatility and negative correlation with past return volatility, in line with the positive $IVol$ anomaly of Duffee (1995) and the negative $IVol$ anomaly of Ang, Hodrick, Xing, and Zhang (2006), the two main disparate empirical anomalies we address in this paper.

In our model, the positive $IVol$ anomaly is explained by the jumps in stock returns of growth option firms experiencing a switch in idiosyncratic operating risk. Similarly, the negative $IVol$ anomaly is explained by the future jumps in stock returns of growth option firms due to the arrival of a switch in idiosyncratic operating risk. In portfolio-based asset pricing tests, sorting and grouping stocks by month-end realized $IVol$ is akin to grouping stocks on the firms' current volatility regime. The highest $IVol$ portfolio oversamples firms with high operating risk while the opposite holds for the lowest $IVol$ portfolios. Portfolios sorted by $IVol$ exhibit differences in future risk-adjusted returns reflecting future reversals in the firms' idiosyncratic operating risk.

Equation (3.11) of the proposition offers the basis for a novel prediction on the relation between risk-adjusted stock returns and idiosyncratic return volatility which we also test empirically in the sequel. The drift of the option's risk-adjusted returns given in equation (3.11) relates positively with idiosyncratic return volatility, i.e. $-\lambda_L \nu_H(P) > -\lambda_H \nu_L(P)$, while the jump term relates inversely, i.e. $\nu_H(P) < 0 < \nu_L(P)$. That is, during normal times absent of jumps, realized return in excess of expectations is continuous and tends to be positive (negative) when volatility is high (low), but a relatively large offsetting 'correction' occurs upon the arrival of a switch in volatility. In the language of asset pricing, realized returns greater than expectation translates to positive *abnormal* returns. The proposition reveals that abnormal stock returns should exhibit continuation that correlates with the level of idiosyncratic return volatility, followed by strong reversals that concur with changes in idiosyncratic return volatility. This model prediction is consistent with the evidence that the negative $IVol$ anomaly is mainly due to strong return reversals after previous increases in stock prices (Fu (2009) and Huang et al. (2009)). We confirm this prediction in the sequel with our own empirical tests.

Some additional comments are in order. The expectation of the right hand side of (3.11) evaluates to zero, hence the implication is that on average abnormal returns vanish for a single firm if the central limit theorem holds. The implication for the cross-section, however, hinges on the stock returns' dependence on the path of their volatility regime. Intuitively, the model implies that the jumps in stock returns within a month constitute the outliers in monthly cross-sectional return regressions that loads on the changes in $IVol$, contributing to the positive statistical relation between $IVol$ and stock returns. Regarding the negative association between $IVol$ and future stock returns, the implication is that large firm valuations due to greater option values precede negative future jumps in returns for stock of firms that experience a switch. In portfolio-based tests, value-weighting stock returns ensures that the negative jumps in high-$IVol$ portfolios receive a greater emphasis than the positive jumps in low-$IVol$ portfolios. Thus, the negative $IVol$ anomaly is a consequence of the low value-weighted portfolio returns of high-$IVol$ stocks, rather than the high portfolio returns of low-$IVol$ stocks. Consistent with this prediction, we find empirically that the negative $IVol$ anomaly is mainly due to the low future portfolio returns of high-$IVol$ stocks and that the anomaly does not hold up if portfolio returns are equally-weighted.[19]

[19] In our sample, the average difference in pricing errors relative to the Fama and French (1993) 3-factor model between the top and bottom quintile portfolios of stocks sorted by $IVol$ is 0.04% per annum and

Lastly, the proposition establishes that the option's beta is regime dependent and it relates inversely with idiosyncratic return volatility, i.e. $\beta_{F,H}(P) < \beta_{F,L}(P)$, and the inequality is amplified by the spread between $\sigma_{P,H}$ and $\sigma_{P,L}$. Intuitively, the positive association between option values and volatility translates to an inverse association between the proportionate systematic component of total firm value and volatility, contributing to an inverse relation between stock return beta and idiosyncratic stock return volatility.[20] Because the risk-adjustment is in relation to the CAPM, the implication of the inverse relation between beta and idiosyncratic volatility is to further strengthen the aforementioned relation of the drift and the jump terms with idiosyncratic return volatility. In contrast to growth options, the beta of mature firms and the assets-in-place of young firms are invariant with respect to the volatility regime.[21]

In conclusion, Proposition 2 shows that our model is able to reconcile the two main conflicting empirical anomalies on the cross-sectional variation of risk-adjusted returns associated with idiosyncratic return volatility, and additionally, offers novel predictions on risk-adjusted return continuation and reversals. We validate these predictions with numerical simulations and empirical tests in the sequel.

$$\textbf{Insert Figure 2 here} \tag{3.13}$$

Figure 2 provides a graphical illustration of Proposition 2 for different set of parameter values of $\sigma_{P,H}$ and $\sigma_{P,L}$. Panel (b) and (c) show that there is a positive difference in the diffusion terms, i.e. $b_H(P) - b_L(P)$, and the continuous drift terms, i.e. $a_H(P) - a_L(P)$.

statistically insignificant if returns are equally-weighted when computing portfolio returns. In contrast, the average difference in pricing errors between the top and bottom quintile $IVol$-portfolios is -8.43% per annum and highly statistically significant if returns are value-weighted, where the top quintile portfolio has an average return of -6.96% and the bottom quintile portfolio has an average return of only 1.47% per annum.

[20] This result is consistent with Galai and Masulis (1976), who show that $\frac{\partial \beta}{\partial \sigma} < 0$ but does not consider stochastic volatility in the firm's assets, and with Johnson (2004), who shows that increasing uncertainty about the value of a firm's assets, while holding the risk premium constant, lowers the expected returns of levered firms.

[21] The beta of mature firms and the assets-in-place is given in the technical appendix.

Panel (d) of the figure shows that there is a negative difference in the jump terms $\nu_H(P) - \nu_L(P)$. All the differences are increasing in the spread between $\sigma_{P,H}$ and $\sigma_{P,L}$, suggesting that the cross-sectional relation between risk-adjusted stock returns and $IVol$ is attributed to firms that experience greater changes in operating risk. Lastly, the difference in all quantities are identically zero if the volatility values are the same in both regimes, which is the usual specification in standard growth option models.

4 Simulations

In this section, we rely on numerical simulations in order to investigate if our model can produce results qualitatively consistent with the main empirical findings in Duffee (1995) and Ang, Hodrick, Xing, and Zhang (2006). Our goal is to create a laboratory as an environment in which to analyze the effects of real options and stochastic idiosyncratic operating risk on the cross-sectional relation between risk-adjusted stock returns and idiosyncratic return volatility.

We simulate a large panel of firm values using the solution equations from our model. All quantities are simulated on a monthly frequency. We begin by simulating a single path of S_t values using the process in equation (2.4), and 2,500 separate paths of P_t and volatility values using processes (2.1) and (2.2) in order to generate a panel.[22] Each simulated path of P_t corresponds to the price series for the output of a firm in the cross-section.[23] We use a time horizon of 50 years for each price series and assume that there are 12 months in each year. That corresponds to a total of 600 (50 years $\times$ 12 months) observations for each firm. Then using equations (2.5), (2.6), (3.1), (3.2) and (3.3), we compute the firm value for each of the 2,500 firms in each month.

The initial firm maturity is drawn from a uniform distribution with equal probabilities for young and old. In order to ensure that mature firms do not dominate the panel overtime,

[22]See Hanson (2007) for a good description of simulations of mixed jump-diffusion processes.

[23]The data in our empirical study contains an average of 2,412 firms with non-missing sales growth observations each month.

we assume that mature firms exit the sample upon the arrival of an independent Poisson event with intensity $\lambda_{exit} = 0.01$ per unit of time or if the firm value reaches zero due to the realization of a low output price. Each month, exiting firms are replaced by new young firms.

In order to carry out the asset pricing tests with the simulated data, we compute monthly risk-adjusted returns for each firm in the panel. The risk-adjustment is relative to the $CAPM$ using the beta of $F_i(P)$ from Proposition 2 and the beta of $G_H(P)$, V_M and the assets-in-place of young firms (equations (7.56) and (7.57) in the Appendix). The young firms' betas are computed as a weighted average of the betas of the firms' assets-in-place and growth options where the weights are based on the proportion of firm value from growth. The beta of mature firms are the beta of the firms' assets-in-place.

Once a full panel of sample equity values and returns is simulated, we use the simulated data to carry out the main empirical analysis conducted by Duffee and Ang et al. and store the results. We then repeat the entire process 100 times in order to arrive at a sample of 100 sets of simulated results and estimates. Then t-tests are carried out the usual way in order to investigate the estimates' statistical significance. The entire simulation process is repeated using three different sets of values for $\sigma_{P,H}$ and $\sigma_{P,L}$ in order to carry out comparative static analysis. The choice of model parameters are summarized in Table 1.

Insert Table 1 here

Figure 3 shows a graphical illustration of a single simulated path of P, the corresponding equity value $V(P)$, realized idiosyncratic operating volatility, realized month-end idiosyncratic return volatility and realized month-end risk-adjusted returns for a firm j using the base case set of model parameters. Panels (a) and (b) of the figure show that P and the equity value $V(P)$ follow a similar patten, as expected. Panels (c) to (d) show that month-end risk-adjusted returns and risk-adjusted return volatility appear to be regime dependent, as expected from Proposition 2.

Insert Figure 3 here

4.1 The Positive Return-Volatility Relation

In this section, we investigate if our model is qualitatively able to reproduce results similar to the main empirical finding in Duffee (1995) (the positive $IVol$ anomaly).

To this end, we fit Fama and MacBeth (1973) monthly cross-sectional regressions of log equity risk-adjusted return r_t^e on $\Delta\sigma_{P,t}$ from our simulated data. The cross sectional regression model for month t is

$$r_t^e = \gamma_{0,t}\iota + \gamma_{1,t}\Delta\sigma_{P,t} + \eta_t \tag{4.1}$$

where r_t^e is a vector of $r_{j,t}^e$ and $\Delta\sigma_{P,t}$ is a vector of $\Delta\sigma_{P,j,t}$ of all the firms $j \in J$, and ι is a vector of ones.

Insert Table 2 here

The results of fitting regression (4.1) are reported in the first column of each panel of Table 2. The table shows that the return-volatility relation is positive and highly statistically significant in the simulated samples where $\sigma_{P,H} > \sigma_{P,L}$. These results confirm the predictions from our model that switches in idiosyncratic operating risk can generate results consistent with Duffee. The table also shows that the positive $IVol$ anomaly is more pronounced for larger spreads between $\sigma_{P,H}$ and $\sigma_{P,L}$, but negligible and insignificant in the samples where $\sigma_{P,H} = \sigma_{P,L}$. This confirms that the positive $IVol$ anomaly is driven by the stochastic nature of the firms' idiosyncratic operating risk, and not by other potentially opaque features of our model.

4.2 The Negative Return-Volatility Relation

In this section, we investigate if our model is qualitatively able to reproduce results similar to the main empirical findings in Ang, Hodrick, Xing, and Zhang (2006) (the negative $IVol$ anomaly) using our simulated data.

To this end, at the end of each month, we sort the simulated firms on σ_P into 10 decile groups. Then, we compute value-weighted one-month portfolio returns for each of the ten portfolios from the equity risk-adjusted returns. The portfolios are rebalanced at the end of each month.

Insert Table 3 here

Table 3 reports the mean portfolio risk-adjusted returns along with the t-statistics for each of the ten portfolios. The decile portfolios are reported across columns. The difference between the highest and lowest volatility portfolios (the long-short portfolio) is reported in the last column. Figure 4 provides a visual illustration of the means of the monthly risk-adjusted returns for the ten decile portfolios reported in the table. The table shows that the long-short portfolio has a highly statistically significant and negative risk-adjusted return for simulated samples where $\sigma_{P,H} > \sigma_{P,L}$, results that are qualitatively consistent with Ang, Hodrick, Xing, and Zhang. Furthermore, the table shows that the negative $IVol$ anomaly is amplified for the simulation sample with a larger spread between $\sigma_{P,H}$ and $\sigma_{P,L}$, but negligible and insignificant for samples where $\sigma_{P,H} = \sigma_{P,L}$. This confirms that the negative $IVol$ anomaly is driven by the stochastic nature of the firm's idiosyncratic operating risk, and not other potentially opaque features of the model.

Insert Figure 4 here

We conduct further analysis by fitting Fama and MacBeth (1973) monthly cross-sectional regressions of equity risk-adjusted returns on lagged $\sigma_{P,i}$ using the simulated data. The regression model for the for month t is

$$r_t^e = \gamma_{0,t}\iota + \gamma_{1,t}\sigma_{P,t-1} + \eta_t \tag{4.2}$$

The results of fitting regression (4.2) are reported in the second column of each panel of Table 2. The table shows that there is a negative and highly statistically significant return-lag volatility relation in the simulated samples where $\sigma_{P,H} > \sigma_{P,L}$. Furthermore, the table shows that this negative relation is amplified by a larger spread in $\sigma_{P,H}$ and $\sigma_{P,L}$, but negligible and insignificant for samples where $\sigma_{P,H} = \sigma_{P,L}$. These results reaffirm the earlier portfolio results.

Taken together, the reported results confirm that the real options in conjunction with stochastic non-systematic operating risk may play a significant role in reconciling the positive and negative $IVol$ anomalies observed in the cross-section of stocks, conflicting empirical puzzles that seem at odds with standard asset pricing arguments.

5 Empirical Analysis

In this section, we empirically test the predictions of our model and show empirical support in the data.

5.1 Data, Variable Descriptions and Summary Statistics

We obtain daily and monthly stock returns from CRSP daily and monthly return files, respectively. Daily and monthly factor returns and risk-free rates are collected from Ken French's website.[24] Our sample period is from January, 1971[25] to December, 2010 for all market-based variables. All our accounting variables are from annual COMPUSTAT files.

[24] http:// mba.tuck.dartmouth.edu/ pages/faculty/ken.french/data library.html

[25] Casual observation reveals that the annual number of firm observations on COMPUSTAT is relatively low prior to the 70's after applying the reported filters in addition with non-missing Sales and Net Income observations.

We consider only ordinary shares traded on the NYSE, AMEX and Nasdaq with primary link to companies on COMPUSTAT with domestic data source and eliminate utility (SIC codes between 4900 and 4999) and financial (SIC codes between 6000 and 6999) companies, observations with a share price of zero, negative book equity values, and returns of firms with less than one year of accounting data on annual COMPUSTAT files. In order to remove the effects of delistings on stock returns, we eliminate return observations within one year of delisting for stocks whose delisting code has the first digit different from 1. After computing monthly idiosyncratic return volatility as described below, our sample size is over 1 million monthly observations with non-missing return and idiosyncratic return volatility values.

5.1.1 Idiosyncratic Volatility

Testing our hypothesis requires a measure of idiosyncratic volatility of the firms' fundamental shock variable. A number of papers, such as Leahy and Whited (1996) and Bulan (2005), and Grullon, Lyandres, and Zhdanov (2010) have used the volatility of stock returns as a proxy for the volatility of the firms' operations. Motivated by the results stated in Proposition 2 that $b_i(P)$ relates to firm-specific operating risk, we adopt the same approach.

Following Ang, Hodrick, Xing, and Zhang (2006) and Ang, Hodrick, Xing, and Zhang (2009), among others, we estimate the idiosyncratic return volatility ($IVol$) for the stock of firm j during the month t as the standard deviation of the firm's daily return during month t, i.e. $\tau \in (t-1, t]$, relative to the Fama and French 3 factor model

$$r_{j,\tau} = \alpha_i + \beta_{j,MKT} MKT_\tau + \beta_{j,SMB} SMB_\tau + \beta_{j,HML} HML_\tau + \varepsilon_{j,\tau} \tag{5.1}$$

More specifically, $IVol$ at month t is defined as $IVol_{j,t} = \sqrt{var(log(1+\varepsilon_{j,\tau}))}$ where $\varepsilon_{j,\tau}$, $\tau \in (t-1, t]$, is the estimated residual from regression (5.1). As in Grullon, Lyandres, and Zhdanov (2010), we use the logarithm of the residuals in order to mitigate the potential mechanical effect of return skewness (see Duffee (1995) and Kapadia (2007)) on the relation between return and contemporaneous return volatility.[26]

We require monthly changes in $IVol$ to test the positive $IVol$ anomaly. To this end, we define the monthly change in $IVol$ as

$$\Delta IVol_{j,t} = IVol_{j,t} - IVol_{j,t-1} \tag{5.2}$$

Our model captures additional testable features related to how extreme firms experience changes in $IVol$. Towards this end, for each stock j in our sample, we define $\overline{IVol_j}$ the difference between the stock's 70th and 30th percentile in-sample values of $IVol$. We consider these values to be the thresholds that determine the volatility regimes for each stock. $\overline{IVol_j}$ captures the spread between volatility regimes, or similarly, how large the changes in $IVol$ tend to be for firm j.

5.1.2 Firm Characteristics

We require several observable firm characteristics shown in the literature to be determinants of stock returns as controls when conducting cross-sectional return regressions. These characteristics are the firms' log market equity, log book-to-market, past stock returns, CAPM beta and trading volume.[27]

[26] Chen, Hong, and Stein (2001) also find that using simple returns induces a pronounced correlation between skewness and contemporaneous volatility.

[27] More specifically, following Fama and French (1993) market value of equity is defined as the share price at the end of June times the number of shares outstanding, book equity is stockholders' equity minus preferred stock plus balance sheet deferred taxes and investment tax credit if available, minus post-retirement benefit asset if available. If missing, stockholders' equity is defined as common equity plus preferred stock par value. If these variables are missing, we use book assets less liabilities. Preferred stock, in order of availability, is preferred stock liquidating value, or preferred stock redemption value, or preferred stock par value. The denominator of the book-to-market ratio is the December closing stock price times the number of shares outstanding. We match returns from January to June of year t with COMPUSTAT-based variables of year $t-2$, while the returns from July until December are matched with COMPUSTAT variables of year $t-1$. This matching scheme is conservative and ensures that the accounting information-based observables are contained in the information set prior to the realization of the market-based variables. We employ the same matching scheme in all our matches involving accounting related variables and CRSP-based variables. We define past returns as the buy-and-hold gross compound returns minus 1 during the six-month period starting from month $t-7$ and ending in month $t-2$. Following Karpoff (1987), trading volume is trading volume normalized by the number of shares outstanding during month t. Lastly, stock CAPM beta is the estimated coefficient from rolling regressions of monthly stock excess returns on the market factor's excess returns. We use a 60-month window every month requiring

5.1.3 Real Option Proxies

We require several firm characteristics to proxy for the firms' reliance on real options. We examine if the relation between the firms' stock returns and $IVol$ is driven by the dependence of real option values on firm-specific operating risk by comparing the strength of this relation across firms with different intensities of real options.

Following the structural model of Merton (1974) and Merton (1992), the equity of a firm is akin to a call option on the firm's assets with the strike price equal to the total face value of the firm's debt. Basing on the knowledge that an option's vega captures the option's sensitivity to the volatility of the underlying state variable, the relation between $IVol$ and stock returns should be stronger for firms with high equity vega. To test this hypothesis, for each firm j and year n, we define the firm's equity vega based on the firm's capital structure and the Black and Scholes' formula

$$vega_{j,n} = V_{j,n} N'(d_{j,n})\sqrt{5} \tag{5.3}$$

where $d_{j,n} = \frac{ln\left(\frac{V_{j,n}}{D_{j,n}}\right) + \left(r_{f,n} - \frac{\sigma^2_{j,n}}{2} \times 5\right)}{\sigma_{j,n}\sqrt{5}}$, $N'(x) = \frac{exp(-x^2/2)}{\sqrt{2\pi}}$, $r_{f,n}$ is the annualized risk free rate, $\sigma_{j,n}$ denotes firm j's annualized six-month rolling window idiosyncratic volatility based on the Fama French 3 factor model, $V_{j,n}$ denotes the sum of the firm's market equity value and book value of debt, and $D_{j,n}$ is the firm's book value of debt. For simplicity, we assume in (5.3) that firms have a debt maturity of 5 years. Because the equity vega is relatively invariant over most of the range of debt and firm values[28], it is useful to classify real option intensity based on equity vega values in relation to other firms in the cross-section. To this end, we identify high equity vega firms as firms with vega values in the top tercile where the breakpoint values are found among NYSE firms in our the sample.

at least 24 monthly return observations in a given window, and use the procedure suggested in Dimson (1979) with a lag of one month in order to remove biases from thin trading in the estimations.

[28]A call option's vega is greatest when the option is at the money, and relatively low and invariant over the remainder of possible prices for the underlying stock (see Hull (2011)).

We follow Grullon, Lyandres, and Zhdanov (2010) in the construction of the remainder of our proxies for real option intensity. The most common type of real options come in the form of future growth opportunities (e.g., Grullon et al., Brennan and Schwartz (1985), MacDonald and Siegel (1986), Majd and Pindyck (1987), and Pindyck (1988) among many others). We consider firm size as an inverse measure of growth opportunities because larger firms tend to be more mature and have larger proportions of their values from assets-in-place, while smaller firms tend to derive value from growth opportunities (e.g., Brown and Kapadia (2007) and Carlson, Fisher, and Giammarino (2004)). We define two measures of firm size: the book value of total assets and the market value of equity. Our third (inverse) proxy for growth options is firm age. Older, more established firms tend to derive larger proportions of profits and firm value from assets-in-place (Lemmon and Zender (2010), Carlson, Fisher, and Giammarino (2004)). Age is defined as the log of the difference between the month of observation and the month in which the stock first appeared in the CRSP monthly return files. We identify young firms and small firms if their age and size are in the bottom tercile where breakpoint values are found among NYSE firms in our sample.

Growth opportunities are revealed in growth measures realized in the future.[29] We define future profit growth as the sum of the growth rates from years $t + 2$ to $t + 5$ of the firms' operating profits.[30] Future sales and investment growth are defined similarly. Then in each year, we categorize firms as high future profit, high future sales, and high future investment growth if they have values that exceed the top tercile breakpoint values among NYSE firms in our sample for each of these three variables. Then we create new categorical variables by combining the existing dummies (e.g. small and growth, small and young, young and high profit growth) to expand our set of growth option proxies.

[29] As in Grullon, Lyandres, and Zhdanov, we are not concerned with potential issues related to look-ahead bias since the focus of our paper is on investigating the relation between $IVol$ and risk-adjusted returns, and not on predicting future stock returns.

[30] As in Grullon, Lyandres, and Zhdanov, we alleviate concerns of spurious correlations between contemporaneous surprises in growth and monthly returns by merging month t returns with growth variables starting two years following the return observation.

Lastly, it is natural to think that firms in certain industries possess more real options than others, and firm valuations from real options may be captured by their industry membership. We consider three main classifications of industry membership based on Fama and French (1997) 49 industries. We define firms with membership in Fama and French (FF) industries 27 (precious metals), 28 (mining), and 30 (oil and natural gas) as natural resource firms. We classify firms in FF industries 22 (electrical equipment), 32 (telecommunications), 35 (computers), 36 (computer software), 37 (electronic equipment), and 38 (measuring and control equipment) as high-tech firms. Membership in FF industries 12 (medical equipment) and 13 (pharmaceutical products) are defined as biotechnology or pharmaceutical firms. Firms with membership in any one of the three aforementioned industry classifications are defined as all-growth industry firms.

5.1.4 Summary Statitics

Table 4 reports summary statistics for the main variables in our study. Mean (median) excess return in our sample is 0.9976% (-0.41%) per month or about 11.9712% (-4.92%) annual. Mean (median) daily idiosyncratic stock return volatility $IVol$ is 2.9476% (2.2782%) or about 44.0171% (34.0208%) annual. Our $IVol$ estimates are similar to those reported in Ang, Hodrick, Xing, and Zhang (2006) and Grullon, Lyandres, and Zhdanov (2010). Mean (median) month-to-month change in $IVol$ is -.0023% (-0.011%). The standard deviation is 2.1096% and similar to the figure reported in Grullon, Lyandres, and Zhdanov (2010).

Insert Table 4 here

5.2 Switches in Idiosyncratic Volatility and Stock Returns

The results of our model offers novel testable predictions on the relation between $IVol$ and risk-adjusted stock returns. Proposition 2 reveals that risk-adjusted returns (3.11) exhibit continuation that correlates with the level of idiosyncratic return volatility. If firm values are partially composed of real options, and subject to changes in volatility, then stocks that experience up switches in idiosyncratic volatility should experience greater risk-adjusted returns after the switch than before the switch. Conversely, stocks that experience a down switch in volatility should experience lower post-switch risk-adjusted returns. The difference between post and pre-switch returns should be amplified among more real option intensive firms and firms that experience more extreme changes in idiosyncratic volatility (the switch effect hereafter). We empirically test these predictions and provide supporting evidence in this section.

To this end, we define an up switch event in $IVol$ in a given month for firm j if the previous month's $IVol$ was below its 30th percentile breakpoint value and if the current month's $IVol$ exceeds its 70th percentile breakpoint value, capturing the notion of a change in volatility regime. A down switch event is defined similarly. Once all the up and down switch events are identified for each stock, we compute the 5-month average of the risk-adjusted returns ending in the month prior to the month of the switch event, and the 5-month average of the risk-adjusted returns beginning from the month after the switch event. Then we investigate if the difference around the switch event-month in average risk-adjusted returns depend on the firms' real option characteristics. More specifically, risk-adjusted returns are based on the Fama and French (1993) 3 factor model

$$r^*_{j,t} = r_{j,t} - r_{f,t} - \sum_{k=1}^{3} \widehat{\beta_{j,k}} F_{k,t} \tag{5.4}$$

where $r_{j,t}$ is the return on stock j in month t, $r_{f,t}$ is the risk-free rate, and $F_{k,t}$ denotes one of the 3 Fama and French factors (market excess return, size, and book-to-market factors) and $k \in [1, 3]$. We estimate the factor loadings $\widehat{\beta_{j,k}}$ for individual stocks using monthly rolling regressions with a 60-month window every month requiring at least 24 monthly return observations in a given window. We use the procedure suggested in Dimson (1979) with a lag of one month in order to remove biases from thin trading in the estimations.

The difference in 5-month averages of the risk-adjusted returns between post and pre-switch events when a switch episode occurs in month t is calculated as

$$r_{j,t}^{Diff} = \frac{1}{5}\sum_{\tau=t+1}^{t+6} r_{j,\tau}^{*} - \frac{1}{5}\sum_{\tau=t-6}^{t-1} r_{j,\tau}^{*} \tag{5.5}$$

We run separate Fama MacBeth cross-sectional regressions of differences in average risk-adjusted returns on the measure of real option for the sample of up switch events and the sample of down switch events. The regression model for month t is

$$r_{t}^{Diff} = \gamma_0 \iota + \gamma_1 RO_{t-1} + \eta_t \tag{5.6}$$

where r_t^{Diff} is a vector of differences in the average of risk-adjusted returns around the switch event month t, ι is a vector of ones, and RO_{t-1} is a vector of measures of real option intensity. Our model's predictions translate to tests that $\gamma_0 > 0$ and $\gamma_1 > 0$ (or $\gamma_1 < 0$ for inverse RO proxies) for the up switch event sample, and $\gamma_0 < 0$ and $\gamma_1 < 0$ (or $\gamma_1 > 0$ for inverse RO proxies) for the down switch event sample.

Insert Table 5 here

The results of estimating (5.6) are presented in Table 5.[31] The table shows that up switches in $IVol$ are positively (i.e. $\gamma_0 > 0$) related to differences in risk-adjusted returns and down switches are negatively (i.e. $\gamma_0 < 0$) related to differences in risk-adjusted returns, the relation being highly statistically significant in all of the specifications using different criteria for real option intensity. This result is consistent with our model's predictions on the switch effect. In order to analyze the effects of real options on this relation, the table shows that the loadings on the inverse real option proxies measured by total asset size, market equity values and age are positive for the down switch sample, and negative for

[31] The results using unadjusted returns are available from the authors upon request, but they are not materially different from the results using risk-adjusted returns.

the up switch sample, indicating that the switch effect is stronger for more growth option intensive firms.

The reported results using categorical variables as proxies for real option intensity are also in favor of our predictions, with greater significance for the up switch sample than for the down switch sample. The only exception occurs when the high vega dummy is used as a real option criteria, whose coefficient estimate is positive and significant for the down switch sample. However, if real option intensity is by the size small size and high vega dummy, the coefficient estimates are significant and consistent with the model's predictions for both the up and down switch event samples. The coefficient estimate for the young and high equity vega dummy is also in favor of our predictions for the up switch sample. In the down switch sample, the estimate is positive but not statistically significant. Based on these results, we argue that vega alone is not a strong measure of firms' real options unless it is combined with other real option characteristics to identify real option reliant firms. Another reason for these results may be that the stocks of younger and smaller firms are more sensitive to the firms' shock variables than older and larger firms even if their vegas are large, in line with the view that the stocks of small and young firms experience larger reaction to operating risk when coupled with heavy borrowing than larger and older firms.[32]

The results reported using industry dummies are weakly in favor of our model. The only industry classification working in favor of the model's predictions is the all-growth option industry, while any one of natural resources, high tech or bio tech classifications alone turns out to be statistically not significant. A possible reason for the relatively weaker results using industry membership is that industry classification alone is a weak proxy for real options since firms within industries may vary widely in their real option intensities.

Next we investigate how the strength of the switch effect is determined by the magnitude of changes in idiosyncratic risk, $\overline{\Delta IVol}$. We run separate monthly Fama MacBeth cross-

[32] Anecdotal evidences seem to point to this possibility.

sectional regressions for each real option criteria and each of the down and up switch event samples. The regression model for month t is

$$r_t^{Diff} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \overline{\Delta IVol} \times RO_{t-1} + \eta_t \tag{5.7}$$

where r_t^{Diff}, ι and RO_{t-1} are as defined previously, and $\overline{\Delta IVol}$ is a vector of $\overline{\Delta IVol_j}$. Our model's predictions translate to tests that $\gamma_1 > 0$ and $\gamma_2 > 0$ (or $\gamma_2 < 0$ for inverse RO proxies) for the up switch event sample, and $\gamma_1 < 0$ and $\gamma_2 < 0$ (or $\gamma_2 > 0$ for inverse RO proxies) for the down switch event sample.

Insert Table 6 here

The results of estimating (5.7) are presented in Table 6. The table shows that the coefficient estimates for $\overline{\Delta IVol}$ (i.e. γ_1) is positive and highly significant for the up switch sample and negative and highly significant for the down switch sample in virtually all of the regression specifications. The only exception occurs in the down switch sample if age is used as a real option criteria where the coefficient estimate is negative but not statistically significant. These results provide conclusive evidence in support of our model's predictions on the switch effect.

The table also reports the coefficient estimates for the interaction terms between $\overline{\Delta IVol}$ and GO (i.e. γ_2) from estimating (5.7). The loadings when firm size is measured by total asset value is positive for the down switch sample and negative for the up switch sample, with significance at the 10% and 5% levels respectively. The loadings when firm size is measured by market capitalization value is negative for the down switch sample but not significant, and negative for the up switch sample and highly statistically significant. We conclude that size as an inverse measure of real options provide results that agree with our model's predictions that the switch effect should be stronger for more real option intensive firms and firms that experience larger changes in $IVol$. The interaction with age, on the other hand, provides inconclusive evidence since the estimated loadings are not significant.

Some of the results reported using categorical variables as proxies for real option intensity are also in favor of the predictions of our model, with greater significance for the up switch sample than for the down switch sample. The coefficient estimates for high future profit growth, high future sale growths, high future investment growth, or small firms with high future growth are all positive for the up switch sample with varying levels of significance. In the down switch sample, none of the estimates are significant even though the estimates are the predicted sign for a number of the specifications. Lastly, using industry dummies as proxies for real option intensity reveals that even though the estimates for most of the specifications are in support of the model's predictions, none of them are statistically significant. We conclude from these results that there is stronger support for the predictions of our model in the up switch sample than in the down switch sample.

5.3 Contemporaneous Relation Between Idiosyncratic Volatility and Returns

In addition to the novel predictions on the switch effect, the model also offers a new explanation for the existing empirical findings on the relation between idiosyncratic return volatility and stock returns. In relation to the first extant empirical finding, our model predicts that to the extent that firms own real options and are subject to changes in idiosyncratic volatility, the positive $IVol$ anomaly should be stronger for more real option intensive firms and firms that experience larger changes in idiosyncratic volatility. We test this hypothesis and provide supporting evidence in this section.

In particular, following Grullon, Lyandres, and Zhdanov (2010) we estimate monthly cross-sectional Fama and MacBeth (1973) regressions of individual stock returns on changes in idiosyncratic volatility and growth options using the various alternative criteria to classify real option intensity. The regression model for the cross section of stocks for time period t is

$$r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \Delta IVol_t + \gamma_2 \Delta IVol_t \times RO_{t-1} + \gamma_3 X_{t-1} + \eta_t \tag{5.8}$$

where r_t is the vector returns, $r_{f,t}$ is the riskless rate, ι is a vector of ones, $\Delta IVol_t$ is a vector of changes in $IVol_t$, RO_{t-1} is a vector of one of the firms' characteristics used to capture real option intensity, and X_{t-1} is a matrix with columns of vectors of controls for firm characteristics related to size, book-to-market, past returns, trading volume and stock beta. Our main hypothesis is that the positive relation between stock returns and idiosyncratic return volatility should be stronger for firms whose value incorporates more real options. Therefore, our hypothesis translates to tests that $\gamma_1 > 0$ and $\gamma_2 > 0$ ($\gamma_2 < 0$ for inverse RO proxies).

Insert Table 7 here

Table 7 reports the coefficient estimates from estimating (5.8) using our proxies for real options. Not surprisingly, and consistent with the majority of the empirical papers, the coefficients on the market factor loading and on the log book-to-market are both significantly positive, while the coefficients on log size are significantly negative in all specifications. The coefficients on contemporaneous volume are positive and highly statistically significant, consistent with Karpoff (1987) and Grullon, Lyandres, and Zhdanov (2010). The coefficients for the past six month cumulative returns are insignificant and negative in all specifications, and consistent with some specifications reported in Cooper, Huseyin, and Schill (2008) and Grullon, Lyandres, and Zhdanov (2010).[33]

The table shows that there is a highly statistically significant positive volatility-return relation in all specifications. In the first two columns of the top panel, in which firm size – measured by firms' equity market value and separately by total asset value – is used as an inverse proxy for real options, the table shows that the estimates of γ_2 are negative and

[33] Grullon, Lyandres, and Zhdanov show that the coefficient on past returns is sensitive to the set of other independent factors included in Fama Macbeth regressions.

highly statistically significant. In the third column, in which age is used as inverse proxy for real options, the table shows that the estimate of γ_2 is negative but not statistically significant.

The table also reports the results when the categorical variables are used as proxies for real option intensity. The estimate of γ_2 for the high equity vega dummy is positive and highly statistically significant. This result is interesting because equity vega is the only proxy for real option intensity under consideration that is not necessarily related to the firms' future growth prospects. In specifications where real options are proxied by future growth opportunities, measured by the high future investment or the high future sales growth dummies, the table reveals that the relation between return and $IVol$ is stronger for more real option intensive firms. The only exception occurs when the dummy for high future profit growth is used, where the estimate is not statistically significant. However, the table shows that when the high future profit growth dummy is combined with the small size dummy, the estimate of γ_2 is positive and highly statistically significant. The table reports a similar pattern of stronger positive relation between returns and volatility if the high future investment growth, the high future sales growth, or the high equity vega dummies are combined with the small size dummy to form proxies of real option intensity. This finding supports the view that combining more than one real option characteristic captures real option intensity better, leading to more favorable results in line with the predictions of our model.

The remainder of the table reports regression results using industry categorical variables as proxies for real option intensity. The table shows that the estimates of the γ_2 coefficient are positive when dummies for natural resources, high technology and bio technology firms are used as proxies for real options, however only the dummy for natural resources gives statistically significant results. The estimate when stocks that belong to any one of the growth option industries is used is positive and highly statistically significant. One explanation for why the γ_2 estimates lack significance when some industry dummies are used is that industry membership alone may be imperfect proxies for real options since firms within industries vary widely in their real option characteristics. In sum, the results reported in Table 7 lend strong support to our model's predictions.

Our model also shows that the positive volatility-return relation should be stronger for more real option intensive firms and firms that experience larger jumps in volatility between volatility regimes. We test this hypothesis in our second set of Fama and MacBeth (1973) regressions. The regression model for the cross section of stocks for time period t is

$$r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \Delta IVol_t + \gamma_3 \overline{\Delta IVol} \times \Delta IVol \times RO + \gamma_3 X_{t-1} + \eta_t \qquad (5.9)$$

where r_t, $r_{f,t}$, ι, $\Delta IVol_t$, RO_{t-1}, X_{t-1} and $\overline{\Delta IVol}$ are as defined previously. Our main hypothesis is that the positive relation between stock returns and idiosyncratic return volatility should be stronger for firms whose values incorporate more real options and firms that experience larger volatility switches. Therefore, our hypothesis translates to tests that $\gamma_3 > 0$ (or $\gamma_3 < 0$ for inverse RO proxies).

Insert Table 8 here

The results of estimating (5.9) are presented in Table 8. The table shows that the estimate of γ_1 is negative and highly statistically significant for all the regression specifications, implying that the the premium for the jump size is negative in the cross section of stock returns. In other words, stocks that experience larger jumps in $IVol$ between volatility regimes on average earn lower stock returns. This finding is interesting and relates to the $IVol$ puzzle of Ang, Hodrick, Xing, and Zhang (2006), an empirical puzzle that is investigated in the context of our model in the next section. The table also shows that the estimate of γ_2 is positive and highly statistically significant for all specifications, in agreement with the results in relation to the estimations of regression equation (5.8).

Table 8 shows that the estimates of γ_3 are positive (negative for inverse real option proxies) and highly statistically significant for virtually all specifications using the different real option proxies. The only exceptions are the coefficient estimates when age, the dummies for young, small and young, and young and high vega are used as real option proxies, which are not statistically significant. The remainder of the table reports regression results using industry membership as proxies for real option intensity. The estimates of the γ_3 coefficient are positive for natural resources, high technology and bio technology firms, however only natural resources is statistically significant. The estimate for all-growth industry is positive and highly statistically significant.

Collectively, the results in Tables 7 and 8 lend strong support for our model's predictions on the positive $IVol$ anomaly.

5.4 The Poor Future Performance of High Idiosyncratic Volatility Stocks

Ang, Hodrick, Xing, and Zhang (2006) report that portfolios of high $IVol$ stocks significantly under-perform their low $IVol$ counterparts on a risk-adjusted basis. Our model also provides an explanation for this finding. Our model predicts that risk-adjusted returns exhibit reversals in tandem with movements in firm-specific operating risks. If firm values incorporate real options and firms are subject to changes in idiosyncratic operating risk, then sorting and grouping stocks on month-end realized $IVol$ is akin to grouping stocks on the firms' most recent idiosyncratic operating volatility regime, and portfolios exhibit differences in risk-adjusted returns. Our model further predicts that the negative $IVol$ anomaly should be amplified for more real option intensive firms and firms that experience more extreme changes in $IVol$. We test these hypotheses and provide empirical support in this section.

At each month-end we sort stocks into terciles ranked on $IVol$. Then, independently, at the end of each June, we separately sort stocks into three terciles on the basis of the real option characteristics age, size (total assets) and size (market equity), and into two groups based on whether they have high equity vega, high future profit growth, high future sales growth, high future investment rate, and whether they are young and small. Then we form value-weighted portfolio returns for each of the two-way classifications and assess their performance over the following month. This approach corresponds to the 1/0/1 (formation period/waiting period/holding period) strategy of Ang, Hodrick, Xing, and Zhang (2006) which most of their analysis is concentrated on.

The performance of the portfolios are assessed on a risk-adjusted basis relative to the Fama and French 3 factor model. More specifically, we estimate the intercepts from running time-series regressions of portfolio excess returns on the three Fama French factors (i.e. market risk premium, size, and book-to-market)

$$r_t - r_{f,t} = \gamma_0 + \gamma_1 MKTRF_t + \gamma_2 SMB_t + \gamma_3 HML_t + \epsilon_t \tag{5.10}$$

where r_t is the portfolio return, $r_{f,t}$ is the monthly riskless rate, $MKTRF$, SMB, and HML are the Fama and French (1993) three factors that proxy for the market risk premium, size and book-to-market factors respectively. In order to investigate the extent to which real option intensity contributes to the negative $IVol$ anomaly, we also estimate the alphas for the zero-cost portfolios along each one of the real option intensity rank classifications. The zero-cost (long-short) portfolio returns along a fixed rank classification are the returns of the portfolio that is long the portfolio of the highest-ranked $IVol$ stocks and short the portfolio of the lowest-ranked $IVol$ stocks. When estimating the alpha of the zero-cost portfolios, we use portfolio returns instead of portfolio excess returns on the left hand side of regression (5.10). All portfolios are rebalanced monthly.

Tables 9 to 11 report average portfolio alphas along with Newey West robust t-statistics after doing two-way independent sorts on $IVol$ and the measure for real option intensity. $IVol$ ranks are listed across columns with the long-short portfolio reported in the last column, while real option ranks are listed down the rows. In order to facilitate interpretation of the economic significance, the reported portfolio alphas are annualized after multiplying the intercept estimates by 12. All other reported statistics are unadjusted.

Insert Table 9 here

The first three panels of Table 9 show that the negative $IVol$ anomaly is more pronounced and statistically more significant for the smaller two tercile groups when firm size is measured by total asset value. The negative $IVol$ anomaly for the largest tercile stocks is not significant. A similar pattern is present for size portfolios, if size is measured by market equity value, and age portfolios. The $IVol$ anomaly is not significant at the 5% level for the largest and oldest tercile stocks, but large and significant for younger and smaller stocks, results that are in agreement with our predictions. Table 9 also reports that the relative poor performance of high $IVol$ stocks are more pronounced and statistically more significant for the high vega firms than for the low vega firms. This finding is enlightening because vega is the only proxy for real option intensity under consideration that is not necessarily related to firms' future growth opportunities.

Insert Table 10 here

More striking are the results in Table 10. The table shows that the negatrive $IVol$ anomaly is even stronger and more significant for small and high equity vega firms than for high equity vega firms alone. These results support the argument that firm values incorporate real options other than future growth opportunities and that the negative $IVol$ anomaly relates to those real options as well. The results also support the notion that the negative $IVol$ anomaly is stronger for stocks of firms that combine more than one real option characteristic, perhaps because these firms have more real options. The other panels in the table point to that conclusion as well. While the negative $IVol$ anomaly is not conclusively stronger for high future profit, high future sale or high future investment growth firms as reported in Table 9, the anomaly is stronger and more significant for these firms if they are also small in size. Similar results also hold for young firms and firms that are both young and have high vega. In sum, the negative $IVol$ anomaly seems to be more pronounced for more real option intensive firms, and for firms that are characterized by more than one real option criteria.

Insert Table 11 here

Table 11 reports the strength of the negative $IVol$ anomaly across real option intensive industries. The first two panels of the table show that the anomaly is more pronounced for natural resources and high technology stocks. The difference in alphas between the high $IVol$ and low $IVol$ portfolios is large and significant at the 5% level for natural resources and high tech stocks, but lower and not significant at the 5% level for stocks that do not belong to these industries. These results are in agreement with the predictions of our model. However, the third panel of the table shows that the negative $IVol$ anomaly is not significant among bio tech stocks, but significant for other stocks. The last panel shows that the anomaly is statistically less significant for stocks with membership in any one of the three growth option industries. From these results, we conclude that the evidence on real option industry membership and the negative $IVol$-anomaly is weakly in favor of the predictions of our model. As mentioned earlier, industry membership alone may be weak a proxy for real option intensity because firms within industries can vary widely in the amount of real options they possess.

In order to investigate how the negative $IVol$ anomaly relates to the size of the changes in $IVol$, in addition to the two-way independent sorts discussed earlier, we also independently sort and rank stocks on $\overline{IVol}$ into terciles. Then we form long-short portfolios for each two-way classification of the real option intensity criteria and $\overline{\Delta IVol}$ and assess their performance over the following month relative to the Fama and French 3 factor model.

The first three panels of Table 12 show that the negative $IVol$ anomaly is monotonically stronger and more significant for the higher $\overline{\Delta IVol}$ tercile than for the lower terciles if real option intensity is proxied by size and age. These results support our predictions that the negative $IVol$ anomaly should be more pronounced for firms that experience more extreme changes in $IVol$. The table also shows that the negative $IVol$ anomaly is stronger and highly statistically significant among the youngest firms and firms that have the largest $\overline{\Delta IVol}$, which also support the predictions our model. However, the negative $IVol$ anomaly seems to be more pronounced for larger firms among the top $\overline{\Delta IVol}$ tercile stocks. These findings are not in direct support of our model. As inverse proxies for real options, our model predicts that the negative $IVol$ anomaly should be stronger for smaller firms. However, consistent with the predictions of our model, the anomaly remains both statistically and economically significant for smaller firms.

If high future profit, high future sale or high future investment growth dummies are used to categorize real option intensity, the main conclusions are similar. While the negative $IVol$ anomaly is stronger for the high $\overline{\Delta IVol}$ stocks independently of the real option characteristics, the anomaly seems to be weaker for high future growth firms. The latter is not in direct support of our model. One possible explanation for the weaker results for high growth firms is that the negative $IVol$ anomaly may be confounded by the strong positive stock returns of the firms with high future growth prospects. This is likely to be a confounding factor that could weaken the negative $IVol$ anomaly to the extent that high future growth correlates with high expected future earnings growth, and investors incorporate earnings growths expectations into stock returns during the portfolios' evaluation month.

Insert Table 13 here

While the negative $IVol$ anomaly does not seem stronger for high future growth stocks within the high $\overline{\Delta IVol}$ stocks, Table 13 shows that the negative $IVol$ anomaly is stronger for high growth firms that are also small. A similar pattern is present for the stocks of firms that are small and young, and firms that are small and have a high vega. These results support the argument made previously that the anomaly is more pronounced for firms that combine more than one real option characteristics.

Insert Table 14 here

Table 14 reports the strength of the anomaly for the different real option intensive industries and $\overline{\Delta IVol}$ ranks. The table shows that the negative $IVol$ anomaly is monotonically stronger and statistically more significant for larger $\overline{\Delta IVol}$ than for smaller $\overline{\Delta IVol}$ stocks independently of industry membership, results that are consistent with our predictions. The table also shows that the strength of the anomaly and its statistical significance correlates with membership in natural resources, bio tech and all-growth industries, and firms that have the larger $\overline{\Delta IVol}$, results that are also in direct support of the predictions of our model. The only industry membership working against the model's predictions is the high tech group, which exhibit a weaker anomaly than the low tech stocks within the high $\overline{IVol}$ group. Although the anomaly is not stronger among the high tech stocks, the anomaly still remains both statistically and economically significant.

Overall, the results in tables 9 to 14 demonstrate that the relative poor performance of high $IVol$ stocks is more pronounced for stocks of firms with characteristics that are more likely to be related with real options and firms that experience more extreme changes in $IVol$, results in line with the predictions of our model.

6 Conclusions

Recent empirical evidences on the correspondence between stock returns and idiosyncratic return volatility at the firm-level have been mixed at best. In this paper, we propose a new economic explanation for the conflicting findings in a simple equity valuation model of firms involving real options and stochastic operating risk. More generally, we motivate why idiosyncratic risks may appear to be priced in the cross-section of stock returns.

In this paper, we introduce a 2-regime Markov switching process for the idiosyncratic volatility of the firms' output price in order to incorporate uncertainty in operating risk. The value of a real option is convex in the output price and its valuation does not distinguish between systematic and idiosyncratic operating risks, a feature that contrasts starkly from the valuation of the firm's assets-in-place. We show that the value of a firm's real options relate positively with the volatility regime, giving rise to regime dependency of the firm's equity return and beta. The time-series dynamics of the realized and expected returns induced by the volatility structure results in an interplay between risk-adjusted returns and idiosyncratic return volatility consistent with what has been observed empirically in the cross-section of stocks. We verify our intuition with numerical simulations, followed with our own predictions and empirical tests. We find that the results are strongly supportive of our model.

To maintain tractability, our model is devoid of a more general structure for the firms' idiosyncratic operating risk. In the 2-regime structure, the operating volatility of a firm leaps between the two values to generate the results aligned with the empirical observations. Qualitatively, our results should persist in a more general structure insofar as idiosyncratic volatility exhibits mean-reversion. Work establishing this conjecture seems to be an interesting extension of our paper.

A number of papers have reported that asset returns must exhibit heteroscedasticity as well as discontinuous movements to fit their empirical distributions. Previously, the literature has relied on liquidity and other microstructure related features to explain the distributional properties of stock returns. Our model suggests that the distributional properties of stock returns stem from the operating environment that firms face. Our model has the capability to parsimoniously generate skewness and fat tails in return distributions, providing fertile grounds for additional research. Further research in this direction is highly merited.

Lastly, we believe that our model imparts a new linkage between corporate investment environments and other stock return regularities. Our model suggests that jumps in stock returns should coincide with large changes in idiosyncratic return volatility in predictable ways, potentially shedding new insights on the three-way relation between stock returns, idiosyncratic return volatility and idiosyncratic expected skewness as shown in Boyer, Mitton, and Vorkink (2010). Additionally, the regime dependency and the time-series pattern of the firm's operating risk in our model help establish predictability akin to return continuation amenable with the findings of Jegadeesh and Titman (1993), and reversals reported in Jegadeesh (1990). We leave these other interesting extensions for future research.

References

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang. 2006. The Cross-Section of Volatility and Expected Returns. *Journal of Finance* .

———. 2009. High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence. *Journal of Financial Economics* 91:1–23.

Berk, Jonathan, Richard Green, and Vasant Naik. 1999. Optimal Investment, Growth Options, and Security Returns. *Journal of Finance.* 54:1553–1607.

Bloom, Nicholas. 2009. The Impact of Uncertainty Shocks. *Econometrica* 77 (3):623–685.

Boyer, Brian, Todd Mitton, and Keith Vorkink. 2010. Expeced Idiosyncratic Skewness. *Review of Financial Studies* 23 (1):169–202.

Brennan, Michael and Eduardo Schwartz. 1985. Evaluating Nature Resource Investments. *Journal of Business* 58:135–157.

Brown, Gregory and Nishad Kapadia. 2007. Firm-Specific Risk and Equity Market Development. *Journal of Financial Economics* 84:358–388.

Bulan, Laarni. 2005. Real Options, Irreversible Investment and Firm uncertainty: New Evidence from U.S. Firms. *Review of Financial Economics* 14:255–279.

Caballero, Ricardo J. and Robert S. Pindyck. 1996. Uncertainty, Investment, and Industry Evolution. *International Economic Review* 37.

Carlson, Murray, Adlai Fisher, and Ron Giammarino. 2004. Corporate Investment and Asset Price Dynamics: Implications for the Cross-Section of Returns. *Journal of Finance.* 59:2577–2603.

Chen, Joseph, Harrison Hong, and Jeremy C. Stein. 2001. Forecasting Crashes: Trading Volume, Past Returns, and Conditional Skewness in Stock Prices. *Journal of Financial Economics* 61:345–381.

Constantinides, George M. 1978. Market Risk Adjustment in Project Valuation. *Journal of Finance* 33:603–616.

Cooper, Michael, Gulen Huseyin, and Michael Schill. 2008. Asset Growth and the Cross-Section of Stock Returns. *Journal of Finance* 1609–1652.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam. 1998. Investor Psychology and Security Market Under- Over-Reactions. *Journal of Finance* 1839–1886.

Dimson, Elroy. 1979. Risk Measurement When Shares are Subject to Infrequent Trading. *Journal of Financial Economics* 7:197–226.

Dixit, Avinash K. and Robert S. Pindyck. 1994. *Investment under Uncertainty.* Princeton University Press.

Duffee, Gregory R. 1995. Stock Returns and Volatility. A Firm-Level Analysis. *Journal of Financial Economics* 37:399–420.

Fama, Eugene and Kenneth R. French. 1992. The Cross-Section of Expected Stock Returns. *Journal of Finance* 47:427–465.

———. 1993. Common Risk Factors in the Returns on Common Stocks and Bonds. *Journal of Financial Economics* 25:2349.

———. 1996. Multifactor Explanations of Asset-Pricing Anomalies. *Journal of Finance* 51:55–84.

———. 1997. Industry Costs of Equity. *Journal of Financial Economics* 49:153–193.

Fama, Eugene and James D. MacBeth. 1973. Risk Return, and Equilibrium: Empirical Tests. *Journal of Political Economy* 71:607–636.

Fu, F. 2009. Idiosyncratic Risk and the Cross-Section of Expected Stock Returns. *Journal of Financial Economics* 91:2437.

Galai, Dan and Ron Masulis. 1976. The Option Pricing Model and the Risk Factor of Stock. *Journal of Financial Economics* 3:53–81.

Garlappi, Lorenzo and Hong Yan. 2008. Financial Distress and The Cross-Section of Equity Returns. *Working Paper* .

———. 2011. Financial Distress and The Cross-Section of Equity. *Journal of Finance* 66:789–822.

Grullon, Gustavo, Evgeny Lyandres, and Alexei Zhdanov. 2010. Real Options, Volatility, and Stock Returns. *Journal of Finance Forthcoming* .

Guo, X. 2001. An Explicit Solution to an Optimal Stopping Problem with Regime Switching. *Journal of Applied Probability* 38:464–481.

Guo, X. and Q. Zhang. 2004. Closed-Form Solutions for Perpetual American Put Options with Regime Switching. *SIAM Journal of Applied Mathematics* 64.

Guo, Xin, Jianjun Miao, and Erwan Morellec. 2005. Irreversible Investment with Regime Shifts. *Journal of Economic Theory* 122:37–59.

Hackbarth, Dirk, Jianjun Miao, and Erwan Morellec. 2006. Capital Structure, Credit Risk, and Macroeconomic Conditions. *Journal of Financial Economics* 82:519–550.

Hanson, Floyd B. 2007. *Applied Stochastic Processes and Control for Jump-Diffusions: Modelling, Analysis and Computation*. Society for Industrial and Applied Mathematics.

Huang, Wei, Qiangqiu Liu, S. Ghon Rhee, and Liang Zhang. 2009. Return Reversals, Idiosyncratic Risk, and Expected Returns. *Review of Financial Studies* .

Hull, John. 2011. *Options, Futures, and Other Derivatives.* Prentice Hall.

Jegadeesh, N. 1990. Evidence of Predictable Behavior of Security Returns. *Journal of Finance* 45:881–898.

Jegadeesh, N. and Sheridan Titman. 1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance* 48:65–91.

Johnson, Timothy C. 2004. Forecast Dispersion and the Cross Section of Expected Returns. *Journal of Finance* 79:1957–1978.

Kapadia, Nishad. 2007. The Next Microsoft? Skewness, Idiosyncratic Volatility, and Expected Returns. *Working Paper* .

Karpoff, Jonathan. 1987. The Relation Between Price Changes and Trading Volume: A Survey. *Journal of Financial and Quantitative Analysis* 45:1161–1187.

Leahy, John and Toni Whited. 1996. The Effect of Uncertainty on Investment: Some Stylized Facts. *Journal of Money, Credit, and Banking* 28:64–83.

Lehmann, Bruce N. 1990. Residual Risk Revisited. *Journal of Econometrics* 45:71–97.

Lemmon, Michael and Jaime Zender. 2010. Debt Capacity and Tests of Capital Structure Theories. *Journal of Financial and Quantitative Analysis* 45:1161–1187.

Lintner, John. 1965. Security Prices, Risk, and Maximal Gains from Diversification. *Journal of Finance* 20:587–616.

MacDonald, Robert and Daniel Siegel. 1985. Investment and Valuation of Firms When There is an Option to Shut Down. *International Economic Review* 26:331–349.

———. 1986. The Value of Waiting to Invest. *Quarterly Journal of Economics* 101:707–727.

Majd, Saman and Robert S. Pindyck. 1987. Time to Build Option Value, and Investment Decisions. *Journal of Financial Economics* 18:7–27.

Malliaris, A. G. 1988. *Stochastic Methods in Economics and Finance.* Elsevier Science.

Merton, Robert C. 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance* 29:449–470.

———. 1976. Option Pricing When Underlying Stock Returns are Discontinuous. *Journal of Financial Economics* 3:125–144.

———. 1992. *Continuous-Time Finance.* Basil Blackwell.

Newey, Whitney K. and Kenneth D. West. 1987. A Simple, Positive Semidefinite, Heteroskedastcity and Autocorrelation Consistent Covariance-Matrix. *Econometrica* 55:703–708.

Pindyck, Robert S. 1988. Irrevesible Investment, Capacity Choice, and the Value of the Firm. *American Economic Review* 78:969–985.

Pontiff. 2006. Costly Arbitrage and the Myth of Idiosyncratic Risk. *Journal of Accounting and Economics* 42:35–52.

Sagi, Jacob S. and Mark S. Seashole. 2007. Firm-Specific Attributes and the Cross-Section of Momentum. *Journal of Financial Economics* 84:389–434.

Schwert, William G. 2003. *Anomalies and Market Efficiency.* George Constantinides, Milton Harris and Rene Stulz in Handbook of the Economics of Finance. North-Holand Amsterdam.

Spiegel, Matthew and Xiaotong Wang. 2006. Cross-sectional Variation in Stock Returns: Liquidity and Idiosyncratic Risk. *Working Paper, Yale School of Management* .

Tinic, Seha M. and Richard R. West. 1986. Risk, Return, and Equilibrium - A Revisit. *Journal of Political Economy* 94:126–147.

Zhang, Lu. 2005. The Value Premium. *Journal of Finance.* 60:67–103.

7 Appendix

This section describes the valuation approach used to value the assets of the firm, followed by the proofs of the propositions stated Section 2 of the paper.

7.1 Valuation Approach

We derive the fundament differential equation that asset values must solve. It simplifies valuation if we reexpress the dynamics of the price (2.1) more concisely by letting

$$\frac{dP}{P} = \mu dt + \sigma_i dB_i \tag{7.1}$$

where $dB_i = \frac{\sigma_{P,i} dB_1 + \sigma_A dB_2}{\sigma_i}$ and $\sigma_i = \sqrt{\sigma_{P,i}^2 + \sigma_A^2}$. Then it can be shown that $\text{Cov}\,(dB_i, dB_2) = \frac{\sigma_A}{\sigma_i} dt$, $\text{Cov}\left(\frac{dP}{P}, \frac{dS}{S}\right) = \sigma_S \sigma_A dt$, and $\rho_i = \frac{\sigma_A}{\sigma_i}$.

Denote $Y(P, \sigma_{P,i}) = Y_i(P)$ the value function of an asset that is twice-differentiable in P where P follows the process in equation (7.1) and $\sigma_{P,i}$ follows the process given by equation (2.2). At this stage, $Y_i(P)$ can be the value of a growth option. The generalized Ito's Lemma (Malliaris (1988)) implies that $Y_i(P)$ has the following process

$$\frac{dY_i(P)}{Y_i(P)} = \frac{\mu P Y_i'(P) + \frac{1}{2} P^2 \sigma_i^2 Y_i''(P)}{Y_i(P)} dt + \frac{\sigma_i P Y_i'(P)}{Y_i(P)} dB_i + \frac{(Y_{i'}(P) - Y_i(P))}{Y_i(P)} dz_i \tag{7.2}$$

The first two terms on the right hand side of the equation are the standard form for Ito's Lemma. The third term is the jump of the value of $Y_i(P)$ when $\sigma_{P,i}$ switches from regime i to regime i'. Equation (7.2) can be written as

$$\frac{dY_i(P)}{Y_i(P)} = [\mu_{Y_i}(P) - \lambda_i \gamma_i(P)]dt + \sigma_{Y_i}(P) dB_i + \gamma_i(P) dz_i \tag{7.3}$$

where

$$\mu_{Y_i}(P) = \left[\frac{\mu P Y_i'(P) + \frac{1}{2}P^2\sigma_i^2 Y_i''(P)}{Y_i(P)}\right] + \lambda_i\gamma_i(P) \tag{7.4}$$

$$\sigma_{Y_i}(P) = \frac{\sigma_i P Y_i'(P)}{Y_i(P)} \tag{7.5}$$

$$\gamma_i(P) = \frac{Y_{i'}(P) - Y_i(P)}{Y_i(P)} \tag{7.6}$$

Following Merton (1976), denote w_1, w_2 and $w_3 = 1-w_1-w_2$ the proportion of portfolio value invested in the market asset S, asset $Y_i(P)$ and the riskless asset M respectively. The instantaneous rate of return on the portfolio is given by

$$\frac{dW}{W} = w_1\frac{dS}{S} + w_2\frac{dY_i}{Y_i} + (1-w_1-w_2)rdt \tag{7.7}$$

$$= \left[w_1(\mu_S - r) + w_2(\mu_{Y_i}(P) - r) + r - w_2\lambda_i\gamma_i(P)\right]dt \tag{7.8}$$

$$+ w_1\sigma_S dB_2 + w_2\sigma_{Y_i}(P)dB_i + w_2\gamma_i(P)dz_i$$

where we have substituted (7.3), (2.3) and (2.4) into (7.7) to arrive at (7.8). It is not possible to make this portfolio riskless[34], instead, we choose the portfolio weights w_1^* and w_2^* to eliminate market risk only. In this case, the expected rate of return for the portfolio equates to the risk free rate, r. This implies that

$$w_1^*(\mu_S - r) + w_2^*(\mu_{Y_i}(P) - r) + r = r \tag{7.9}$$

and

$$w_1^*\sigma_S + w_2^*\sigma_{Y_i}(P)\frac{\sigma_A}{\sigma_i} = 0 \tag{7.10}$$

where we have used the knowledge that $dB_i = \frac{\sigma_{P,i}dB_1 + \sigma_A dB_2}{\sigma_i}$. After solving for w_1^* and w_2^*,

[34] As in Merton (1976), the jump risk in the hedge portfolio is unhedgeable.

equation (7.9) together with equation (7.10) implies that

$$\sigma_S \mu_{Y_i}(P) = -\frac{r\sigma_A \sigma_{Y_i}(P)}{\sigma_i} + \frac{\sigma_A \mu_S \sigma_{Y_i}(P)}{\sigma_i} + r\sigma_S \tag{7.11}$$

Substituting equations (7.4) and (7.5) into (7.11), and simplifying gives the fundamental valuation equation

$$\frac{1}{2}P^2\sigma_i^2 Y_i''(P) + (\mu - \sigma_A\lambda)\, PY_i'(P) + \lambda_{i'}\left(Y_{i'}(P) - Y_i(P)\right) = rY_i(P) \tag{7.12}$$

where we have substituted in the market Sharpe ratio $\lambda = \frac{\mu_S - r}{\sigma_S}$. The differential equation (7.12) serves as the backbone for the derivation of all valuations in Section II of the paper.[35]

7.2 Value of a Mature Firm

To value a mature firm, it requires only that we discount the operating cash flows under the risk neutral measure $\mathbb{Q}$, which implies

$$V_M(P) = \mathbb{E}^{\mathbb{Q}}\left[\int_0^\infty e^{-rs}\xi_M(P_{t+s} - c)ds\right] \tag{7.15}$$

[35] An alternative and more direct approach to deriving the valuation equation (7.12) for any asset $Y_i(P)$ is based on Constantinides (1978). The first step in the approach calls for the replacement of the drift of $\frac{dS}{S}$, μ, by $\mu^* = \mu - \lambda \text{Corr}\left(\frac{dP}{P}, \frac{dS}{S}\right)\sigma_i = \mu - \lambda\rho_i\sigma_i = \mu - \lambda\sigma_A$. The second step evaluates the stream of cash flows of $Y_i(P)$ as if the market price of risk were zero, i.e., discount expected cash flows at the riskfree rate. To this end, the Bellman equation for asset Y_i is given by

$$Y_i(P) = \frac{1}{1 + r\triangle t} E\left[Y(P + \triangle P, \sigma_i + \triangle\sigma)\right] \tag{7.13}$$

The expectation on the right hand side evaluates to

$$E\left[Y_i(P + \triangle P)\right] = \left\{\lambda_{i'}\triangle t E\left[Y_{i'}(P + \triangle P)\right] + (1 - \lambda_{i'}\triangle t)E\left[Y_i(P + \triangle P)\right]\right\} \tag{7.14}$$

The first term is the asset's probability weighted expected value if there is a switch in volatility regime and the second term corresponds to the asset's probability weighted expected value under the current volatility regime. One can arrive at equation (7.12) after substituting equation (7.14) into (7.13), multiplying both sides by $1 + r\triangle t$, letting $\triangle t$ go to zero, applying Ito's Lemma, and substituting μ by μ^*. Looked another way, the traded assets M and S allow us to define a new measure under which the process $dB_i^* = \rho_i\lambda dt + dB_i$ is a brownian motion under the $\mathbb{Q}$ measure. Under this risk neutral measure, the price dynamics follows $\frac{dP}{P} = \mu^* dt + \sigma_i dB_i^*$, where $\mu^* = \mu - \sigma_i\rho_i\lambda = \mu - \sigma_A\lambda$.

where $\mathbb{E}^{\mathbb{Q}}$ is the expectation under the $\mathbb{Q}$ measure. Evaluating the integral results in the value function (2.5). In the valuation of the growth option, a similar function constitutes the reward when a young firm invests I and increases production scale, with the exception that ξ_M is substituted by ξ.

7.3 Value of a Growth Firm

Let $G_i(P)$ denote the value of a growth option in the region where $P \in (P_1, P_2)$ and when the volatility regime i is in effect. In this region of P values, the growth option is in-the-money only if the low volatility regime is in effect, in which case is exercised immediately and it's payoff is

$$G_L(P) = \xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I \tag{7.16}$$

Using valuation equation (7.12), the value of a growth option in the high volatility regime obeys the following differential equation

$$\frac{1}{2}P^2\sigma_i^2 G_H''(P) + (\mu - \sigma_A\lambda)\, PG_H'(P) + \lambda_L\left[\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I - G_H(P)\right] = rG_H(P) \tag{7.17}$$

Equation (7.17) is the standard valuation equation commonly seen in the growth option literature with the exception of the last term. The last term corresponds to the probability weighed change in the value of the option due to a change in regime from high to low and immediate activation. The payoff from activation, net of investment cost I and opportunity cost $G_H(P)$, is $\left[\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I - G_H(P)\right]$.

Now we address the region of low values of P, i.e. $P \in (0, P_1)$. In this region the option is out-of-the money in both volatility regimes and kept alive. Let $F_i(P)$ denote the value of a growth option in the region where $P \in (0, P_1)$ and the i regime is in effect. Following the same steps as above that led to the differential equation (7.17), we arrive at the following pair of differential equations

$$\frac{1}{2}P^2\sigma_H^2 F_H''(P) + (\mu - \sigma_A\lambda)\, PF_H'(P) + \lambda_L\left(F_L(P) - F_H(P)\right) = rF_H(P) \tag{7.18}$$

$$\frac{1}{2}P^2\sigma_L^2 F_L''(P) + (\mu - \sigma_A\lambda)\, PF_L'(P) + \lambda_H\left(F_H(P) - F_L(P)\right) = rF_L(P) \tag{7.19}$$

As before, the differential equations are similar to those in standard diffusion models with the exception that they include an additional component that captures the possibility of a change in the volatility regime of the decision variable P. This term equals $\lambda_L\left(F_L(P) - F_H(P)\right)$ if the high volatility state is in effect, and $\lambda_H\left(F_H(P) - F_L(P)\right)$ otherwise.

With this last pair of valuation equations, we have all the tools required for all the valuations in the paper.

7.4 Proof of Proposition 1

The solution method follows Guo (2001) and Guo and Zhang (2004). Consider first the system of differential equations composed of equations (7.18) and (7.19). It is easy to show that the system has the following characteristic function

$$q_H(\beta) \times q_L(\beta) - \lambda_L\lambda_H = 0 \tag{7.20}$$

where $q_H(\beta)$ and $q_L(\beta)$ are given by the following quadratic equations

$$q_H(\beta) = -\beta\mu^* - \frac{1}{2}(\beta-1)\beta\sigma_H^2 + \lambda_L + r \tag{7.21}$$

$$q_L(\beta) = -\beta\mu^* - \frac{1}{2}(\beta-1)\beta\sigma_L^2 + \lambda_H + r \tag{7.22}$$

The characteristic function has four distinct roots[36] $\beta_{2,1} > \beta_{2,2} > 0 > \beta_{2,3} > \beta_{2,4}$ (Guo (2001)) such that the general form of the solutions to (7.18) and (7.19) are given by

$$F_H(P) = \sum_{i=1}^{4} B_{H,i}(P)P^{\beta_{2,i}} \quad \text{and} \quad F_L(P) = \sum_{i=1}^{4} B_{L,i}(P)P^{\beta_{2,i}}$$

The valuation problem is greatly simplified if we reduce the number of terms in the general solutions. Given the signs of $\beta_{2,1}$, $\beta_{2,2}$, $\beta_{2,3}$, and $\beta_{2,4}$, and the property that the option value must approach zero if P approaches zero, the constants multiplying the negative powers of P must be zero. Therefore, the solutions take the simplified form given by

$$F_H(P) = B_{H,1}P^{\beta_{2,1}} + B_{H,2}P^{\beta_{2,2}} \tag{7.23}$$

$$F_L(P) = B_{L,1}P^{\beta_{2,1}} + B_{L,1}P^{\beta_{2,2}} \tag{7.24}$$

Substituting equations (7.23) and (7.24) into the differential equations (7.18) and (7.19) results in the following equations

$$0 = P^{\beta_{2,1}}\left(\lambda_L B_{L,1} - q_H(\beta_{2,1})B_{H,1}\right) + P^{\beta_{2,2}}\left(\lambda_L B_{L,2} - q_H(\beta_{2,2})B_{H,2}\right)$$

$$0 = P^{\beta_{2,1}}\left(\lambda_H B_{H,1} - q_L(\beta_{2,1})B_{L,1}\right) + P^{\beta_{2,2}}\left(\lambda_H B_{H,2} - q_L(\beta_{2,2})B_{L,2}\right)$$

The next step involves solving for the unknown constants. Equation (3.1) in the proposition is found by solving for $B_{H,1}$ and $B_{H,2}$ and substituting into $F_H(P)$.

Now turning out attention to equation (7.17), the solution has the following form

$$G_H(P) = C_{H,1}P^{\beta_{1,1}} + C_{H,2}P^{\beta_{1,2}} + \phi(P) \tag{7.25}$$

where $C_{H,1}$ and $C_{H,2}$ are constants of integration, $\phi(P)$ is a particular solution and $\beta_{1,1}$ and

[36] The roots of quartic equations can be found in standard math textbooks.

$\beta_{1,2}$ are the two real roots of the following quadratic equation

$$q_H(\beta) = -\beta\mu^* - \frac{1}{2}(\beta - 1)\beta\sigma_H^2 + \lambda_L + r$$

In particular, if $q_H(0) = r + \lambda_L \neq 0$ one can choose

$$\phi(P) = \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P}{r - \mu^*} - \frac{c}{r}\right) - I\right) \tag{7.26}$$

and the complete solution is given by (3.3) in the proposition.

It remains to determine the constants of integration $B_{L,1}$, $B_{L,2}$, $C_{H,1}$, $C_{H,2}$, and the exercise policies P_1 and P_2. To complete the solution, we make use of the following boundary conditions

$$H(P_1) - I = F_L(P_1) \tag{7.27}$$

$$H'(P)|_{P=P_1} = F_L'(P)|_{P=P_1} \tag{7.28}$$

$$H(P_2) - I = G_H(P_2) \tag{7.29}$$

$$H'(P)|_{P=P_2} = G_H'(P)|_{P=P_2} \tag{7.30}$$

$$G_H(P_1) = F_H(P_1) \tag{7.31}$$

$$G_H'(P)|_{P=P_1} = F_H'(P)|_{P=P_1} \tag{7.32}$$

where $H(P) = \xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right)$. The value matching conditions (7.27) and (7.29) impose an equality between the option's intrinsic value and the option's value at the optimal exercise values of P in the two volatility regimes. These conditions merely mean that upon activation the owner foregoes the value of the option in exchange for the net benefits of exercising the option, $H(P) - I$. The smooth pasting conditions (7.28) and (7.30) ensure the optimality of the exercise policies P_1 and P_2 (Dixit and Pindyck (1994)). Lastly, the conditions (7.31) and (7.32) ensure that the value of the option is continuous and smooth around P_1.

We turn to each of the conditions (7.27)–(7.32) above. At $P = P_1$ the value matching condition is given by $H(P_1) - I = F_L(P_1)$ and the smooth pasting condition is given by $H'(P)|_{P=P_1} = F_L'(P)|_{P=P_1}$. More explicitly, the two conditions (7.27) and (7.28) can be written as

$$\xi\left(\frac{P_1}{r-\mu^*} - \frac{c}{r}\right) - I = B_{L,1}P_1^{\beta_{2,1}} + B_{L,2}P_1^{\beta_{2,2}} \tag{7.33}$$

$$\frac{\xi P_1}{r-\mu^*} = \beta_{2,1}B_{L,1}P_1^{\beta_{2,1}} + \beta_{2,2}B_{L,2}P_1^{\beta_{2,2}} \tag{7.34}$$

At $P = P_2$ the value matching condition is given by $H(P_2) - I = G_H(P_2)$ and the smooth pasting condition is given by $H'(P)|_{P=P_2} = G_H'(P)|_{P=P_2}$. More explicitly, the two conditions (7.29) and (7.30) can be written as

$$\xi\left(\frac{P_2}{r-\mu^*} - \frac{c}{r}\right) - I = C_{H,1}P_2^{\beta_{1,1}} + C_{H,2}P_2^{\beta_{1,2}} + \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P_2}{r-\mu^*} - \frac{c}{r}\right) - I\right) \tag{7.35}$$

$$\frac{\xi P_2}{r-\mu^*} = \beta_{1,1}C_{H,1}P_2^{\beta_{1,1}} + \beta_{1,2}C_{H,2}P_2^{\beta_{1,2}} + \frac{\lambda_L}{\lambda_L + r}\left(\frac{\xi P_2}{r-\mu^*}\right) \tag{7.36}$$

We can use the first four conditions (7.33)–(7.36) to solve for $B_{L,1}$, $B_{L,2}$, $C_{H,1}$ and $C_{H,2}$. The expressions in closed form are

$$B_{L,1} = -P_1^{-\beta_{2,1}}\left(\frac{\beta_{2,2}}{\beta_{2,1} - \beta_{2,2}}\right)\left[\frac{\xi P_1\left(1 - \frac{1}{\beta_{2,2}}\right)}{(r-\mu^*)} - \frac{\xi c}{r} - I\right] \tag{7.37}$$

$$B_{L,2} = P_1^{-\beta_{2,2}}\left(\frac{\beta_{2,1}}{\beta_{2,1} - \beta_{2,2}}\right)\left[\frac{\xi P_1\left(1 - \frac{1}{\beta_{2,1}}\right)}{(r-\mu^*)} - \frac{\xi c}{r} - I\right] \tag{7.38}$$

$$C_{H,1} = -P_2^{-\beta_{1,1}}\left(\frac{\beta_{1,2}}{\beta_{1,1} - \beta_{1,2}}\right)\left[\frac{\xi P_2\left(1 - \frac{1}{\beta_{1,2}}\right)}{(r-\mu^*)} - \frac{\xi c}{r} - I\right] \tag{7.39}$$

$$C_{H,2} = P_2^{-\beta_{1,2}}\left(\frac{\beta_{1,1}}{\beta_{1,1} - \beta_{1,2}}\right)\left[\frac{\xi P_2\left(1 - \frac{1}{\beta_{1,1}}\right)}{(r-\mu^*)} - \frac{\xi c}{r} - I\right] \tag{7.40}$$

(7.41)

Continuity and smoothness of the value functions at $P = P_1$ requires that $F_H(P_1) = G_H(P_1)$ and $F'_H(P)|_{P=P_1} = G'_H(P)|_{P=P_1}$. These conditions are the equation (3.5) and (3.6) in the proposition.

Conditions (3.5) and (3.6) and the constants of integration $B_{L,1}$, $B_{L,2}$, $C_{H,1}$ and $C_{H,2}$ are expressed in terms of the exercise boundaries P_1 and P_2. Therefore, the equations compose a system of two equations and two unknowns variables, P_1 and P_2, which are solved numerically for each set of parameters of the model. This completes the proof of Proposition 1 of the paper.

To implement the solution of the model, it is required to only determine the values of P_1 and P_2 numerically for any reasonable set of parameter values. ∎

7.5 Proof of Proposition 2

Direct application of Ito's Lemma on $F_H(P)$ and $F_L(P)$ results in equation (3.7) where

$$a_H(P) = \left(\frac{1}{2}\sigma_H^2 P^2 F''_H(P) + \mu P F'_H(P)\right) \Big/ F_H(P) \tag{7.42}$$

$$b_H(P) = \sigma_H P F'_H(P)/F_H(P) \tag{7.43}$$

$$a_L(P) = \left(\frac{1}{2}\sigma_L^2 P^2 F''_L(P) + \alpha P F'_L(P)\right) \Big/ F_L(P) \tag{7.44}$$

$$b_L(P) = \sigma_L P F_L(P)/F_L(P) \tag{7.45}$$

$$\nu_H(P) = (F_L(P) - F_H(P))/F_H(P) \tag{7.46}$$

$$\nu_L(P) = (F_H(P) - F_L(P))/F_L(P) \tag{7.47}$$

Substituting in the value functions (3.1) and (3.2) and their derivatives into expressions (7.42) to (7.47) results in

$a_H(P) =$

$$\frac{\beta_{2,1}B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1})\left(\frac{1}{2}\left(\beta_{2,1}-1\right)\sigma_H^2+\mu\right)+\beta_{2,2}B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})\left(\frac{1}{2}\left(\beta_{2,2}-1\right)\sigma_H^2+\mu\right)}{B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1})+B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})} \tag{7.48}$$

$$b_H(P) = \frac{\sigma_H\left(\beta_{2,1}B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1})+\beta_{2,2}B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})\right)}{B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1})+B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})} \tag{7.49}$$

$$\nu_H(P) = \frac{B_{L,1}P^{\beta_{2,1}}\left(\lambda_H-q_L(\beta_{2,1})\right)+B_{L,2}P^{\beta_{2,2}}\left(\lambda_H-q_L(\beta_{2,2})\right)}{B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1})+B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})} \tag{7.50}$$

$$a_L(P) = \frac{\beta_{2,1}B_{L,1}P^{\beta_{2,1}}\left(\frac{1}{2}\left(\beta_{2,1}-1\right)\sigma_L^2+\mu\right)+\beta_{2,2}B_{L,2}P^{\beta_{2,2}}\left(\frac{1}{2}\left(\beta_{2,2}-1\right)\sigma_L^2+\mu\right)}{B_{L,1}P^{\beta_{2,1}}+B_{L,2}P^{\beta_{2,2}}} \tag{7.51}$$

$$b_L(P) = \frac{\sigma_L\left(\beta_{2,1}B_{L,1}P^{\beta_{2,1}}+\beta_{2,2}B_{L,2}P^{\beta_{2,2}}\right)}{B_{L,1}P^{\beta_{2,1}}+B_{L,2}P^{\beta_{2,2}}} \tag{7.52}$$

$$\nu_L(P) = \frac{B_{L,1}P^{\beta_{2,1}}\left(q_L(\beta_{2,1})-\lambda_H\right)+B_{L,2}P^{\beta_{2,2}}\left(q_L(\beta_{2,2})-\lambda_H\right)}{\lambda_H\left(B_{L,1}P^{\beta_{2,1}}+B_{L,2}P^{\beta_{2,2}}\right)} \tag{7.53}$$

The dynamics for $G_H(P)$, $V_M(P)$ and the assets-in-place of young firms are omitted but they can be derived in the same way.

The conditional CAPM beta can be found by forming a replicating portfolio with state dependent and time varying weights in the traded assets S and M that exactly reproduces the systematic risk of the option.[37] The proportion of portfolio value held in S determines the beta of the option. To this end, take equation (7.2) and substitute in $dB_i = \frac{\sigma_{P,i}dB_1+\sigma_A dB_2}{\sigma_i}$ and $Y_i(P) = F_i(P)$. By inspection we can see that the diffusion term of the common risk factor can be eliminated by holding $\frac{F_i'(P)\sigma_A P}{\sigma_S S}$ units of the stock in the hedge portfolio. Multiplying the number of stocks by S and dividing by the portfolio value $F_i(P)$, we get the weight of the hedge portfolio invested in the tradeable asset. Since the tradeable asset has a beta of one, the beta of the growth option is given by $\beta_{F,i}(P) = \frac{F_i(P)'\sigma_A P}{\sigma_S S}\frac{S}{F_i(P)} = \frac{F_i'(P)\sigma_A P}{\sigma_S F_i(P)}$.

[37]Alternatively, one can find the conditional CAPM beta by computing the option's return elasticity with respect to the returns of the tradeable asset. The elasticity is $\frac{\mathrm{Cov}[\mathrm{dF}_i/F_i,\mathrm{dS}/S]}{\mathrm{Var}[\mathrm{dS}/S]} = \frac{\sigma_{F,i}\sigma_A}{\sigma_i\sigma_S}$. Substituting in (7.5), $F_i(P)$ from equations (3.1) and (3.2), and their derivative gives (7.54) and (7.55). There is yet a third approach as shown in Sagi and Seashole (2007) to compute the CAPM beta. Sagi and Seashole show that the expected excess return is given by $(\mu-\mu^*)\frac{\partial \mathrm{log} F_i(P)}{\partial \mathrm{log} P} = (\mu-\mu^*)\frac{F_i'(P)}{F_i(P)}$ where $(\mu-\mu^*)$ is the difference between the unadjusted and risk-adjusted mean returns of P. In our set up, $(\mu-\mu^*) = \rho_i\sigma_i\lambda = \sigma_A\lambda$. Substituting in $F_i(P)$ from equations (3.1) and (3.2), and dividing by $\sigma_S\lambda$ results in (7.54) and (7.55).

Substituting in $F_i(P)$ from equations (3.1) and (3.2) and their derivative gives

$$\beta_{F,H}(P) = \frac{\sigma_A}{\sigma_S}\left[\frac{\beta_{2,1}B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1}) + \beta_{2,2}B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})}{B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1}) + B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})}\right] \tag{7.54}$$

$$\beta_{F,L}(P) = \frac{\sigma_A}{\sigma_S}\left[\frac{\beta_{2,1}B_{L,1}P^{\beta_{2,1}} + \beta_{2,2}B_{L,2}P^{\beta_{2,2}})}{B_{L,1}P^{\beta_{2,1}} + B_{L,2}P^{\beta_{2,2}})}\right] \tag{7.55}$$

The beta of $G_H(P)$, $V_M(P)$ and the assets-in-place of young firms can be derived in the same way. The beta of $G_H(P)$ is

$$\beta_{G,H}(P) = \frac{\sigma_A\left(\beta_{1,1}C_{H,1}P^{\beta_{1,1}} + \beta_{1,2}C_{H,2}P^{\beta_{1,2}} + \frac{\xi P\lambda_L}{(r-\mu^*)(\lambda_L+r)}\right)}{\sigma_S\left(C_{H,1}P^{\beta_{1,1}} + C_{H,2}P^{\beta_{1,2}} + \frac{\lambda_L\left(\frac{\xi P}{r-\mu^*} - \frac{c\xi}{r} - I\right)}{\lambda_L+r}\right)} \tag{7.56}$$

and the beta of the mature firm and the assets-in-place of young firms is

$$\beta_M = \beta_{\text{assets-in-place}} = \frac{\sigma_A Pr}{\sigma_S\left(Pr + c(\mu^* - r)\right)} \tag{7.57}$$

The proof for expression (3.11) follows immediately from the expressions for $\beta_i(P)$, $b_i(P)$ and $\nu_i(P)$. Expression (3.11) is consistent with the expected return given by the CAPM. To show this, substitute in (7.49), (7.52), (7.50) and (7.53) into the right hand side of (eq:abret) for $i = H$ and $i = L$. The resulting expressions equate to the expressions from substituting (7.54) and (7.55) into the left hand side of (eq:abret) and simplifying.

The proofs for (3.8), (3.9), (3.10) and $\beta_H^{CAPM}(P) < \beta_L^{CAPM}(P)$ are trivial and merely rely on the knowledge that $\beta_{2,1} > \beta_{2,2}$ and $P < P_1$ if the investment option has not been extinguished.

This completes the proof. ∎

Table 1: Model Inputs. This table reports the parameter values used to solve and simulate the real option/stochastic idiosyncratic volatility model developed in Section 3. Base case parameter values are distinguished with an asterisk $*$ if more than one value is reported for a variable.

Model Parameters		
Price Dynamics	Variable Description	Values
$\sigma_{P,H}$	Output price idiosyncratic volatility in the high regime	0.3,0.4,0.5*
$\sigma_{P,L}$	Output price idiosyncratic volatility in the low regime	0.1*,.02,0.3
λ_H	Transition parameter from low volatility regime to high volatility regime	0.1
λ_L	Transition parameter from high volatility regime to low volatility regime	0.1
μ	Drift of output price process	0.04
σ_A	Market volatility of output price process	0.15
Market	Variable Description	Values
r	Riskless rate	0.05
μ_S	Drift of tradeable asset (Market)	0.1
σ_S	Diffusion of tradeable asset (Market)	0.25
Firm's Profit Function	Variable Description	Values
c	Variable cost per unit of output	10
ξ_Y	Production scale for young Firms	1
ξ	Difference in production scale between mature and young Firms	1.1
I	Investment cost of young firm to become mature	$\frac{1.5\times(\xi-1)\times c}{r-\mu^*}$
Simulations	Variable Description	Values
N	Number of samples	100
n	Number of firms in each sample	2500
T	Number of years	50
nt	Number of months in each year	12
λ_{exit}	Exit parameter for mature firms	0.01

Table 2: Simulation Results. The table reports coefficient estimates along with their t-statistics for the regression model $r_t^e = \gamma_{0,t}\iota + \gamma_{1,t}\Delta\sigma_{P,t} + \eta_t$ in the first column of panels (a), (b) and (c), and estimates for the regression model $r_t^e = \gamma_{0,t}\iota + \gamma_{1,t}\sigma_{P,t-1} + \eta_t$ in the second column of panels (a), (b) and (c) using equity return data simulated from the real option/stochastic idiosyncratic volatility model developed in Section 3 of the paper. Panels (a), (b) and (c) report separate model estimates corresponding to the simulated samples where $\sigma_{P,H} = 0.5$,$\sigma_{P,L} = 0.1$, $\sigma_{P,H} = 0.4$,$\sigma_{P,L} = 0.2$ and $\sigma_{P,H} = \sigma_{P,L} = 0.3$ respectively. T-statistics are reported in square brackets.

	(a) $\sigma_{P,H} = 0.5$,$\sigma_{P,L} = 0.1$		(b) $\sigma_{P,H} = 0.4$,$\sigma_{P,L} = 0.2$		(c) $\sigma_{P,H} = 0.3$,$\sigma_{P,L} = 0.3$	
Intercept	0.0346	0.0346	-0.0059	0.0030	-0.0063^{***}	-0.0098^{***}
	[1.6031]	[1.6031]	[-1.8594]	[0.4584]	[-2.2178]	[-4.9173]
$\Delta\sigma_{P,t}$	0.1073^{***}		0.0882^{***}		0.0008	
	[23.6857]		[8.2872]		[0.5774]	
$\sigma_{P,t-1}$		-0.0542^{***}		-0.0170^{***}		0.0071
		[-2.4840]		[-2.5375]		[1.2149]

Table 3: Simulation Results. The table reports the mean portfolio risk-adjusted returns along with their t-statistics using the return data simulated based on the real option/stochastic idiosyncratic volatility model developed in Section 3 of the paper. Equity returns are sorted into decile portfolios based on the level of realized idiosyncratic volatility σ_P over the past month. Then, value-weighted one-month holding period mean portfolio returns are computed using the monthly risk-ajdusted returns. The portfolios are rebalanced at the end of each month. σ_P ranks are listed across columns with the difference in risk-adjusted returns between the highest and lowest σ_P decile portfolios reported in the last column. The table reports separate portfolio risk-adjusted returns corresponding to the simulated samples where $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$, $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$ and $\sigma_{P,H} = \sigma_{P,L} = 0.3$. T-statistics are reported in square brackets.

	σ_P Decile Portfolios										
	1	2	3	4	5	6	7	8	9	10	10-1
$\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$	0.0006***	0.0006***	0.0006***	0.0006***	0.0012	−0.0129***	−0.0136***	−0.0136***	−0.0136***	−0.0142***	−0.0148***
	[8.7598]	[9.2058]	[14.3916]	[13.9203]	[1.0769]	[-49.9083]	[-50.6209]	[-50.6509]	[-45.2303]	[-69.5785]	[-68.0273]
$\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$	−0.0018***	−0.0017***	−0.0017***	−0.0017***	−0.0018***	−0.0087***	−0.0087***	−0.0088***	−0.0088***	−0.0096***	−0.0078***
	[-9.4463]	[-19.5809]	[-16.2214]	[-22.4159]	[-12.3837]	[-51.4181]	[-40.6960]	[-49.1244]	[-47.0627]	[-25.5529]	[-18.8114]
$\sigma_{P,H} = 0.3, \sigma_{P,L} = 0.3$	−0.0051***	−0.0055***	−0.0055***	−0.0054***	−0.0054***	−0.0052***	−0.0053***	−0.0051***	−0.0053***	−0.0055***	−0.0004
	[-26.7324]	[-42.1925]	[-43.1618]	[-39.9578]	[-42.1094]	[-32.1852]	[-38.8333]	[-23.1704]	[-35.8675]	[-39.8603]	[-1.4912]

Table 4: This table reports summary statistics for excess stock returns, return volatilities $IVol$, month-to-month changes in $IVol$, or $\Delta IVol$, and proxies for firms' real option intensity. Stock return data are from CRSP. The sample period is from January, 1971 to December, 2010 for all market based variables. All our accounting variables are from annual COMPUSTAT files. Utilities (SIC codes between 4900 and 4999) and financials (SIC codes between 6000 and 6999) are excluded. A stock's excess returns is the difference between its monthly stock return and the risk-free rate. Volatility and its change refer to monthly volatility of log daily risk-adjusted returns where risk-adjustment is based on the Fama and French 3-factor model. Market equity and total assets are in millions of dollars. Age is in months since first appearance in monthly CRSP files. Future investment, profit and sales growth are the sum of the growth rates from year $t+2$ to $t+5$ of firms' investments in property plant and equipment, of firm's operating profits, and of firms' sales, respectively. $vega$ is computed for each firm according to (5.3) in the paper.

market variables	Mean	StdDev	P5	Median	P95	N
excess return	0.009976	0.180828	-0.22309	-0.0041	0.272627	1041266
$IVol$	0.029476	0.024979	0.0079	0.022782	0.072884	1038601
$\Delta IVol$	-2.3E-05	0.021096	-0.02552	-0.00011	0.026111	1035935

Real Option variables	Mean	StdDev	P5	Median	P95	N
log(market equity)	4.694734	2.106019	1.5389081	4.521163	8.389149	1040478
log(total assets)	4.804593	2.009753	1.789757	4.62188	8.352702	1041266
log(age)	3.953142	1.540425	0	4.290459	5.746203	1041266
investment growth	0.996235	18.22423	-0.64226	0.225036	2.237907	871778
profit growth	-0.55037	80.99137	-6.71653	0.353252	4.689659	871779
sales growth	1.579677	79.57993	-0.46927	0.29381	1.83045	868519
$vega$	2.84E-69	1.49E-67	9.63E-110	9.89E-81	1.88E-70	1041104

Table 5: This table reports coefficient estimates along with their t-statistics of Fama and MacBeth (1973) cross sectional regressions of firm level differences in the 5-month average of the risk-adjusted returns between post and pre-$IVol$ switch monthly events on the real option proxies. The regression equation is $r_t^{Diff} = \gamma_0 \iota + \gamma_1 RO_{t-1} + \eta_t$. The construction of the real option proxies are described in the paper. The estimates for separate real option proxies used in the regression are reported across columns. Separate regression estimates are reported for up and down switch samples. The reported estimates are time series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

		RO Proxy									
switch	Coeff.	size (total asset)	size (mkt equity)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
down	Intercept	-0.0201***	-0.0175***	-0.0111**	-0.0062***	-0.0068***	-0.0079***	-0.0079***	-0.0080***	-0.0076***	-0.0074***
		[-6.3508]	[-5.4812]	[-2.0936]	[-6.8905]	[-6.6695]	[-7.8954]	[-7.7187]	[-8.7388]	[-8.6914]	[-8.1784]
	RO	0.0024***	0.0018***	0.0005	0.0063***	-0.0016	0.0021	-0.0007	0.0025	-0.0116**	-0.0071**
		[4.4752]	[3.1156]	[0.3552]	[-3.1565]	[-1.0178]	[1.2240]	[-0.3579]	[1.5724]	[-2.5072]	[-2.1897]
	RSQR	0.0326***	0.0329***	0.0265***	0.0268***	0.0266***	0.0252***	0.0288***	0.0244***	0.0482***	0.0376***
up	intercept	0.0158***	0.0182***	0.0150***	0.0052***	0.0040***	0.0036***	0.0044***	0.0030**	0.0048***	0.0047***
		[5.3102]	[6.9155]	[3.2173]	[5.2992]	[3.0968]	[3.0411]	[3.9123]	[2.4548]	[4.9601]	[4.7785]
	RO	-0.0022***	-0.0027***	-0.0024**	0.0023	0.0039***	0.0059***	0.0053***	0.0076***	0.0082**	0.0095***
		[-4.4951]	[-6.2128]	[-2.0830]	[1.2730]	[2.7997]	[3.8766]	[3.1845]	[5.2559]	[2.2245]	[3.4609]
	RSQR	0.0283***	0.0279***	0.0285***	0.0252***	0.0251***	0.0269***	0.0255***	0.0261***	0.0408***	0.0349***

		RO Proxy					*RO* Industries			
switch	Coeff.	small high profit	small high sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Industry
down	Intercept	-0.0062***	-0.0068***	-0.0069***	-0.0069***	-0.0077***	-0.0083***	-0.0085***	-0.0079***	-0.0082***
		[-7.1963]	[-7.6766]	[-7.6615]	[-8.1033]	[-8.3749]	[-9.2471]	[-8.5055]	[-8.8771]	[-8.5319]
	RO	-0.0088***	-0.0045	-0.0108***	-0.0036	0.001	-0.0015	0.0013	-0.001	0.0002
		[-3.1267]	[-1.4208]	[-3.5391]	[-0.8044]	[0.2391]	[-0.3588]	[0.5947]	[-0.3122]	[0.1036]
	RSQR	0.0394***	0.0388***	0.0384***	0.0439***	0.0357***	0.0333***	0.0273***	0.0288***	0.0296***
up	Intercept	0.0045***	0.0043***	0.0051***	0.0036***	0.0054***	0.0056***	0.0057***	0.0056***	0.0041***
		[3.9771]	[4.0485]	[5.1847]	[3.2879]	[5.2951]	[5.1394]	[5.5452]	[5.5774]	[3.8737]
	RO	0.0088***	0.0107***	0.0077***	0.0164***	0.0079***	0.002	0.0025	0.0028	0.0043**
		[3.0261]	[3.4846]	[2.6551]	[4.7291]	[2.7409]	[0.5846]	[1.3052]	[0.8773]	[2.1847]
	RSQR	0.0365***	0.0368***	0.0324***	0.0397***	0.0325***	0.0299***	0.0237***	0.0265***	0.0290***

Table 6: This table reports coefficient estimates along with their t-statistics of Fama and MacBeth (1973) cross sectional regressions of firm level differences in the 5-month average of the risk-adjusted returns between post and pre-$IVol$ switch monthly events on the difference between the high and low $IVol$ values, and the real option proxies. The regression equation is $r_t^{Diff} = \gamma_0\iota + \gamma_1\overline{\Delta IVol} + \gamma_2\overline{\Delta IVol} \times RO_{t-1} + \eta_t$. The construction of the real option proxies are described in the paper. The results using different real option proxies in the regression are reported across columns. We estimate separate monthly regressions for the up and down switch samples. The reported estimates are the time series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

		RO Proxy									
switch	Coeff.	size (total asset)	size (mkt equity)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
down	Intercept	-0.0018	0.0011	0.0015	0.0002	-0.0003	0	0.0023	0.0004	0.0014	0.0011
		[-0.6873]	[0.4085]	[0.7341]	[0.1085]	[-0.1137]	[0.0180]	[1.0268]	[0.1644]	[0.6783]	[0.5057]
	$\overline{\Delta IVol}$	-1.0054***	-0.8526***	-0.2454	−0.6433***	-0.5962**	-0.7080***	-1.0003***	-0.7249***	-0.8262***	-0.8361***
		[-3.8117]	[-3.4519]	[-0.4000]	[-2.8142]	[-2.5609]	[-3.1400]	[-4.6543]	[-3.2998]	[-3.9831]	[-3.8818]
	$\overline{\Delta IVol} \times RO$	0.1072*	-0.0046	-0.1626	-0.1921	0.0088	0.1432	0.2439	0.1334	-0.2349	0.0228
		[1.7705]	[-0.0742]	[-1.0051]	[-1.2213]	[0.0611]	[0.9226]	[1.4789]	[0.8459]	[-1.0029]	[0.1089]
	RSQR	0.0882***	0.0912***	0.0877***	0.0871***	0.0967***	0.0964***	0.0898***	0.0958***	0.0959***	0.0962***
		[19.6250]	[16.1655]	[17.3481]	[17.1345]	[18.9544]	[19.4351]	[17.9502]	[17.5652]	[17.1672]	[19.9459]
up	Intercept	-0.0003	0.0037	-0.0035	-0.0043*	-0.0066***	-0.0056**	-0.003	-0.0059**	-0.0038*	-0.0024
		[-0.1080]	[1.4158]	[-1.6219]	[-1.8809]	[-2.8511]	[-2.4964]	[-1.2945]	[-2.4858]	[-1.7371]	[-1.1340]
	$\overline{\Delta IVol}$	0.9565***	0.9889***	1.3251**	0.8610***	0.9418***	0.8203***	0.6773***	0.7773***	0.7808***	0.6350***
		[3.9990]	[4.4507]	[2.1528]	[4.0885]	[4.4276]	[4.2999]	[3.1748]	[3.9858]	[3.9437]	[3.2559]
	$\overline{\Delta IVol} \times RO$	-0.1229**	-0.2472***	-0.2528	-0.0527	0.2297*	0.4044***	0.1471	0.6317***	0.0379	0.2846
		[-2.2675]	[-3.9057]	[-1.0236]	[-0.3575]	[1.7174]	[2.7888]	[1.2512]	[4.7608]	[0.1882]	[1.5988]
	RSQR	0.0721***	0.0775***	0.0740***	0.0736***	0.0821***	0.0810***	0.0689***	0.0810***	0.0809***	0.0784***

		RO Proxy					*RO* Industry			
switch	Coeff.	small high profit	small high sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Industries
down	Intercept	-0.0007	0.0001	0	-0.0002	0.0015	0.0009	0.0003	0.0006	0.0005
		[-0.2803]	[0.0229]	[0.0216]	[-0.0799]	[0.6820]	[0.4145]	[0.1555]	[0.2746]	[0.2131]
	$\overline{\Delta IVol}$	-0.5243**	-0.6459***	-0.7449***	-0.6130***	-0.8933***	-0.7969***	-0.7817***	-0.7619***	0.8022***
		[-2.2498]	[-2.8497]	[-3.5372]	[-2.8459]	[-4.1053]	[-3.6453]	[-3.6182]	[-3.5135]	[-3.6652]
	$\overline{\Delta IVol} \times RO$	-0.209	-0.0493	-0.1334	-0.0914	0.3063	-0.0335	0.0778	0.0886	0.1131
		[-1.0722]	[-0.2414]	[-0.7354]	[-0.3688]	[1.3479]	[-0.0943]	[0.4351]	[0.3732]	[0.7114]
	RSQR	0.1005***	0.1007***	0.0930***	0.1045***	0.0941***	0.0937***	0.0913***	0.0888***	0.0934***
		[20.2486]	[20.3574]	[18.3941]	[18.4894]	[16.2996]	[17.5190]	[19.7699]	[19.0352]	[17.9798]
up	Intercept	-0.0059**	-0.0054**	-0.0032	-0.0048**	-0.0031	-0.0037*	-0.0042*	-0.0037	-0.0032
		[-2.3840]	[-2.1412]	[-1.4104]	[-2.0238]	[-1.4008]	[-1.6704]	[-1.6530]	[-1.4657]	[-1.4908]
	$\overline{\Delta IVol}$	0.9227***	0.8880***	0.7471***	0.7969***	0.7031***	0.7761***	0.8545***	0.7441***	0.6572***
		[4.2713]	[4.2814]	[3.7131]	[4.1374]	[3.4753]	[4.0162]	[3.7309]	[3.6893]	[3.5573]
	$\overline{\Delta IVol} \times RO$	0.1712	0.3161*	0.0841	0.6159***	0.0601	0.339	0.0486	0.0432	0.1858
		[0.9805]	[1.7296]	[0.5751]	[3.2721]	[0.3100]	[1.0305]	[0.3155]	[0.1791]	[1.1833]
	RSQR	0.0847***	0.0841***	0.0701***	0.0842***	0.0777***	0.0753***	0.0723***	0.0711***	0.0773***

Table 7: This table reports coefficient estimates along with their t-statistics of Fama and MacBeth (1973) cross sectional regressions of firm level excess returns on the estimated loading on the market factor (β^{CAPM}), beginning of year log book-to-market ($Log(BM)$), log market equity ($Log(ME)$), six-month lagged return from months -7 to -2 relative to the month of observation ($Lag(r)$), monthly trading volume normalized by the number of shares outstanding ($trade$), month-to-month change in firm level idiosyncratic volatility ($\Delta Ivol$), real option proxy, and the interaction between the real option proxy and $\Delta Ivol_t$. The construction of the real option proxies are described in the paper. The regression model is $r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \Delta IVol_t + \gamma_2 \Delta IVol_t \times RO_{t-1} + \gamma_3 X_{t-1} + \eta_t$ in the paper. The real option proxies used in the regression are reported across columns. The reported estimates are the time series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

	RO Proxy									
Coeff.	size (mkt equity)	size (total assets)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
Intercept	0.0402***	0.0404***	0.0405***	0.0404***	0.0417***	0.0414***	0.0404***	0.0417***	0.0403***	0.0403***
	[6.7509]	[6.8148]	[6.8501]	[6.8507]	[7.0746]	[7.0291]	[6.8406]	[7.0813]	[6.8330]	[6.8257]
$Log(BM)$	0.0057***	0.0059***	0.0057***	0.0057***	0.0052***	0.0052***	0.0057***	0.0052***	0.0057***	0.0058***
	[7.7458]	[8.0586]	[7.7807]	[7.6285]	[6.8186]	[6.8100]	[7.7920]	[6.7996]	[7.7387]	[7.7815]
$Log(ME)$	-0.0034***	-0.0035***	-0.0035***	-0.0035***	-0.0037***	-0.0037***	-0.0035***	-0.0037***	- -0.0035***	-0.0035***
	[-5.5311]	[-5.6697]	[-5.7620]	[-5.7513]	[-6.1040]	[-6.0733]	[-5.7558]	[-6.1205]	[-5.7169]	[-5.7682]
β^{CAPM}	0.0015**	0.0015**	0.0015**	0.0016**	0.0014*	0.0015*	0.0015**	0.0014*	0.0016**	0.0015**
	[2.2340]	[2.1974]	[2.0700]	[2.2294]	[1.8310]	[1.9049]	[2.1691]	[1.8073]	[2.2730]	[2.1722]
$trade$	0.0157***	0.0156***	0.0155***	0.0155***	0.0154***	0.0153***	0.0155***	0.0154***	0.0155***	0.0155***
	[14.4667]	[14.3834]	[14.2095]	[14.3831]	[13.4373]	[13.4022]	[14.2209]	[13.4620]	[14.3394]	[14.2345]
$Lag(r)$	-0.0031	-0.0031	-0.0031	-0.0031	-0.0031	-0.003	-0.0031	-0.0032	-0.0032	-0.003
	[-1.1424]	[-1.1612]	[-1.1747]	[-1.1768]	[-1.0903]	[-1.0371]	[-1.1534]	[-1.1049]	[-1.1820]	[-1.1219]
$\Delta IVol$	2.3623***	2.2474***	1.1341***	1.0903***	1.2728***	1.1797***	1.1865***	1.1680***	1.1066***	1.2181***
	[17.5356]	[15.8777]	[6.9660]	[9.8669]	[12.1527]	[11.3973]	[10.1845]	[10.9610]	[10.5451]	[12.3035]
$RO \times \Delta Ivol$	-0.3125***	-0.2647***	-0.0094	0.3748***	0.0217	0.2497***	0.0321	0.3273***	0.5297***	0.3139***
	[-10.790]	[-9.8134]	[-0.1847]	[5.0322]	[0.3632]	[3.7902]	[0.3178]	[5.0967]	[5.2866]	[3.4861]
RSQR	0.1150***	0.1128***	0.1099***	0.1107***	0.1104***	0.1111***	0.1099***	0.1110***	0.1127***	0.1107***

	RO Proxy					*RO* Industry			
Coeff.	small high profit	small highs sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Option Industries
Intercept	0.0416***	0.0413***	0.0405***	0.0416***	0.0403***	0.0403***	0.0403***	0.0403***	0.0404***
	[7.0506]	[7.0154]	[6.8674]	[7.0715]	[6.8187]	[6.8169]	[6.8440]	[6.8730]	[6.8530]
$Log(BM)$	0.0053***	0.0052***	0.0058***	0.0052***	0.0058***	0.0057***	0.0057***	0.0058***	0.0057***
	[6.8579]	[6.8384]	[7.8430]	[6.7970]	[7.7727]	[7.7483]	[7.7597]	[7.8780]	[7.7787]
$Log(ME)$	-0.0037***	-0.0036***	-0.0035***	-0.0037***	-0.0035***	-0.0035***	-0.0035***	-0.0035***	-0.0035***
	[-6.0680]	[-6.0359]	[-5.7614]	[-6.0951]	[-5.7286]	[-5.7625]	[-5.7303]	[-5.7558]	[-5.7524]
β^{CAPM}	0.0015*	0.0015**	0.0016**	0.0014*	0.0015**	0.0016**	0.0015**	0.0015**	0.0015**
	[1.8461]	[2.0167]	[2.2027]	[1.8043]	[2.1244]	[2.2400]	[2.1177]	[2.0997]	[2.1287]
$trade$	0.0154***	0.0153***	0.0155***	0.0154***	0.0155***	0.0154***	0.0155***	0.0155***	0.0154***
	[13.4108]	[13.4492]	[14.2725]	[13.4390]	[14.1888]	[14.1937]	[14.2384]	[14.3645]	[14.2776]
$Lag(r)$	-0.003	-0.0029	-0.003	-0.003	-0.003	-0.0032	-0.0032	-0.0031	-0.0031
	[-1.0544]	[-1.0095]	[-1.1569]	[-1.0560]	[-1.1339]	[-1.1729]	[-1.1851]	[-1.1741]	[-1.1711]
$\Delta IVol$	1.1829***	1.1282***	1.1573***	1.1504***	1.2322***	1.2437***	1.2796***	1.2852***	1.1634***
	[11.4545]	[10.8841]	[11.6522]	[10.8892]	[12.0615]	[12.0660]	[12.8921]	[12.8305]	[11.8727]
$RO \times \Delta Ivol$	0.4097***	0.6017***	0.3011***	0.5906***	0.1640**	0.3629***	0.0536	0.0483	0.2804***
	[4.6175]	[7.4742]	[3.5690]	[7.7597]	[2.0284]	[3.5133]	[0.6618]	[0.4384]	[4.2373]
RSQR	0.1116***	0.1122***	0.1111***	0.1122***	0.1108***	0.1106***	0.1102***	0.1101***	0.1102***

Table 8: This table reports coefficient estimates along with their t-statistics of Fama and MacBeth (1973) cross sectional regressions of firm level excess returns on the estimated loading on the market factor (β^{CAPM}), beginning of year log book-to-market ($Log(BM)$), log market equity ($Log(ME)$), six-month lagged return for months -7 to -2 relative to the month of observation ($Lag(r)$), monthly trading volume normalized by the number of shares outstanding ($trade$), month-to-month change in firm level idiosyncratic volatility ($\Delta Ivol$), the difference between the high and low $IVol$ regimes, the real option proxy, and the interaction between the real option proxy and the difference in $IVol$ regime $\overline{\Delta Ivol}$. The construction of the real option proxies are described in the paper. The regression model is $r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \Delta IVol_t + \gamma_3 \overline{\Delta IVol} \times \Delta IVol \times RO + \gamma_3 X_{t-1} + \eta_t$ in the paper. The real option proxies used in the regression are reported across columns. The reported estimates are the time series average of the monthly coefficient estimates. Newey-West robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

	RO Proxy									
Coeff.	size (mkt equity)	size (total assets)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
Intercept	0.0620***	0.0627***	0.0635***	0.0636***	0.0628***	0.0622***	0.0633***	0.0626***	0.0635***	0.0633***
	[12.6667]	[12.9225]	[13.0910]	[13.1015]	[13.0393]	[12.8871]	[12.9775]	[12.9719]	[13.0683]	[12.9677]
$Log(BM)$	0.0043***	0.0044***	0.0044***	0.0043***	0.0039***	0.0040***	0.0043***	0.0039***	0.0043***	0.0044***
	[5.9892]	[6.0253]	[6.0236]	[5.8810]	[5.2759]	[5.3703]	[6.0379]	[5.3164]	[5.9482]	[6.1117]
$Log(ME)$	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***
	[-11.668]	[-11.886]	[-11.888]	[-11.951]	[-12.321]	[-12.203]	[-11.845]	[-12.268]	[-11.906]	[-11.880]
β^{CAPM}	0.0016**	0.0017**	0.0017**	0.0017***	0.0016**	0.0017**	0.0017**	0.0017**	0.0017***	0.0017***
	[2.3965]	[2.4694]	[2.5253]	[2.5956]	[2.1575]	[2.2686]	[2.4378]	[2.1917]	[2.5962]	[2.5885]
$Lag(r)$	-0.0042	-0.0041	-0.0041	-0.0041	-0.0039	-0.0038	-0.0041	-0.004	-0.0041	-0.0041
	[-1.6123]	[-1.5606]	[-1.5916]	[-1.5743]	[-1.4138]	[-1.3623]	[-1.6018]	[-1.4368]	[-1.5860]	[-1.5573]
$trade$	0.0158***	0.0158***	0.0159***	0.0159***	0.0157***	0.0157***	0.0158***	0.0157***	0.0159***	0.0158***
	[14.8382]	[14.7683]	[14.8815]	[14.9759]	[14.1370]	[14.0990]	[14.8001]	[14.1068]	[14.9459]	[14.8546]
$\overline{\Delta IVol}$	-0.9410***	-0.9714***	-1.0111***	-1.0136***	-0.9507***	-0.9369***	-0.9915***	-0.9412***	-1.0108***	-1.0003***
	[-9.4354]	[-9.6958]	[-10.129]	[-10.100]	[-8.9019]	[-8.7162]	[-9.9101]	[-8.8688]	[-10.092]	[-9.9407]
$\Delta IVol$	1.8416***	1.6986***	1.0350***	1.1713***	1.2416***	1.1677***	1.2704***	1.1767***	1.1665***	1.2368***
	[16.2397]	[12.7028]	[8.1893]	[10.8873]	[12.1765]	[11.3403]	[12.5016]	[11.2056]	[11.3075]	[12.3132]
$\overline{\Delta IVol} \times \Delta IVol \times RO$	-10.149***	-7.0635***	-2.5813	10.4454***	5.7354*	15.9548***	-1.0437	18.5515***	13.4981***	11.4536***
	[-6.4869]	[-5.3356]	[-0.5099]	[2.6592]	[1.6515]	[4.5518]	[-0.2828]	[5.1675]	[3.1356]	[2.6497]
RSQR	0.1176***	0.1164***	0.1166***	0.1180***	0.1170***	0.1175***	0.1169***	0.1173***	0.1191***	0.1173***

	RO Proxy					*RO* Industry			
Coeff.	small high profit	small highs sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Option Industries
Intercept	0.0629***	0.0623***	0.0636***	0.0628***	0.0634***	0.0627***	0.0627***	0.0627***	0.0631***
	[13.0392]	[12.9103]	[13.0859]	[12.9952]	[13.0304]	[12.9122]	[12.8832]	[13.0103]	[13.0997]
$Log(BM)$	0.0040***	0.0040***	0.0044***	0.0039***	0.0044***	0.0044***	0.0044***	0.0044***	0.0044***
	[5.2952]	[5.3880]	[6.0411]	[5.3075]	[5.9657]	[6.0308]	[6.0725]	[6.1201]	[6.0726]
$Log(ME)$	-0.0059***	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0058***	-0.0059***
	[-12.312]	[-12.215]	[-11.882]	[-12.270]	[-11.959]	[-11.903]	[-11.811]	[-11.871]	[-11.952]
β^{CAPM}	0.0017**	0.0017**	0.0017**	0.0017**	0.0018**	0.0018***	0.0018**	0.0016**	0.0017**
	[2.1808]	[2.3133]	[2.4464]	[2.1928]	[2.5693]	[2.6381]	[2.5412]	[2.3988]	[2.4645]
$Lag(r)$	-0.0038	-0.0037	-0.0041	-0.0039	-0.0041	-0.0042	-0.0042	-0.0041	-0.0041
	[-1.3929]	[-1.3437]	[-1.5979]	[-1.4052]	[-1.5618]	[-1.6174]	[-1.5995]	[-1.6019]	[-1.5898]
$trade$	0.0157***	0.0157***	0.0158***	0.0157***	0.0158***	0.0158***	0.0157***	0.0158***	0.0158***
	[14.1190]	[14.1104]	[14.8476]	[14.1089]	[14.8011]	[14.8076]	[14.8051]	[14.9660]	[14.8848]
$\overline{\Delta IVol}$	-0.9580***	-0.9432***	-1.0027***	-0.9531***	-1.0019***	-0.9704***	-0.9810***	-0.9709***	-0.9873***
	[-8.9675]	[-8.7742]	[-9.9681]	[-8.9661]	[-9.9511]	[-9.6355]	[-9.7776]	[-9.5971]	[-9.6676]
$\Delta IVol$	1.2115***	1.1590***	1.2214***	1.1849***	1.2578***	1.2651***	1.2737***	1.2776***	1.1733***
	[11.9304]	[11.2169]	[12.0872]	[11.3227]	[12.4545]	[12.3600]	[12.7952]	[12.8883]	[11.9514]
$\overline{\Delta IVol} IVol \times RO$	14.6836***	24.4176***	5.9097	22.5754***	3.2036	19.3544***	5.77	8.6091	13.0368***
	[3.3967]	[6.0754]	[1.3540]	[5.6070]	[0.7889]	[3.3040]	[1.4192]	[1.2756]	[4.2008]
RSQR	0.1178***	0.1180***	0.1176***	0.1179***	0.1173***	0.1158***	0.1168***	0.1160***	0.1167***

Table 9: The table reports Fama and French (1993) alphas along with robust Newey and West (1987) t-statistics in square brackets for the portfolios of stocks sorted by idiosyncratic return volatility $IVol$ and real option measures. $IVol$ terciles are reported across the columns. Real option classifications are reported down the rows. The column labeled '3-1' refers to FF-3 alphas for the portfolios that are long and short the top and bottom tercile $IVol$ portfolios respectively within each classification of real option. Idiosyncratic return volatility is computed relative to FF-3. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted by idiosyncratic return volatility relative to the FF-3 model. Then, independently, at the end of each June, we sort stocks into three tercile on the basis of the real option characteristics age, size (total assets) and size (mkt equity), and into high/low vega, high/low future profit growth, high/low future sales growth, young/old, high/low future investment rate based on 33th/66th NYSE breakpoints. The row labeled 'Mean' corresponds to the FF-3 alpha for the portfolio equally-weighted in the real option portfolios within each $IVol$ classification. All portfolios of stocks are value-weighted and rebalanced monthly.

		Ivol Rank			
RO Proxy	*RO* Proxy Rank	1	2	3	3-1
size (total assets)	1	4.0975**	4.0300**	-6.5365***	-10.6340***
		[2.4911]	[1.9908]	[-2.8647]	[-4.3472]
	2	4.1199***	2.5739*	-5.4489***	-9.5688***
		[4.0848]	[1.8638]	[-2.7191]	[-4.1640]
	3	1.1176**	1.0809	-3.3977	-4.5153
		[1.9880]	[0.7594]	[-1.0931]	[-1.4095]
	Mean	3.1117***	2.5616**	-5.1277**	-8.2394***
		[3.9365]	[2.1641]	[-2.5398]	[-3.7403]
size (mkt equity)	1	3.4891**	7.1679***	-1.4118	-4.9009**
		[2.1164]	[4.2677]	[-0.6068]	[-2.0704]
	2	2.0518**	2.4464***	-5.7961***	-7.8479***
		[2.2206]	[2.6878]	[-3.0515]	[-3.3100]
	3	1.1703**	1.1891	-4.3537	-5.5241*
		[2.0816]	[0.8592]	[-1.5122]	[-1.8321]
	Mean	2.2371***	3.6012***	-3.8539**	-6.0910***
		[2.7423]	[3.7955]	[-1.9974]	[-2.7314]
age	1	0.7141	1.4974	-5.4860**	-6.2001**
		[0.5620]	[0.8703]	[-2.0730]	[-2.2095]
	2	0.0977	2.2573	-6.8932***	-7.0886***
		[0.0690]	[1.1694]	[-2.9674]	[-3.0333]
	3	0.2338	1.0879	-1.3004	-0.999
		[0.1729]	[0.7286]	[-0.4450]	[-0.3091]
	Mean	0.4196	1.6689	-4.5701**	-4.9897**
		[0.5937]	[1.3031]	[-2.1691]	[-2.2072]
high vega	0	1.4045**	2.0348*	-2.361	-3.7655*
		[2.2822]	[1.7913]	[-1.1747]	[-1.6816]
	1	-1.1113	-2.3174	-10.4032***	-9.2920***
		[-0.7644]	[-1.2174]	[-3.6044]	[-2.9722]
	Mean	0.1466	-0.1413	-6.3821***	-6.5287***
		[0.1741]	[-0.1068]	[-2.9884]	[-2.7877]
high profit	0	0.9203	-0.3679	-3.9303	-4.8505*
		[1.3045]	[-0.3151]	[-1.6076]	[-1.8132]
	1	2.1091*	2.2445	-2.0649	-4.1739
		[1.8580]	[1.1791]	[-0.7984]	[-1.5930]
	Mean	1.5147**	0.9383	-2.9976	-4.5122*
		[2.3362]	[0.7270]	[-1.2911]	[-1.8501]
high sale	0	0.5376	-2.7208**	-8.0188***	-8.5564***
		[0.7579]	[-2.2648]	[-3.3133]	[-3.3929]
	1	4.8059***	6.1847***	2.9237	-1.8821
		[3.9921]	[3.2661]	[1.0957]	[-0.6379]
	Mean	2.6718***	1.732	-2.5475	-5.2193**
		[3.9133]	[1.3646]	[-1.0939]	[-2.0502]
high inv	0	-1.6825**	-5.2831***	-11.3864***	-9.7039***
		[-2.3719]	[-3.9863]	[-4.4539]	[-3.5168]
	1	8.6144***	10.8817***	8.8514***	0.237
		[9.5657]	[6.3204]	[3.5488]	[0.0895]
	Mean	3.4659***	2.7993**	-1.2675	-4.7334**
		[5.8032]	[2.2492]	[-0.5834]	[-2.0071]

Table 10: The table reports Fama and French (1993) alphas along with robust Newey and West (1987) t-statistics in square brackets for the portfolios of stocks sorted by idiosyncratic return volatility $IVol$ and real option measures. $IVol$ terciles are reported across the columns. Real option classifications are reported down the rows. The column labeled '3-1' refers to FF-3 alphas for the portfolios that are long and short the top and bottom tercile $IVol$ portfolios respectively within each classification of real option. Idiosyncratic return volatility is computed relative to FF-3. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted by idiosyncratic return volatility relative to the FF-3 model. Then, independently, at the end of each June, we sort stocks into three tercile on the basis of the real option characteristics age, size (total assets) and size (mkt equity), and into high/low vega, high/low future profit growth, high/low future sales growth, young/old, high/low future investment rate based on 33th/66th NYSE breakpoints. The row labeled 'Mean' corresponds to the FF-3 alpha for the portfolio equally-weighted in the real option portfolios within each $IVol$ classification. All portfolios of stocks are value-weighted and rebalanced monthly.

		Ivol Rank			
RO Industry	*RO* Industry dummy	1	2	3	3-1
young	0	1.3132	1.1363	-3.6190*	-4.9322**
		[1.2116]	[0.6678]	[-1.7697]	[-2.1477]
	1	0.4351	-0.1468	-7.3830***	-7.8181***
		[0.3662]	[-0.0634]	[-2.7331]	[-2.9091]
	Mean	0.8742	0.4948	-5.5010***	-6.3752***
		[1.0866]	[0.3128]	[-2.6674]	[-2.8934]
small high vega	0	1.3118**	1.443	-4.1226**	-5.4345**
		[2.2125]	[1.2015]	[-2.0369]	[-2.4447]
	1	7.2256***	-1.3516	-9.4742***	-16.6999***
		[3.1348]	[-0.5017]	[-2.7450]	[-5.1945]
	Mean	4.2687***	0.0457	-6.7984***	-11.0672***
		[3.5833]	[0.0289]	[-3.0578]	[-5.0682]
small growth	0	1.3399**	1.1866	-3.5023	-4.8422**
		[2.3010]	[0.9753]	[-1.5847]	[-2.0239]
	1	2.622	-1.3573	-17.2875***	-19.9095***
		[1.2730]	[-0.4825]	[-5.9209]	[-7.4748]
	Mean	1.9809*	-0.0854	-10.3949***	-12.3758***
		[1.8978]	[-0.0533]	[-4.8105]	[-5.8970]
small high profit	0	1.1321*	0.9205	-4.8299**	-5.9620***
		[1.9005]	[0.7740]	[-2.2867]	[-2.6013]
	1	6.6400***	5.1844	-5.9994	-12.6394***
		[3.2080]	[1.5296]	[-1.4277]	[-3.1488]
	Mean	3.8861***	3.0525	-5.4146*	-9.3007***
		[3.5113]	[1.6193]	[-1.9624]	[-3.4349]
small high sale	0	1.1484*	0.8917	-4.9013**	-6.0497**
		[1.8996]	[0.7600]	[-2.2452]	[-2.5556]
	1	6.9615***	7.2097***	-4.2678	-11.2293***
		[3.5611]	[2.6765]	[-1.1800]	[-3.1720]
	Mean	4.0549***	4.0507***	-4.5846*	-8.6395***
		[3.9697]	[2.6368]	[-1.9041]	[-3.5687]
small young	0	1.3810**	1.2084	-3.6206*	-5.0016**
		[2.3321]	[1.0050]	[-1.7557]	[-2.2157]
	1	3.2628	-0.4925	-12.5558***	-15.8186***
		[1.4685]	[-0.2145]	[-3.7289]	[-5.5878]
	Mean	2.3219**	0.358	-8.0882***	-10.4101***
		[2.0742]	[0.2500]	[-3.4986]	[-5.0069]
small high inv	0	1.0777*	0.8581	-5.3113**	-6.3890***
		[1.7810]	[0.7173]	[-2.4564]	[-2.7210]
	1	13.4407***	15.6392***	-0.1405	-13.5812***
		[6.5670]	[4.2039]	[-0.0450]	[-4.3761]
	Mean	7.2592***	8.2486***	-2.7259	-9.9851***
		[6.8613]	[3.9692]	[-1.2189]	[-4.5802]
young high vega	0	1.2383**	1.8331	-3.2394	-4.4778*
		[2.1660]	[1.4618]	[-1.5166]	[-1.9169]
	1	-2.2457	-1.7817	-11.4735***	-9.2278***
		[-1.0604]	[-0.6608]	[-3.4854]	[-2.6007]
	Mean	-0.5037	0.0257	-7.3565***	-6.8528***
		[-0.4390]	[0.0147]	[-3.2038]	[-2.8329]

Table 11: The table reports Fama and French (1993) alphas along with robust Newey and West (1987) t-statistics in square brackets of the portfolios of stocks sorted on idiosyncratic return volatility $IVol$ and real option proxies. $IVol$ Ranks are reported across the columns and the ranks of the real option proxies are reported down the rows. The column labeled '3-1' refers to FF-3 alphas of the portfolios that are long and short the top and bottom $IVol$ portfolios respectively within each rank of the real option proxy. Idiosyncratic return volatility is computed relative to FF-3. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted by idiosyncratic volatility relative to the FF-3 model. Then, independently, at the end of each June, we sort stocks into whether they belonged to the Natural Resources, High Technology, and Bio Technology industries, and whether they belonged to any one of the three industries (All G.O. Industries) on the basis of Fama and French (1997) industry classifications. The row labeled 'Mean' corresponds to the FF-3 alpha for the portfolio equally weighted in the industry sorted portfolios within each $IVol$ rank group. All portfolios are value weighted.

	Zero-Cost $IVol$ Portfolio Intercepts					
RO Proxy Rank	size (total assets)	age	small young	small high inv.	small high vega	All Growth Industries
1	-10.6340***	-6.2001**				
	[-4.3472]	[-2.2095]				
2 (0)	-9.5688***	-7.0886***	-4.8422**	-6.3890***	-5.4345**	-5.4061***
	[-4.1640]	[-3.0333]	[-2.0239]	[-2.7210]	[-2.4447]	[-2.9169]
3 (1)	-4.5153	-0.999	-19.9095***	-13.5812***	-16.6999***	-5.8266*
	[-1.4095]	[-0.3091]	[-7.4748]	[-4.3761]	[-5.1945]	[-1.8956]
Mean	-8.2394***	-4.9897**	-12.3758***	-9.9851***	-11.0672***	-5.6163***
	[-3.7403]	[-2.2072]	[-5.8970]	[-4.5802]	[-5.0682]	[-2.7146]

Table 12: The table reports Fama and French (1993) alphas, along with robust Newey and West (1987) t-statistics in square brackets, of the portfolios that are long and short the top and bottom tercile $IVol$ portfolios respectively within each classification of real option and difference in $IVol$ values between the high and low regimes, $\overline{\Delta IVol}$. The columns labeled 'Mean(1 to 3)' and 'Mean(0 to 1)' correspond to the FF-3 alpha for the equally-weighted portfolio in the real option sorted portfolios within each classification of $\overline{\Delta IVol}$. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value-weighted. We use daily data over the previous month and rebalance monthly. We sort stocks by idiosyncratic volatility relative to the FF-3 model into three portfolios. At the end of each June, we independently sort stocks into three terciles on the basis of their real option characteristics age, size (total assets) and size (mkt equity), and into high/low vega, high/low future profit growth, high/low future sales growth, young/old, high/low future investment rate based on 33th/66th NYSE breakpoints. Independent sorts of $\overline{\Delta IVol}$ are done once using in-sample values of $IVol$ for each stock and then grouping stocks on the basis of the 33th/66th NYSE $\overline{\Delta IVol}$ breakpoint values.

		RO Proxy Rank			
RO Proxy	$\overline{\Delta Ivol}$ Rank	1	2	3	Mean(1 to 3)
size (total assets)	1	1.2953	1.9408	-1.7919	0.4814
		[0.7783]	[0.9964]	[-0.8756]	[0.3398]
	2	-1.1459	-5.6068**	-5.9637**	-4.2388**
		[-0.3936]	[-2.1743]	[-2.0455]	[-2.0982]
	3	-9.1933**	-16.1224***	-14.8518***	-13.3892***
		[-2.5293]	[-5.0924]	[-3.6702]	[-4.9292]
size (mkt equity)	1	-0.1546	0.8774	-0.7373	-0.0048
		[-0.1097]	[0.6937]	[-0.3732]	[-0.0040]
	2	0.6286	0.2541	-6.2849**	-1.8008
		[0.2874]	[0.1028]	[-2.2975]	[-1.0100]
	3	-6.3840**	-12.7608***	-19.9126***	-13.0192***
		[-2.0049]	[-4.3610]	[-5.4313]	[-5.0374]
age	1	1.1583	-2.4428	-3.479	-0.2096
		[0.4656]	[-0.8672]	[-1.6132]	[-0.1219]
	2	0.5376	-9.0395***	-3.8205	-3.8516*
		[0.1828]	[-3.0131]	[-1.1623]	[-1.9194]
	3	-13.2842***	-9.7927**	-10.4336**	-11.6222***
		[-3.2330]	[-2.5055]	[-2.3812]	[-3.8341]

		RO Proxy Rank			
RO Proxy	$\overline{\Delta Ivol}$ Rank	0	1		Mean(0 to 1)
high vega	1	0.147	-8.1421		-3.2983
		[0.0618]	[-1.3157]		[-0.9137]
	2	-3.505	-5.6724		-4.5887*
		[-1.4618]	[-1.4065]		[-1.9547]
	3	-6.8298*	-5.0545		-5.9422*
		[-1.9367]	[-1.2537]		[-1.8772]
high profit	1	-0.943	-7.1614		-3.9736
		[-0.3316]	[-1.5875]		[-1.3484]
	2	-1.9006	1.2129		-0.3438
		[-0.6468]	[0.3879]		[-0.1404]
	3	-12.9659***	1.8703		-5.5478
		[-3.1592]	[0.4470]		[-1.6406]
high sale	1	-5.8249**	3.3435		-1.2823
		[-1.9994]	[0.6184]		[-0.3764]
	2	-1.9143	-1.607		-1.7606
		[-0.6081]	[-0.5251]		[-0.7201]
	3	-12.3146***	-2.904		-7.6093**
		[-2.7325]	[-0.7771]		[-2.2865]
high inv	1	-6.8576**	2.6811		-2.2096
		[-2.0994]	[0.6673]		[-0.7627]
	2	-4.5404	0.7809		-1.8797
		[-1.5994]	[0.2560]		[-0.8361]
	3	-9.8621**	-4.5532		-7.2076**
		[-2.3589]	[-1.1472]		[-2.2730]

Table 13: The table reports Fama and French (1993) alphas, along with robust Newey and West (1987) t-statistics in square brackets, of the portfolios that are long and short the top and bottom $IVol$ portfolios respectively within each rank of the real option proxy and difference in $IVol$ values between the high and low regimes, $\overline{\Delta IVol}$. The column labeled 'Mean(0 to 1)' corresponds to the FF-3 alpha for the portfolio equally weighted in the real option proxy sorted portfolios within each $\overline{\Delta IVol}$ rank group. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value weighted. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted on idiosyncratic volatility relative to the FF-3 model. At the end of each June, we independently sort stocks on the basis of the real option characteristics age and size (total assets), and into high/low vega, high/low future profit growth, high/low future sales growth, young/old, high/low future investment rate based on 33th/66th NYSE breakpoints. Independent sorts of $\overline{\Delta IVol}$ are done once using in-sample values of $IVol$ for each stock and then grouping stocks on the basis of the 33th/66th NYSE $\overline{\Delta IVol}$ breakpoint values.

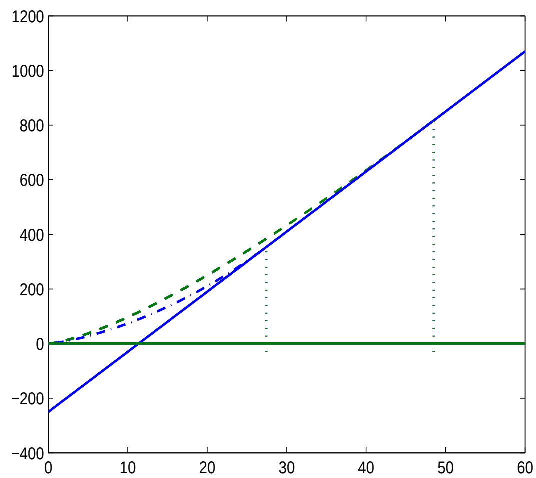
		RO Proxy Rank		
RO Proxy	$\overline{\Delta Ivol}$ Rank	0	1	Mean(0 to 1)
young	1	-2.6092	1.1289	-1.1093
		[-0.9174]	[0.2094]	[-0.3338]
	2	-5.2328**	-1.3772	-3.3045
		[-1.9887]	[-0.4292]	[-1.4599]
	3	-8.4579**	-8.0120**	-8.6963***
		[-2.3154]	[-2.2161]	[-3.2494]
small high vega	1	-2.4541	-27.9701*	-9.3070**
		[-1.0159]	[-1.8613]	[-2.5311]
	2	-2.187	-25.0583***	-13.5731***
		[-0.9645]	[-3.7224]	[-3.7903]
	3	-9.4226***	-12.0750***	-10.7488***
		[-2.7477]	[-3.3645]	[-3.7550]
small growth	1	-1.2082	4.5421	-0.8958
		[-0.4787]	[0.5815]	[-0.2379]
	2	-3.1672	-11.2681***	-7.2177***
		[-1.2881]	[-2.6043]	[-2.8752]
	3	-7.5165**	-16.2439***	-11.8802***
		[-1.9739]	[-5.2359]	[-4.4760]
small high profit	1	-3.5166	-4.1494	-11.5094***
		[-1.3669]	[-0.4946]	[-3.2741]
	2	-2.8234	-7.9637	-5.3935*
		[-1.1582]	[-1.3867]	[-1.7095]
	3	-6.7369*	-6.2536	-6.4952*
		[-1.7856]	[-1.4220]	[-1.9543]
small high sale	1	-4.1531	-5.0725	-9.9128***
		[-1.5812]	[-0.6850]	[-2.8132]
	2	-2.8776	-5.5105	-4.194
		[-1.1796]	[-1.0362]	[-1.3573]
	3	-7.0473*	-7.3758*	-7.2115**
		[-1.8367]	[-1.8315]	[-2.4433]
small young	1	-1.8499	-0.1049	-5.7142
		[-0.7736]	[-0.0137]	[-1.4315]
	2	-2.7867	-8.3822*	-5.5845**
		[-1.1933]	[-1.7600]	[-2.0902]
	3	-8.4557**	-10.3204***	-9.3881***
		[-2.2478]	[-2.8822]	[-3.3053]
small high inv	1	-3.6428	-0.432	-10.0648***
		[-1.3841]	[-0.0552]	[-2.9128]
	2	-2.9884	-8.5901*	-5.7893**
		[-1.2031]	[-1.7891]	[-2.0616]
	3	-7.0524*	-9.6920**	-8.3722***
		[-1.8104]	[-2.5248]	[-2.9166]
young high vega	1	-0.8212	-12.0280*	-5.0852
		[-0.3315]	[-1.7938]	[-1.3339]
	2	-3.0975	6.1775	1.54
		[-1.2775]	[1.1195]	[0.5106]
	3	-8.8177***	0.4351	-4.1913
		[-2.5898]	[0.0944]	[-1.3614]

Table 14: The table reports Fama and French (1993) alphas, along with robust Newey and West (1987) t-statistics in square brackets, of the portfolios that are long and short the top and bottom $IVol$ portfolios respectively within each classification of the real option industries and difference in $IVol$ values between the high and low regimes, $\overline{\Delta IVol}$. Industry classifications are reported across columns and $\overline{\Delta IVol}$ classifications are reported down the rows. The column labeled 'Mean(0 to 1)' corresponds to the FF-3 alpha for the portfolio equally-weighted in both industry portfolios within each $\overline{\Delta IVol}$ classification. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value-weighted. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted on idiosyncratic volatility relative to the FF-3 model. At the end of each June, we independently sort stocks into whether they belonged to the Natural Resources, High Technology, and Bio Technology industries, and whether they belonged to any one of the three industries (All G.O. Industries) on the basis of Fama and French (1997) industry classifications. Independent sorts of $\overline{\Delta IVol}$ are done once using in-sample values of $IVol$ for each stock and then grouping stocks on the basis of the 33th/66th NYSE $\overline{\Delta IVol}$ breakpoint values.

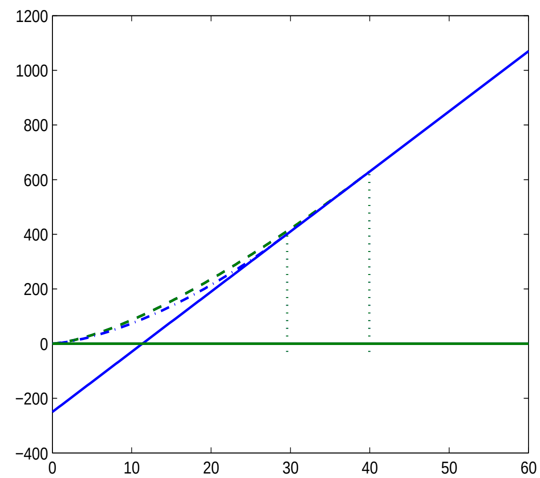
		RO Proxy Rank		
RO Proxy	$\overline{\Delta Ivol}$ Rank	0	1	Mean(0 to 1)
Natural Resources	1	-1.6231	6.8209	3.1442
		[-0.5745]	[1.3028]	[0.9702]
	2	-1.7178	-2.7294	-2.2236
		[-0.6962]	[-0.6421]	[-0.9270]
	3	-7.0048**	-11.2290**	-8.9211***
		[-2.1141]	[-2.5374]	[-3.5191]
High Tech	1	0.3569	0.7912	0.5179
		[0.1429]	[0.1257]	[0.1458]
	2	-0.8473	-8.2843**	-4.5658*
		[-0.3146]	[-2.2199]	[-1.9168]
	3	-10.9527***	-8.6281*	-9.7904***
		[-3.1950]	[-1.6827]	[-3.2519]
Bio Tech	1	-0.8241	7.5278	4.0748
		[-0.2945]	[1.2412]	[1.2224]
	2	-3.0634	-4.2983	-3.6546
		[-1.2751]	[-1.0584]	[-1.4655]
	3	-7.7035**	-12.9701**	-9.6440***
		[-2.1873]	[-2.2989]	[-2.7023]
All G.O. Industries	1	-0.8772	6.3261	2.709
		[-0.4017]	[1.0666]	[0.8223]
	2	-0.7051	-6.5709**	-3.6380*
		[-0.3291]	[-2.1681]	[-1.7147]
	3	-12.3964***	-13.3725***	-12.8844***
		[-4.1275]	[-3.3579]	[-4.8065]

Figure 1: Model Results. The figure shows the option values, and the option exercise policies P_1 and P_2 in P space for the low and the high volatility regimes. The 45 degree solid line corresponds to the intrinsic value of the growth option. Option values in the high and low volatility states are depicted by dashed and dashed dotted curves respectively. The exercise thresholds are depicted by the vertical dotted lines where the lower threshold corresponds to the exercise threshold P_1 if the option is the low volatility regime, and the higher threshold corresponds to the exercise threshold P_2 if the option is the high volatility regime. Panel (a) depicts the model solution corresponding to parameters $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$, panel (b) depicts the model solution corresponding to parameters $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$, and panel (c) depicts the model solution corresponding to parameters $\sigma_{P,H} = 0.3, \sigma_{P,L} = 0.3$.

(a) $\sigma_H = 0.5$, $\sigma_L = 0.1$



(b) $\sigma_H = 0.4$, $\sigma_L = 0.2$



(c) $\sigma_H = 0.3$, $\sigma_L = 0.3$

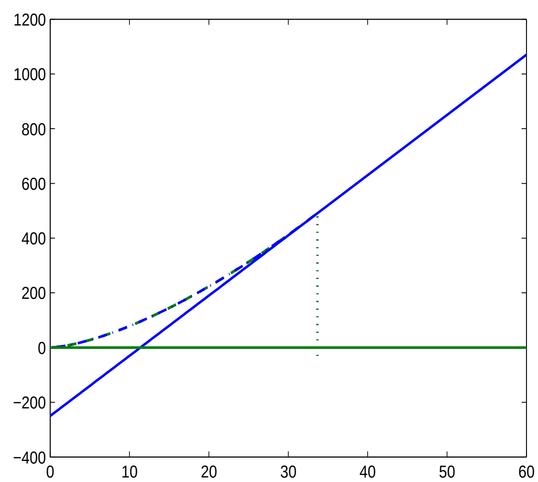
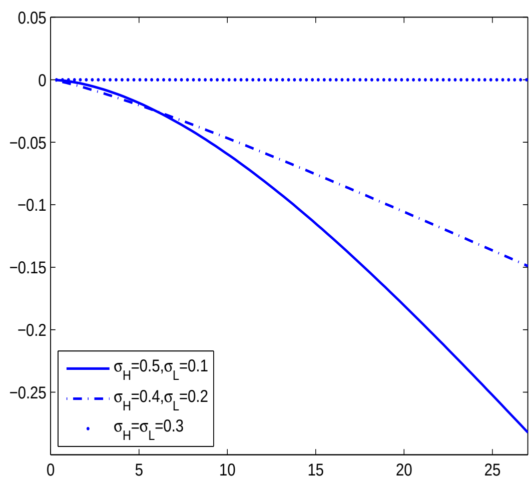
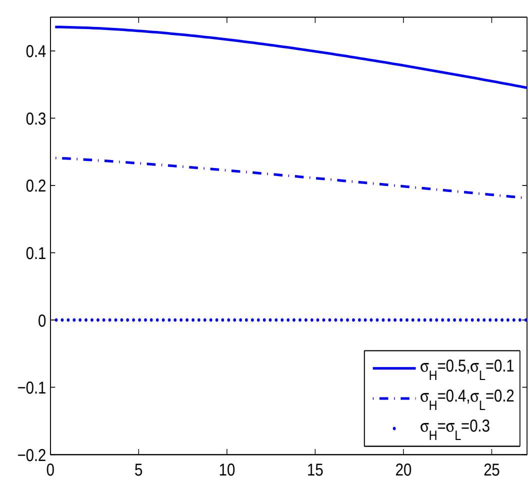


Figure 2: Model Solution. The figure shows the differences in growth option beta, the continuous drift, the jump, and the diffusion terms of the jump-diffusion process (3.7) between the high and low volatility regimes in P space based on the model developed in Section 3 of the paper. Panel (a) shows the differences in option betas, panel (b) shows the differences in the diffusion term, panel (c) shows the differences in the continual drift term, and panel (d) shows the differences in the sporadic jump term. The figure shows separate results corresponding to different set of model parameters, i.e. $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$, $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$ and $\sigma_{P,H} = \sigma_{P,L} = 0.3$.

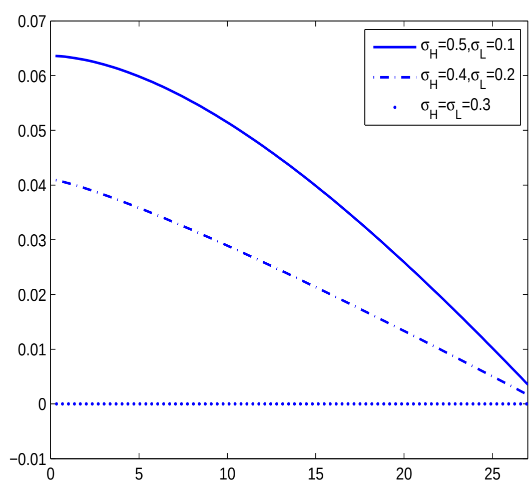
(a) $\beta_H(P) - \beta_L(P)$



(b) $b_H(P) - b_L(P)$



(c) $a_H(P) - a_L(P)$



(d) $\nu_H(P) - \nu_L(P)$

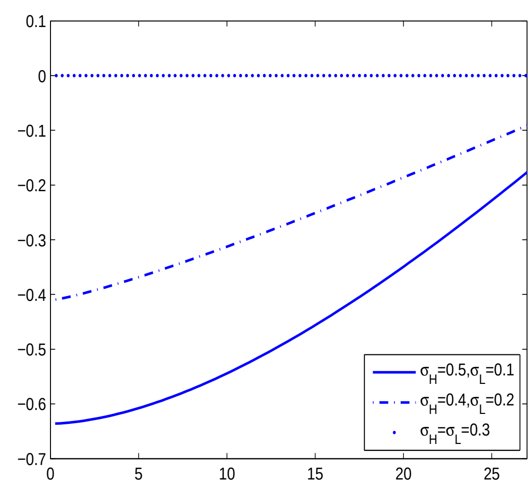


Figure 3: Simulation Results. The figure shows a sample path of simulated variables based on the model developed in Section 3 of the paper for $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$. Panel (a) shows a sample path of the output price P at the end of each month, panel (b) shows the corresponding equity value $V(P)$, panel (c) shows P's month-end idiosyncratic volatility regimes where $i = 1$ represents the high volatility regime and $i = 0$ represents the low volatility regime, panel (d) shows month-end realized idiosyncratic return volatility, and panel (e) shows month-end realized risk-adjusted returns.

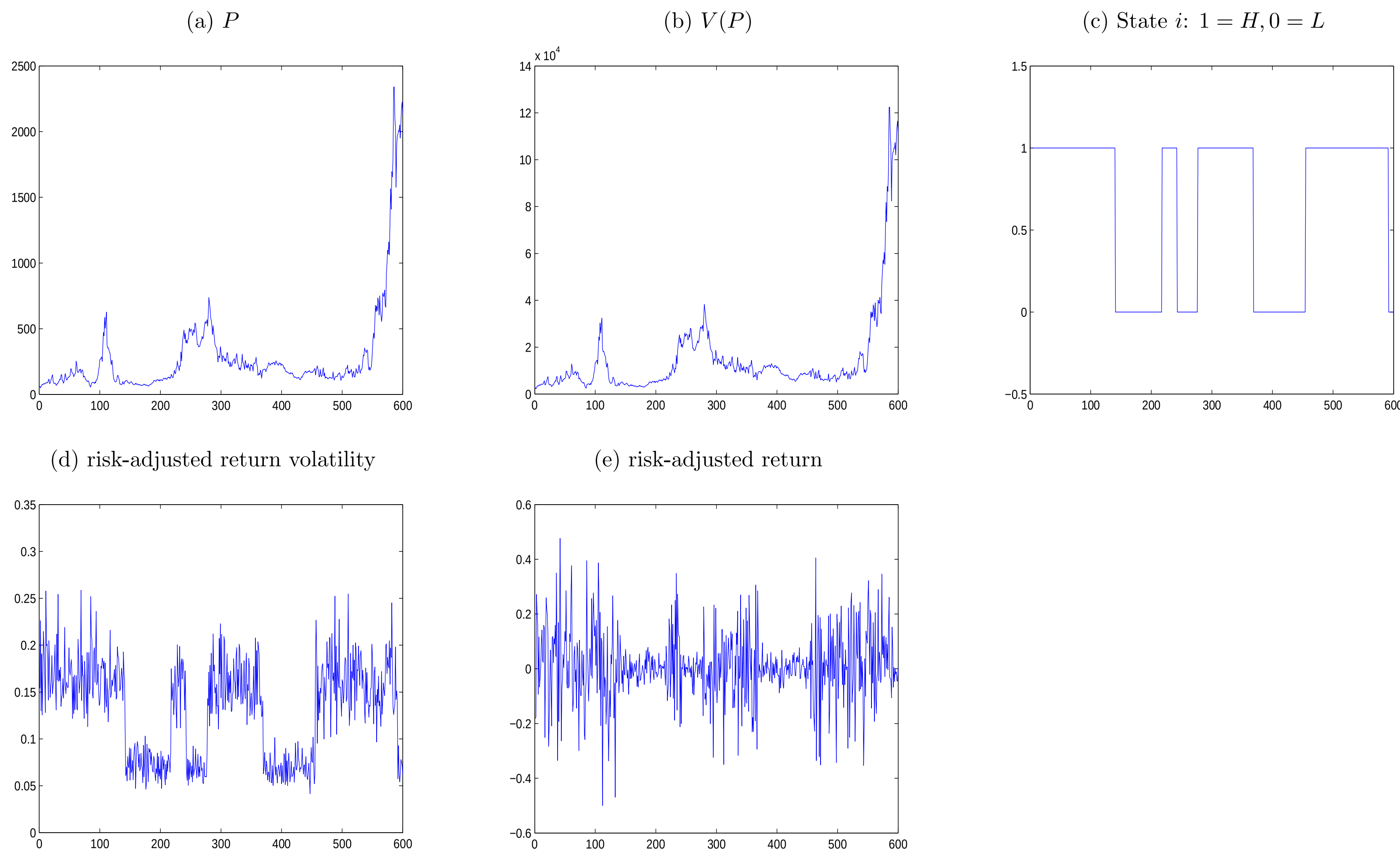


Figure 4: Simulation Results. The figure shows the average risk-adjusted value-weighted portfolio returns of the ten decile portfolios formed after sorting equity returns by $\sigma_{P,i}$ using the simulated data based on the model developed in Section 3 of the paper. Risk-adjusted equity returns are sorted into decile portfolios based on the idiosyncratic volatility regime at the end of the previous month. Then, the average of the value-weighted one-month holding period portfolio returns are computed using the monthly risk-adjusted returns on the firms' equity. The portfolios are rebalanced at the end of each month. The figure shows separate portfolio returns corresponding to the simulated samples where $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$, $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$ and $\sigma_{P,H} = \sigma_{P,L} = 0.3$.